DYNAMICS OF GRAVITATING SYSTEMS OF COLLIDING PARTICLES IN PLANETARY DISCS

André Brahic
Observatoire de Paris, 92190 Meudon et
Université Paris VII, 75005 Paris
France

1. INTRODUCTION

During this symposium on the dynamics of the solar system, we have mainly studied the movements of the bodies of the solar system submitted to gravitational perturbations. The next step is to take into account the physical collisions. Indeed, there can be little doubt that collisions between "macroscopic bodies" are of frequent occurence in the Universe. All kinds of quite different objects undergo such collisions: these may range from large interstellar clouds to small solid bodies in the solar system. Collisions have surely played an important role in the formation of planets and satellites and continue to play a central role in the behaviour of the planetary discs. For example for Saturn's rings, one can see intuitively that until the optical depth drops much below unity, the rings are still evolving. Each orbiting particle can be taken as occupying a kind of torus, and collisions will continue until there is only one particle in each such "orbital tube"; this corresponds to a very small optical depth.

Since the time of Poincaré (1911), it is known that inelastic collisions tend to flatten any system; inelastic collisions tend to damp out the motions perpendicular to the plane of the disc as well as the radial motions, so that the orbits become more and more circular and coplanar. With the help of Hénon, I am studying systematically three-dimensional gravitating systems of colliding particles by numerical simulation and analytically. This work finds an immediate application to the dynamics of planetary discs for example. These simulations lend particularly well to astrophysical problems in which the mean free path of a given particle is of the order of or larger than the dimensions of the system.

In order to understand first the basic mechanics of the process, I have considered the simplest models in which attraction between particles has been neglected (and so particles orbits are keplerian around a central mass point), and in which particles have the same masses and radii. In a collision, the grazing component of velocity is conserved and the perpendicular component is multiplied by a coefficient k which lies between 0 and −1. The evolution of these first models have been

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already published (Brahic, 1977a) and can be briefly summarized in the following way: after a very fast flattening of the order of twenty collisions per particle, the system reaches a quasi-equilibrium state in which the thickness of the newly formed disc is finite and in which collisions still occur. Under the combined effect of differential rotation and inelastic collisions, the disc spreads very slowly, particles move both inwards and outwards carrying out some angular momentum. For Saturn's rings, the time scale of flattening is of the order of a few weeks, the time scale of the quasi-equilibrium state is of the order of 10^{14} years and the system reaches a third hypothetical collisionless state in a time scale of the order of 10^{21} years. This result disagrees strongly with the time scale obtained by Jeffreys (1947). His paper contains an erroneous calculation as noted by Hénon (1975).

Now, I have considered different collision laws, particles of different masses and radii,..... I have no place here to give all the results which will be published in the near future. During the few minutes of this communication, I shall give you just the results concerning the existence of a new transition in the (k,k') diagram and the results concerning the mutual evolution of two different types of particles under various conditions.

2. A NEW TRANSITION IN THE (k,k') DIAGRAM

If the grazing component of the relative velocity of two colliding particles is reduced after each collision by a factor k' which lies between 0 and +1, an instability phenomenon appears. For very inelastic collisions and contrary to the behaviour of the first models (Figure 1, zone B), the disc is completely flattened and the system become a two-dimensional configuration in which collisions continue to occur (Figure 1, zone A). A sharp transition L_1 separates zones A and B. A linear stability analysis of the observed behaviour has been made: differential rotation constantly feeds energy into horizontal motions and thus maintains a finite dispersion of horizontal velocities, whilst vertical motions are only a by-product of the horizontal activity and do not necessarily arise (Brahic, 1977b).

There exists a second sharp transition L_2 between zone B and zone C (Figure 1). In zone B, the thickness of the disc decreases and reaches a value of the order of r/R, where r is the radius of a particle and R some characteristic dimension of the system. In zone C, the thickness of the disc increases. On the one hand, keplerian motion tends to establish an anisotropic distribution of velocities with a velocity two times larger in the radial direction than in the transverse direction. On the other hand, collisions tend to establish an isotropic distribution of velocities. If the degree of inelasticity is small (Figure 1, zone C), a large quantity of energy is extracted from the ordered motion and expended in the collisions. If the degree of inelasticity is larger (Figure 1, zones A and B), a smaller quantity of energy is extracted from the ordered motion and the orbits become more and more circular and coplanar.

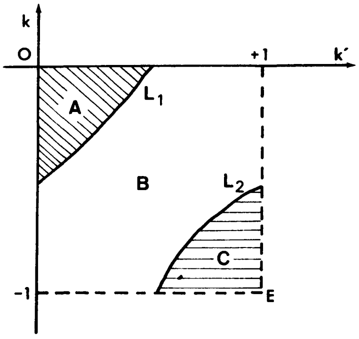

Figure 1. The flattening of the system is a function of the rebound coefficients k and k'. In zone A, the system is completely flattened. In zone B, the thickness of the disc decreases towards a value of the order of a few times the size of a particle. In zone C, the thickness of the disc increases. There exist two sharp limits L_1 and L_2 between the three zones A, B and C.

3. PARTICLES OF DIFFERENT MASSES AND RADII

In a real system, most collisions would be between particles of very unequal size. I shall give here just the results concerning the mutual evolution of two different types of particles. Generally speaking one can say that the two groups of particles evolve in essentially the same way. There is no real equipartition of energy, but rather a kind of separation. The mean inclination and the mean eccentricity of the large particles is smaller than the mean inclination and the mean eccentricity of the small particles (Figure 2). The evolution is essentially similar to that of the first models. The more elastic the collisions, the bigger the separation; the larger the mass ratio, the bigger the separation; there would not seem to be any separation in the plane of the flattening.

Generally speaking, the massive particles evolve faster than the light ones. The relation ($a\ r^2 < 3.10^{10}$) between the size r of the particles and the age a of the system (resp. in meters and years), proved for systems of identical particles (Brahic, 1977a), remains valid. To the extend that Saturn's rings may be modelled in this way, then if the rings are as old as the solar system, the maximum size of the particles is 2.5 meters. A system, which to day is essentially made of large particles, must presumably have been created much later.

4. CONCLUSIONS

The rings of Uranus could be made of very small colliding particles. If the above relation is applied to Uranus'rings, rings having the age of the solar system would be composed of particles whose size is less

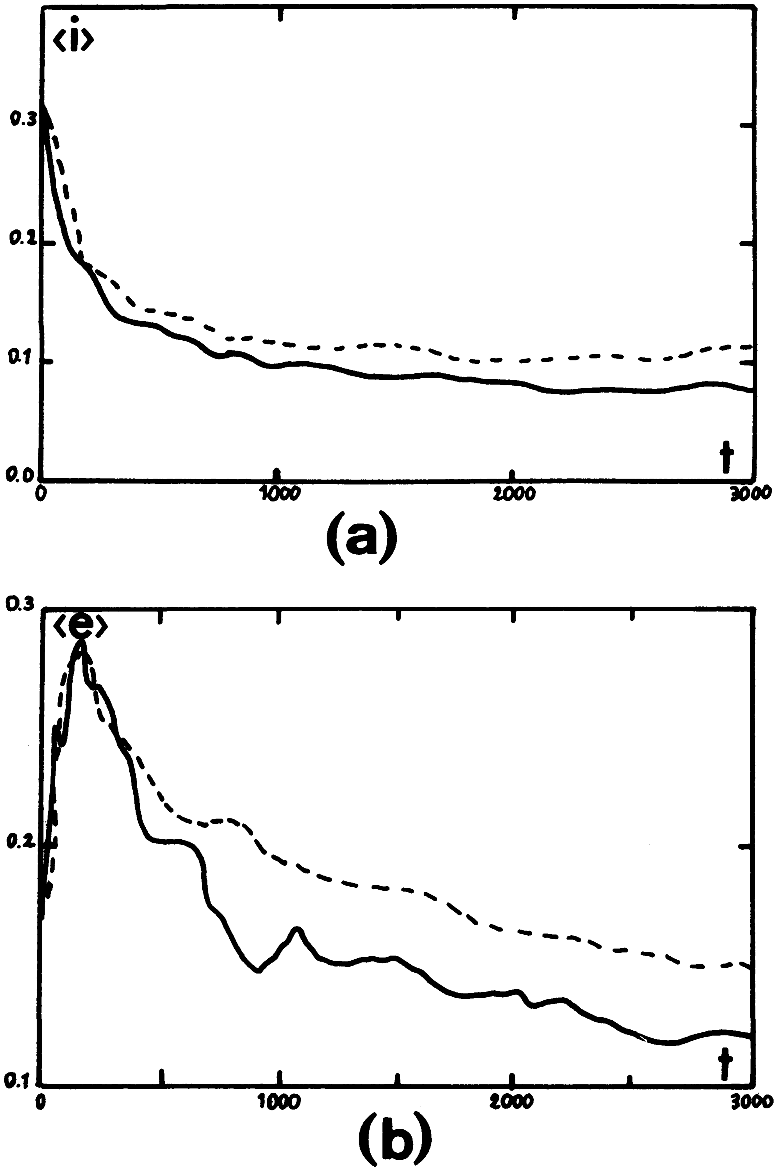

Figure 2 (a). Variations as a function of time of the mean inclination of the particles (which is a good measure of the flattening of the system). The large particles are in full line and the small particles are in dashed line. The particles have the same density, the large particles are four times more massive and about 1.5 larger than the small particles.

(b). Variations as a function of time of the mean eccentricity of the same system.

than five millimeters. The limiting size given by the Poynting-Robertson effect is curiously of the same order of magnitude: smaller particles have fallen onto Uranus. The rings could be very young, allowing particles of larger size.

In the near future, new data (observations of Saturn's rings from spacecrafts and observations of some occultations of stars by Uranus' rings) will be particularly helpful to test theories. Note that planetary rings is a case where on the one hand the input of the theory is relatively free of uncertainties, and of the other hand detailed observations will be soon available, so that observations and theories can be compared fruitfully.

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DISCUSSION

Carusi: Have you taken into account the possibility of adhesion or fragmentation of particles in the ring? Which is the initial velocity distribution function?

Brahic: The particles are indestructible spheres, and so there is no fragmentation after a collision. This would seem reasonable; relative velocities are small in real physical cases as Saturn's rings, and also for the major part of the evolution of our models. Coalescence is not considered here. Nevertheless, we can apply the above results; in this case, the total volume $N(r/R)^3$ of the particles is conserved, and $N(r/R)^2$ varies as R/r. Thus evolution will be accelerated by fragmentation in the proportion $1/r$. The initial conditions are set up by selecting at random the six elements of the Keplerian orbit of each particle in such a way that the initial trajectories are all ellipses, lying between two spheres of radius R_1 and R_2, respectively. In order to start from a system already a little flattened, the initial inclinations i of the orbits are taken between zero and some maximal value i_{max}.

Marchal: Do some bodies escape to infinity?

Brahic: I have assumed that particles on hyperbolic trajectories escape at once. Even for very inelastic collisions one of the particles involved can acquire a hyperbolic velocity. For typical values of the parameters, this happens on the average of once every 1000 collisions, and twenty-two particles have fallen onto the central body.

Marchal: What is the importance of the oblateness of the planet?

Brahic: The oblateness of the central planet has no very important effect on the evolution of the system. Even with an oblateness of the central body ten times larger than in the case of Saturn, I have not been able to see any difference.

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