PART III
METEORS AND METEOROIDS

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RADIATION PRESSURE AND POYNTING-ROBERTSON DRAG FOR SMALL SPHERICAL PARTICLES

S. SOTER, J. A. BURNS and P. L. LAMY

A new heuristic derivation of the radiation pressure and Poynting-Robertson drag forces is presented for particles with general optical properties; previous derivations considered only perfectly absorbing materials. The equation of motion for a particle of mass m and geometrical cross-section A, moving with velocity v through a radiation field of energy flux F, is (to terms of order v/c)

$$m\dot{v} = (FA/c)Q_{pr}[(1 - \dot{r}/c)\hat{r} - v/c],$$

where $\hat{r}$ *is the radial unit vector,* $\dot{r}$ *is the radial velocity, and c is the speed of light. The radiation pressure efficiency factor* Q_{pr} *includes both scattering and absorption; it is evaluated using Mie theory for small spherical particles with measured optical properties that are irradiated by the actual solar spectrum. Very small particles (<0.01 μm) are not substantially affected by radiation forces.*

INTRODUCTION

Small particles in interplanetary space, such as cometary dust grains or micrometeoroids, are not only attracted to the Sun by gravity but are also repelled from it by the radiation pressure due to the momentum carried by solar photons. The orbits of such particles are also modified by the velocity-dependent Poynting-Robertson effect, the nature of which has been the subject of considerable controversy and misunderstanding since the beginning of the century, receiving its correct treatment by Robertson (1973). However, Robertson's derivation is unnecessarily difficult because it relies on the formalism of general relativity even though the effect can be understood in classical terms without invoking spacetime dilatation.

Furthermore, Robertson's expression contains an important assumption which is usually overlooked: he considered only perfectly absorbing particles, whereas in general small particles scatter or transmit most of the energy incident on them. This latter shortcoming of Robertson's expression has been noted by Mukai *et al.* (1974) and Lamy (1976), both of whom (incorrectly) considered the drag to be caused by the absorbed energy only; we shall see below that the scattered energy is just as important.

Perhaps it is in part due to Robertson's abstruse derivation and in part to the subsequent incorrect treatment of scattering that most textbooks explanations

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of the Poynting-Robertson effect have been confusing and/or inaccurate. Even though the effect is fundamental to an understanding of the orbital evolution of small interplanetary (and circumplanetary) particles, we are not aware of any previous presentation of a simple but accurate derivation.

In order to clarify the problem, we present our heuristic derivation in two stages, first considering only perfectly absorbing particles in order to obtain Robertson's expression (but on a much simpler physical basis), and then applying a similar approach to find the correct expression for a particle that in general scatters, transmits and absorbs light. Finally, we summarize numerical results on the solar radiation forces felt by small particles of cosmochemically important compositions.

RADIATION FORCES ON A PERFECTLY ABSORBING PARTICLE

The force on a perfectly absorbing particle due to solar (photon) radiation is composed of two parts: (2) that due to the initial interception by the particle of the incident momentum in the beam, and (b) that due to the effective rate of mass loss from the moving particle as it continuously re-radiates the incident energy.

The total amount of energy intercepted from a radiation beam of integrated flux F (erg cm^{-2} sec^{-1}) by a stationary perfectly absorbing particle of geometrical cross-section A is FA. If the particle is moving relative to the Sun with velocity v, we must replace F by

$$F' = F(1 - \dot{r}/c),$$

where $\dot{r} = v \cdot \hat{r}$ is the radial velocity, $\hat{r}$ is a unit vector in the direction of the incident beam, and c is the speed of light; the factor in parentheses is due to the Doppler effect, which alters the incident energy flux by shifting the received wavelengths. The momentum removed per second from the incident beam as seen by the particle is then $(F'A/c)\hat{r}$. This is the radiation pressure force.

The absorbed energy F'A is continuously re-radiated from the particle, which is moving at velocity v. Since the re-radiation is isotropic (small particles are effectively isothermal), then there is no net force exerted thereby on the particle in its own frame. However, there *is* an equivalent mass loss rate of $F'A/c^2$ from the particle moving at velocity v, which gives rise to a force on the particle of $-(F'A/c^2)v$ as seen in the inertial frame of the Sun. This is the drag force.

The net force on the particle is then the sum of the forces due to the incident radiation momentum and the re-radiation from the moving particle or, for a particle of mass m,

$$m\dot{v} = (F'A/c)\hat{r} - (F'A/c^2)v$$

$$\cong (Fa/c)\left[(1 - \dot{r}/c)\hat{r} - v/c\right],$$

to terms of order v/c. This last expression is equivalent to Robertson's result.

RADIATION FORCES WITH SCATTERING

Our discussion of scattering is partly based on van de Hulst (1957). In general, a small (spherical) particle of geometrical cross-section A will scatter an amount of light equivalent to that incident on an area AQ_{sca}, and absorb that incident on AQ_{abs}, where Q_{sca} and Q_{abs} are defined as the scattering and absorption coefficients, respectively, and can be calculated from Mie theory. For a given particle, Q_{sca}, Q_{abs}, and Q_{pr} (to be defined) depend on wavelength; in what

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follows, we take them to be average values, computed by integrating over the actual solar spectrum.

The total amount of energy intercepted from the beam of integrated flux F by the moving particle is now $F'A(Q_{abs} + Q_{sca})$, where F' is defined as before. The momentum removed per second from the beam as seen by the particle is then $(F'A/c)(Q_{abs} + Q_{sca})\hat{r}$. Of this, the part proportional to Q_{abs} is not returned to the beam, but the part proportional to Q_{sca} is partially returned to the beam by the forward component of the momentum in the scattered light.

The scattering diagram resulting from unpolarized light incident on a spherical particle has rotational symmetry about the radial direction $\hat{r}$. The intensity of the scattered light thus depends only on the angle α it makes with the incident beam direction (α = 0 is forward scattering). The energy scattered into the incremental solid angle annulus $d\omega = 2\pi \sin\alpha d\alpha$ in the direction α is proportional to $f(\alpha)d\omega$, where $f(\alpha)$ is the phase function of the scattering particle, normalized so that $\int f(\alpha)d\omega$ taken over all directions is unity. Then the net momentum scattered per second into the *forward* direction is $(F'A/c)Q_{sca}\langle\cos\alpha\rangle\hat{r}$, where

$$\langle\cos\alpha\rangle \equiv \int f(\alpha)\cos\alpha d\omega$$

can be calculated, again from Mie theory. This part of the incident momentum, removed from and immediately returned to the beam, cannot affect the dynamics of the particle. It is equivalent to an energy flow of $F'AQ_{sca}\langle\cos\alpha\rangle$ to which the particle is dynamically "transparent."

The remainder of the energy flow in the incident beam as seen by the particle is $F'AQ_{pr}$, where

$$Q_{pr} \equiv Q_{abs} + Q_{sca}(1 - \langle\cos\alpha\rangle).$$

This part is associated with the momentum removed from the beam and *not* returned by forward scattering. This momentum is given to the particle. It follows that the incident beam itself exerts a net force on the particle of $(F'A/c)Q_{pr}\hat{r}$, which is the radiation pressure force on a scattering and absorbing particle. Note that for perfect forward scattering, $Q_{pr} = Q_{abs}$; for isotropic scattering, $Q_{pr} = Q_{abs} + Q_{sca}$; and for perfect backscattering, $Q_{pr} = Q_{abs} + 2Q_{sca}$.

The input energy $F'AQ_{pr}$ is continuously re-radiated and scattered from the moving particle. The re-radiation is isotropic (as before) and so contains zero net momentum as seen by the particle. The scattered light, proportional to $Q_{sca}(1 - \langle\cos\alpha\rangle)$, also contains no net momentum in the particle frame; its momentum in the $+\hat{r}$ direction cancels that in the $-\hat{r}$ direction. This is because all the asymmetrically scattered momentum, proportional to $Q_{sca}\langle\cos\alpha\rangle$, was returned to the beam without dynamically affecting the particle, as we have seen. Thus there is no net force exerted upon the particle *in its own frame* either by re-radiation or by scattered light. However, there *is* the equivalent mass loss rate of $(F'A/c^2)Q_{pr}$ from the particle moving at velocity v, which gives rise to the drag force $-(F'A/c^2)Q_{pr}v$ as seen in the inertial frame of the Sun.

The net force on the particle is then the sum of the forces due to (a) the incident radiation momentum (neglecting the forward scattered part), and that due to (b) the re-radiation and symmetrical scattering of energy from the moving particle, or

$$m\dot{v} = (F'A/c)Q_{pr}\hat{r} - (F'A/c^2)Q_{pr}v$$
$$\cong (FA/c)Q_{pr}[(1 - \dot{r}/c)\hat{r} - v/c],$$

again to terms of order v/c. This is identical to Robertson's expression except

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for the inclusion of the important factor Q_{pr}. Our result has been confirmed by use of the energy and momentum transformation laws of special relativity (Burns *et al.* 1977). Note that since $v = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$, where $\hat{\theta}$ is the unit vector normal to $\hat{r}$ in the orbit plane, we can also write the radiation force as

$$m\dot{v} \cong (FA/c)Q_{pr}[(1 - 2\dot{r}/c)\hat{r} - (r\dot{\theta}/c)\hat{\theta}].$$

MAGNITUDE OF THE FORCES

Both the radiation pressure and drag forces on small particles are proportional to the radiation pressure coefficient Q_{pr}, which is itself a function of particle size and composition. It is convenient to express this factor in terms of the ratio β of the radiation pressure force to the gravitational attraction force, or

$$\beta = (FAQ_{pr}/c) / (GM_{\odot}m/R^2) = 5.7 \times 10^{-5}Q_{pr}/\rho s,$$

where G is the gravitational constant, $M_{\odot}$ is the solar mass, and ρ, s and R are the particle's density, radius, and distance from the Sun, respectively. Here we have made use of the fact that $FR^2 = F_1R_1^2$, where $F_1 = 1.36 \times 10^6$ erg cm^{-2} sec^{-1} is the solar constant and R_1 is one astronomical unit. Note that β is independent of R.

The radiation pressure coefficient Q_{pr} is the average of the monochromatic coefficient $Q_{pr}(s,\lambda)$ integrated over the actual solar spectrum, or

$$Q_{pr} = (1/F) \int_0^{\infty} F(\lambda)Q_{pr}(s,\lambda)d\lambda,$$

where $F \equiv \int_0^{\infty}F(\lambda)d\lambda$, and $F(\lambda)$ is the monochromatic solar flux in the wavelength increment $d\lambda$ about λ.

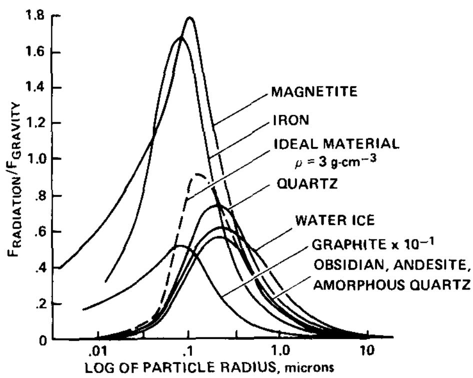

Figure 1. The ratio of the radiation pressure force to the gravitational attraction as a function of particle size s. The radiation pressure is found from Mie theory calculations of Q_{pr} using the actual solar spectrum and laboratory-measured optical properties for the various materials as found in the literature. The ideal material is defined as one that absorbs only radiation of wavelengths $\lambda \leq 2\pi s$. After Burns et al. *(1977).*

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In Fig. 1 we plot β as a function of the (spherical) particle radius for several cosmochemically important materials [cf. review by Burns *et al.* (1977)]. We see that large particles are little affected by the solar radiation field; the s^{-1} term in the expression for β dominates. Also for most very small particles (r < 0.01 μm) the radiation force is insignificant because the scattering cross-section goes like s^4 (*i.e.*, Q_{pr} in the numerator of β goes to zero more rapidly than s in the denominator). Figure 1 shows that, except for conducting particles and the very absorbent graphite, the repulsive pressure force never exceeds the attraction of gravity (β < 1). Only for particles whose size is comparable to a characteristic wavelength of the solar radiation field is the radiation pressure--and therefore the Poynting-Robertson drag--a significant perturbation. As described by Burns *et al.* (1977), very small particles are perturbed more by the solar wind and probably by the interplanetary magnetic field, while the orbits of large particles are most affected by the Yarkovsky effect and perhaps by interparticle collisions.

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