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ARTICLE

Estimating sentiment and risk in a consumption model: a factor analysis approach[†]

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Abstract

This empirical paper deals with the impacts of sentiment about the future, short-run risk, and long-run risk in a dynamic economic model of optimal consumption decisions with Schroder and Skiadas [(1999) *Journal of Economic Theory* 89, 68–126.] continuous-time stochastic recursive preferences. The empirical strategy combines both a latent factor method and a democratic orthogonalization technique. The latent factor method is applied to a large database of macroeconomic indicators, and a democratic orthogonalization technique is used to separate the relative importance of sentiment about the future and long-run risk channels in shaping optimal consumption decisions. The empirical results suggest that consumers with recursive preferences are not indifferent to long-run uncertainty shocks to future consumption prospects. Endogenous consumption variations are driven by a multicomponent mechanism, where on average, the sentiment component accounts for 15.33%, the short-run risk accounts for 16.89%, and the long-run risk pertains to 34.51%.

Keywords: Optimal consumption; recursive utility; sentiment; risk

1. Introduction

Sentiment and attitudes towards long-run uncertainty constitute an important dashboard for gauging by how much consumers feel optimistic or pessimistic about future economic prospects. Over the past decades, much interest has been aroused in understanding the role played by sentiment about the future in shaping the dynamics of economic decisions (Driscoll and Holden (2014); Milani (2017); Salamaliki and Venetis (2019); Pan (2020)). Among the popular measures of sentiment about future economic prospects, the University of Michigan's Survey of Consumers and the Conference Board's Consumer Confidence Index are closely watched by news media and decision-makers.[1] They are leading indicators of future developments of households' consumption (Bram and Ludvigson (1998); Dominitz and Manski (2004); Ludvigson (2004); Curtin (2007)), based upon answers regarding their sentiment about the general economic situation, unemployment, and capability of savings. Early preoccupation about the role played by expectations about future prospects in dynamic economic analysis goes back at least to Pigou (1927), and it continues to spark the interest of market participants, business managers, and policy makers (Barro (1994); Biolsi and Du (2020); Benhabib and Spiegel (2018)). Despite the important progress made in the empirical literature assessing the link between consumer's sentiment about

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the future and consumption expenditures (Dominitz and Manski (2004); Ludvigson (2004)), an approach that builds on a formal structural economic growth model and combines both sentiment and long-run risks remains under-explored. Bram and Ludvigson (1998) and Ludvigson (2004) rightfully note that there is a need for empirical studies grounded in a theoretical economic model that explicitly incorporates sentiment as a driving factor of consumer decisions. While a great deal of effort has been devoted to surveying consumers' sentiment about the future, explicitly linking empirical research and theoretical economic models related to consumers' optimizing behavior, again, remains largely uncharted. One possible explanation for the gap is that the previous empirical works on the relationship between consumption and sentiments about the future did not fully incorporate insights from an optimal consumption model with recursive utility. In fact, only recently have stochastic dynamic models that include sentiment about the future among the explanatory factors of consumer behavior appeared in the optimal growth-related literature. For example, the paper compiled by Kakeu and Byron (2016) is among the few papers that explicitly point to the role of sentiment about the future as a factor affecting optimal consumption choices in a dynamic stochastic model of optimal consumption with recursive utility.[2] The work of these authors features several factors that dynamically impact optimal consumption choices over time, including sentiment about the future, as well as short-run and long-run risks. Previous empirical inquiries have not explicitly incorporated these insights in their econometric specification. Building on the theoretical optimal consumption model by Kakeu and Byron (2016), our work complements the previous empirical literature by explicitly incorporating sentiment about the future as well as various risk factors in the analysis.

The forward-looking feature of recursive utility allows a potential role for sentiment to matter endogenously in the consumer's decision (Cochrane (2005); Sargent (2007); Hansen (2010, 2012)).[3] Sentiment about the future refers to changes in future expectations (Hansen (2010, 2012); Kakeu and Byron (2016)). A positive sentiment about the future reflects an increase in expectations about future prospects whereas a negative sentiment reflects a decrease in expectations about future prospects. The paper by Kakeu and Byron (2016) shows that the optimal consumption path is shaped in a nontrivial way by factors such as changes in expectations about future prospects (sentiment), short-run risks, and long-run risks. The rich framework offered by recursive preferences provides greater flexibility for understanding plausible channels by which long-term uncertainty matters in current decision-making (Kreps and Porteus (1978); Duffie and Epstein (1992); Sargent (2007); Skiadas (2007); Hansen (2010, 2012)). The implications of assuming a preference for the temporal resolution of uncertainty have proven useful for explaining several puzzles in the macro-finance literature.[4] Recursive utility allows for adopting a flexible approach to capturing how agents behave regarding the temporal resolution of uncertainty alongside the concomitant implications for the attitudes toward long-run risks (Kreps and Porteus (1978); Sargent (2007); Hansen et al. (2008); Kaltenbrunner and Lochstoer (2010); Ericson and Fuster (2011); Swanson (2012); Strzalecki (2013); Epstein et al. (2014)).

To estimate the dynamic structural model, we use consumption and total productivity data available at the St. Louis Federal Reserve Bank, data on capital stocks from the Penn World Table, and sentiment data from the University of Michigan's Survey of Consumers. The University of Michigan's Survey focuses on consumer's prospects for their own financial situation, their prospects about the economy over the near term, and their prospects about the economy over the long term.

From an empirical point of view, our analysis combines both the latent factor method and the democratic orthogonalization method in empirically exploring an economic growth model that delivers sentiment and risk as drivers of the consumption path. Consumption variations are decomposed into sentiment factors and risk factors by using the democratic orthogonalization method developed by (Löwdin (1970). The democratic decomposition method developed by Löwdin (1970) is a statistical method that allows to transform a matrix of correlated variables into

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an information-equivalent matrix of variables that are noncorrelated.[5] The democratic decomposition allows to isolate the specific contribution of each component of the matrix of variables, important in light of common variation, permitting a clear interpretation of the role of each individual variable. Using this procedure allows us to compute the relative individual contribution of sentiment, short-run risk, and long-run risk to changes in the expected consumption growth. The results provide empirical evidence that sentiment about the future, short-run risk, and long-run risk are factors that matter for understanding the optimal consumption path. The sentiment factor accounts for 13.8% of variations in consumption while macroeconomic risk factors account for 50.86%, of which 15.8% pertains to the short-run risk and 35% pertains to the long-run risk.

Our paper makes at least three contributions to the literature. First, previous studies have used an estimation procedure that imposes a linear relationship between consumption and sentiment without reference to dynamic preference structures. From a methodological point of view, this paper differs from standard approaches that look at the relationship between sentiment about the future and consumption by empirically estimating both sentiment and risk impacts from an economic growth model perspective. Second, our work offers a bridge that connects the empirical literature that looks at sentiments in macroeconomics and the literature devoted to economic growth models in a recursive utility framework. Third, the empirical strategy uses both the latent factor method and the democratic orthogonalization method. The methodology of latent factor analysis is a technique that allows a large panel of indicators on consumer future expectations to be summarized by a relatively small number of estimated factors. In the past few years, a growing number of applied works in finance and macroeconomics have used latent factor models [see for instance Bai and Ng (2002); Bernanke and Boivin (2003); Bernanke et al. (2005); Favero et al. (2005); Boivin and Giannoni (2006); Forni et al. (2009); Ludvigson and Ng (2009); Bouaddi and Taamouti (2012, 2013); Chen et al. (2013); Kakeu and Bouaddi (2017); and Thimme (2017)]. To provide endogenous approximation of the value function of the consumer problem, it is replaced with a log linear function of sentiment latent factors and parameters to be estimated through the dynamics of the optimal consumption. This strategy is in line with Cochrane (2017), [p. 955] who suggests that while the utility index itself is not observable, one way to deal with the estimation is to substitute it as a function of quantifiable variables, such as latent factors, related to the state of the economy. While estimating the risk factors involved in the dynamic of consumption behavior, we make use of the Engle (2002) Dynamic Conditional Correlation model approach. A similar tool is used by Bali and Engle (2010). As mentioned earlier, we also make use of the democratic orthogonalization method developed by (Löwdin (1970)) in the Quantum Chemistry literature to compute the relative importance of sentiments and risk factors in contributing to variations in consumption.

The remainder of this paper is structured as follows. Section 2 briefly presents the stochastic growth model by Kakeu and Byron (2016) that features sentiment and long-run risk factors associated with future growth prospects in optimal consumer behavior. Section 3 describes an econometric methodology that incorporates the latent factor analysis for estimating the optimal consumer behavior. Section 4 presents the data. Section 5 presents empirical results. The last section offers concluding comments.

2. A stochastic growth model with recursive utility

This empirical paper builds on the stochastic growth model by Kakeu and Byron (2016) that analyzes how uncertainty and expectations about future prospects affect optimal consumer behavior. The representative consumer is endowed with stochastic recursive preferences à la Duffie and Epstein (1992). Recursive preferences give rise to an "aggregator" function for current utility at each time t, $f(c(t), J(t))$, that combines current consumption, $c(t)$, and an index of future utility, $J(t)$. The index $J(t)$ may also be viewed as prospective utility (Koopmans (1960)),

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which expresses expectations about future consumption prospects. In combining both current consumption and expectations about future consumption prospects, recursive utility allows a potential role for sentiment about future consumption to matter in the consumer's decision (Cochrane (2005); Sargent (2007); Hansen (2010, 2012)). The consumer's optimization problem consists in choosing a consumption process $c = \{c(t) : t \geq 0\}$ so as to maximize the present discounted value of the entire stream of net benefits subject to the capital accumulation constraint. That is, the consumer's decision problem is

$$\max_{\{c(t):\, t \geq 0\}} E_0\Big[\int_0^\infty f(c(t), J(t))dt\Big], \tag{1}$$

subject to:

$$dK(t) = \Big[F(K(t)) - c(t) - \delta K(t)\Big]dt + \sigma(K(t))dB(t), \tag{2}$$

$$c(t) \geq 0, \tag{3}$$

$$K(t) \geq 0, \tag{4}$$

$$K(0) = K_0 > 0. \tag{5}$$

As mentioned earlier, the forward-looking flexible feature of recursive utility allows a potential role for sentiment about future consumption to matter endogenously in the consumer's decision (Cochrane (2005); Sargent (2007); Hansen (2010, 2012)). Sentiment about the future is shaped dynamically by changes in uncertainty shocks to the economy as well as changes in the state of the economy. In analyzing how uncertainty and expectations about future consumption prospects affect optimal consumer behavior, Kakeu and Byron (2016) show that the expected consumption growth rate is given by

$$\begin{aligned}
\frac{1}{c(t)}\frac{1}{dt}E_t dc(t) = \left(\frac{-c(t)f_{cc}(t)}{f_c(t)}\right)^{-1}\cdot\\
\left[\underbrace{f_J(c(t), J(t))}_{\text{Endogenous discount rate}} + \underbrace{F_K(t) - \delta}_{\text{Marginal product}} + \underbrace{\left[\frac{J(t)f_{cJ}(t)}{f_c}\right]\left(\frac{1}{J(t)}\frac{1}{dt}E_t dJ(t)\right)}_{\text{Sentiment effect}}\right]\\
+\frac{1}{2}\left[\underbrace{\frac{-c^2(t)f_{ccc}(t)}{c(t)f_{cc}(t)}\sigma_c^2}_{\text{Short-run risk}} - \underbrace{\frac{J^2(t)f_{cJJ}(t)}{c(t)f_{cc}(t)}\sigma_J^2(t) - \frac{2c(t)J(t)f_{ccJ}(t)}{c(t)f_{cc}(t)}\sigma_{cJ}(t)}_{\text{Long-run risk}}\right]
\end{aligned} \tag{6}$$

Equation (6) shows the link among the expected growth rate of consumption, sentiments about future prospects, short-run risk factors, and long-run risk factors.

In equation (6), the term $\left[\frac{-c(t)f_{cc}(t)}{f_c(t)}\right]^{-1}$ is the inverse of the Arrow–Pratt measure of relative risk aversion associated with consumption.

The term $-f_J(c, J(t))$ highlights the role of an endogenous discount rate on the optimal expected consumption growth rate.

The term $F_K(t) - \delta$ is the net marginal product of the capital stock.

The term $\left(\frac{J(t)f_{cJ}(t)}{f_c(t)}\right)\left(\frac{1}{J(t)}\frac{1}{dt}E_t dJ(t)\right)$ contains the growth rate of the future utility index which updates the expectations about future consumption prospects. It gives the overall effect of sentiment on the consumer's future consumption prospects.

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In equation (6), the weight $-\frac{c^2(t)f_{ccc}(t)}{c(t)f_{cc}(t)}$ is a measure of prudence regarding short-run uncertainty. The weight $-\frac{J^2(t)f_{cJJ}(t)}{c(t)f_{cc}(t)}$ is a measure of prudence regarding long-run uncertainty. The weight $\frac{c(t)J(t)f_{ccJ}(t)}{c(t)f_{cc}(t)}$ is a measure of cross-prudence regarding both short-run and long-run uncertainty.

In the very particular case where the aggregator takes the form $f(c, J) = U(c) - \beta J$, which is the aggregator of the time-additive utility,[6] then equation (6) reduces to:

$$\frac{1}{c(t)}\frac{1}{dt}E_t dc(t) = \left(\frac{-c(t)u_{cc}(t)}{u_c(t)}\right)^{-1}[(F_K(t) - \delta) - \beta] - \frac{1}{2}\left[-\frac{c^2(t)u_{ccc}(t)}{c(t)u_{cc}(t)}\right]\sigma_c^2(t). \tag{7}$$

It is worth noting that in such a case, neither sentiment nor long-run risk appears in equation (7). With a time-additive utility function, risks involving future growth prospects do not affect the optimal consumption decision. If in addition, it is assumed that there is no uncertainty, then the optimal consumption rule equation (7) reduces to the following well-known Keynes—Ramsey rule:

$$\frac{\dot{c}}{c} = \left[-c\frac{U_{cc}(c)}{U_c(c)}\right]^{-1}[(F_K(t) - \delta) - \beta], \tag{8}$$

where a single dot over a variable signifies its first derivative with respect to time. In this very particular case, where the aggregator is additive with respect to the future utility index, $f_{cJ}(t) = 0$. Therefore, sentiments about the future and long-run risk do not affect the optimal consumption path.

2.1. A parametric example

The aggregator $f(c, J)$ encodes tradeoffs between current consumption and the future utility index. To better understand how sentiment about the future as well as short-run and long-run risk affect the expected growth rate of consumption, we consider the aggregator of a parametric recursive utility for which existence and unicity are discussed by Schroder and Skiadas (1999). The time-additve expected utility is a special case of the Schroder and Skiadas (1999) recursive utility.

2.1.1. Aggregator of the Schroder and Skiadas (1999)

To estimate equation (6), we rely on the aggregator of the Schroder and Skiadas (1999)'s parametric recursive utility given by

$$f(c, J) = (1 + \alpha)\left[\frac{c^\gamma}{\gamma}J^{\frac{\alpha}{1+\alpha}} - \beta J\right], \tag{9}$$

with $\alpha > -1$ and $\gamma < \min\left(1, \frac{1}{1+\alpha}\right)$. This parametric recursive utility is more general than the time-additive expected utility. Additionally, it simplifies the exposition of the econometric model and the interpretation of the parameters to be estimated.

The discount parameter β is assumed to be positive. In a time-additive utility model, β would simplify the rate of time preference. Time preference is generally endogenous in nonadditive recursive utility settings. The ratio $\frac{1}{1-\gamma}$ is the elasticity of intertemporal substitution and the parameter α captures the dependency of current utility to future utility J.[7] The curvature of the aggregator $f(x, J)$ with respect to the future utility argument, J, characterizes preferences for the timing of resolution of uncertainty (Kreps and Porteus (1978); Duffie and Epstein (1992); Schroder and Skiadas (1999); Strzalecki (2013); Epstein et al. (2014); Zhao (2017)), with a convex aggregator favoring early resolution, and a concave aggregator favoring late resolution (Skiadas (1998)). An additive temporal aggregator then corresponds to indifference towards the timing of

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resolution. The sign of the product term $\gamma\alpha$ expresses the curvature of the aggregator with respect to the second argument, and it, therefore, captures attitudes towards the temporal resolution of uncertainty.[8] As will be later evidenced in subsections 2.1.3 and 3.1, the parameter α calls attention to the role played by sentiment about the future as well as long-run risks in shaping the dynamic of optimal consumption behavior. Additionally, the sign of the product parameter $\alpha\gamma$ helps understand consumers' attitudes towards the temporal resolution of uncertainty. A value of $\gamma\alpha$ different from zero expresses nonindifference towards the temporal resolution of uncertainty. A negative sign, $\gamma\alpha < 0$, expresses a preference for early resolution of uncertainty whereas a positive sign, $\gamma\alpha > 0$, expresses a preference for late resolution of uncertainty. In the very particular case where $\alpha = 0$, which corresponds to the indifference towards the timing of resolution, the aggregator of the standard time-additive expected utility is obtained as $f(x, V) = \frac{x^\gamma}{\gamma} - \beta J$.

Let us mention that in a deterministic setting, a world without uncertainty ($\sigma = 0$), concerns for temporal resolution of uncertainty is not relevant ($\alpha = 0$), and the Schroder and Skiadas (1999) parametric recursive utility reduces to the aggregator of the time additive expected utility, that is $f(x, J) = \frac{x^\gamma}{\gamma} - \beta J$.

2.1.2. Production function

Let us assume the per capital production function is given by

$$F(K(t)) = A(t)(K(t))^\nu, \tag{10}$$

where $0 < \nu \leq 1$ and parameter $A(t) > 0$ is the total factor productivity (TFP) at time t. This functional form includes the $A(t)K(t)$ production function as a special case, ($\nu = 1$).

2.1.3. Optimal consumption path

Substituting the production function (10) and the parametric aggregator specification (9) in the expected consumption growth (6) gives

$$\begin{aligned}\frac{1}{c}\frac{1}{dt}E_t dc = \frac{1}{1-\gamma}&\left[\left(-\beta(1+\alpha) + \alpha\frac{c^\gamma}{\gamma}J^{\frac{-1}{1+\alpha}}\right) + (\nu A K^{\nu-1} - \delta) + \underbrace{\left[\frac{\alpha}{1+\alpha}\right]\left(\frac{1}{J}\frac{1}{dt}E_t dJ\right)}_{\text{Sentiment}}\right]\\ &+ \frac{1}{2}\left[\underbrace{(2-\gamma)\sigma_c^2}_{\text{Short-run risk}} + \underbrace{\frac{\alpha}{(1+\alpha)^2(\gamma-1)}\sigma_J^2(t) - \frac{2\alpha}{1+\alpha}\sigma_{cJ}(t)}_{\text{Long-run risk}}\right]\end{aligned} \tag{11}$$

It is also worth mentioning that the special case where $\alpha = 0$ which relates to a time-additive expected utility, expresses indifference towards the resolution of uncertainty. In that case, equation (11) reduces to

$$\frac{1}{c}\frac{1}{dt}E_t dc = \frac{1}{1-\gamma}\left[-\beta + (\nu A K^{\nu-1} - \delta) + \frac{1}{2}\underbrace{(2-\gamma)\sigma_c^2}_{\text{Short-run risk}}\right] \tag{12}$$

As we can see in equation (12), the special case where $\alpha = 0$ would imply that consumption variations are not impacted neither by sentiment component accounts nor by long-run risks. The statistical significance of the preference parameters in estimating the consumption path (11) will be of paramount interest in the empirical section.

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3. Empirical strategy based on latent factor method

This section describes our data, calibration approach and presents details regarding the main estimation procedure for the consumption growth equation. A combination of calibrated and estimated parameters is used.

3.1. Estimation of the parameters

The estimation of the parameters α and γ requires to obtain the proxies of variables in both the right-hand side of the optimal consumption rule (11). The left-hand side corresponds to the optimal consumption. It will be estimated using the method of estimating continuous time diffusion process of Nowman (1997). On the other hand, the variables that appear on the right-hand side of the optimal consumption (11) include the value function and its growth rate. While the value function is not directly observable, it depends on expectations about future prospects of the economy. To emphasize this point, Koopmans (1960) refers to the value function as the prospective utility. The value function can be estimated by using a small numbers of factors $F = (F_1, \ldots, F_L)$ called latent variables that capture expectations about future prospects of the economy. The estimation of latent factors is based on the procedure proposed by Bai and Ng (2002) and presented in Appendix A. Other studies that have used the latent factor approach include Bai and Ng (2002); Bernanke and Boivin (2003); Bernanke et al. (2005); Favero et al. (2005); Boivin and Giannoni (2006); Forni et al. (2009); Ludvigson and Ng (2009); Bouaddi and Taamouti (2012); Bouaddi and Taamouti (2013); Kakeu and Bouaddi (2017).

In general, there is no closed-form solution to the value function. An approximation of the value function is used for empirical purposes. The empirical estimate of models with a recursive utility framework is still difficult due to the latency of the value function (Chen et al. (2013); Thimme (2017)), and researchers must use approximations. While the utility index itself is not observable, one way to deal with the estimation is to substitute it as a function of quantifiable variables, such as latent factors, related to the state of the economy (Cochrane (2017)). In a recursive utility framework, the value function depends upon expectations about future consumption growth. Proxying the recursive utility index by using latent factor techniques that capture expectations about the future of the economy is well grounded following tradition in empirical works using recursive utility (Cochrane (2005); Hansen (2010, 2012); Chen et al. (2013); Kakeu and Bouaddi (2017)). For instance, Chen et al. (2013) use latent factor techniques to explicitly estimate the unobservable continuation value of the future consumption plan in a discrete time Epstein and Zin (1989) recursive utility framework. We use a Schroder and Skiadas (1999) continuous-time stochastic recursive preferences, and we assume that the logarithm of the value function is a linear function of the estimated latent factors related to sentiment about the future.[9] We will use the Michigan consumer sentiment index which is a monthly assessment of consumer expectations about the future. The Michigan survey is a leading indicator that attempts to predict economic conditions a full year into the future.

Our approach uses a latent factor approximation of the value function which takes the following functional form.

$$J(t) = e^{\left(\theta + \sum_{i=1}^{L} \phi_i F_i(t)\right)}. \tag{13}$$

Equation (13) can be viewed as a way of log linearizing the model. Note that the differential form of equation (13) implies that changes in the value function can be approximated linearly by changes in the latent factors. Log-linear approximations are often used in macroeconomic and financial models (Bansal and Yaron (2004); Ljungqvist and Sargent (2004); Cochrane (2005); Lau and Ng (2007); Restoy and Weil (2011)). The estimation of the latent factors is done in the first step of the econometric procedure. The approximated value function $\widetilde{J}(t) = J(\widetilde{F}_{1t}, \ldots, \widetilde{F}_{Lt})$ is a function of those estimated latent factors obtained from sentiment variables. The estimated value function $\widetilde{J}(t)$ is a consistent estimate of the value function $J(t)$ that is not observable.

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When the value function is replaced by its approximate in the pricing equation, the estimation of the parameters θ, and $(\phi_i)_{i=1,..L}$ in the expression (13) are done simultaneously with the preference parameters α, β, and γ described in the expression (9). That is, this specification allows the value function to be endogenously and jointly estimated with the preference parameters in the structural model.[10]

The optimal consumption rule (11) reduces to

$$\frac{1}{c}\frac{1}{dt}E_t dc = \frac{1}{1-\gamma}\left[\left(-\beta(1+\alpha)+\alpha\frac{c^{\gamma}}{\gamma}\left(e^{\left(\theta+\sum_{i=1}^{L}\phi_i F_i(t)\right)}\right)^{\frac{1}{1+\alpha}}\right)+(\nu A K^{\nu-1}-\delta)\right.$$
$$\left.+\underbrace{\frac{\alpha}{1+\alpha}\left(\sum_{i=1}^{L}\phi_i F_i(t)+\theta\right)}_{\text{Sentiment}}\right]$$
$$+\frac{1}{2}\left\{\underbrace{(2-\gamma)\sigma_c^2(t)}_{\text{Short-run risk}}+\underbrace{\frac{\alpha}{(1+\alpha)^2(\gamma-1)}\sum_{i=1}^{L}\phi_i^2F_i^2(t)\sigma_{F_i}^2(t)-\frac{2\alpha}{1+\alpha}\sum_{i=1}^{L}\phi_i F_i(t)\sigma_{cF_i}(t)}_{\text{Long-run risk}}\right\} \tag{14}$$

The factor decomposition provided by equation (14) gives a description of the relationship between the expected optimal consumption, sentiment, and long-run risks. Following Hansen (2010, 2012), this shows that recursive preferences provide a channel for sentiment to matter in consumption decision-making. The present work aims at quantifying the impact of sentiments and long-run risks on optimal consumption decisions. The long-run risk components in equation (14) encapsulate the uncertainty shocks to the latent factors, which follow stochastic processes. Movements in the latent factors should be traced to movements in sentiment indicators. The sentiment indicators are forward-looking indicators as they capture information about the state of the economy as well as expectations about future prospects of the economy.

3.2. Description of the data on consumption per capita

Consumption data come from the St. Louis Federal Reserve Bank and cover the period from February 1980 to December 2014. We used the real personal consumption expenditures of services, the real personal consumption expenditures of nondurable goods, and capital stock at constant national prices.[11] We also use the total civilian population to get the real consumption per capita and the real capital stock per capita.[12] We computed the per capita real consumption as the sum of the real personal consumption expenditures of services and real personal consumption expenditures of nondurable goods over civilian population to get the per capita consumption level. Similarly, we compute the per capita capital as the real capital stock over civilian population. We then use the growth rate (log-difference) on real per capita personal consumption expenditures of services and nondurable goods (See Figure 1).

3.2.1. Estimation of the expected growth rate of consumption $\frac{1}{c}\frac{1}{dt}E_t dc$

The consumption is expected to follow a stochastic process

$$dC(t)=\mu_C(t,C(t))dt+\sigma_C(t,C(t))d\zeta(t), \tag{15}$$

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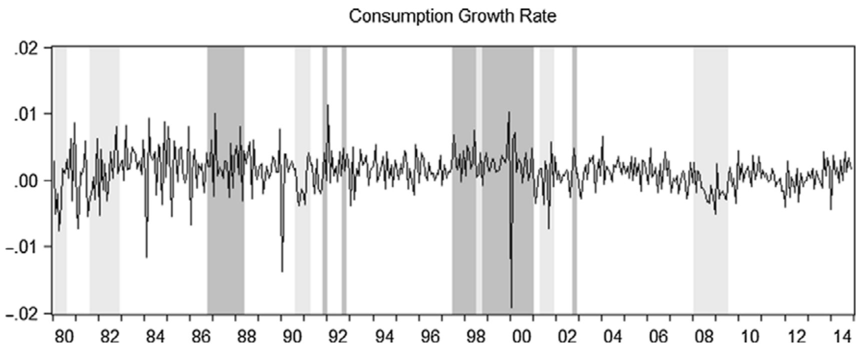

Figure 1. Consumption per capita growth rate.

where

$$C(t) = \int_0^t \frac{dc(\tau)}{c(\tau)}$$

represent the cumulative consumption, and $\mu_C(t, C(t)) = \frac{1}{dt}E_t dC = \frac{1}{c}\frac{1}{dt}E_t dc$ represents the expected growth rate of consumption.

For estimation purposes, let us assume the following parametric specification

$$dC(t) = (\alpha_1 + \alpha_2 C(t))dt + \delta C(t)^{\alpha_3} d\zeta(t), \tag{16}$$

for which a discrete approximation was developed by Nowman (1997) and based on some results found in Bergstrom (1983) as follows:

$$C_t = e^{\alpha_2} C_{t-1} + \frac{\alpha_1}{\alpha_2}(e^{\alpha_2,} - 1) + \eta_t \tag{17}$$

where the conditional distribution of the error term η_t satisfies

$$E_{t-1}(\eta_s \eta^t) = \begin{cases} 0 & \text{if } s \neq t, \\ \frac{\delta^2}{2\alpha_2}(e^{\alpha_2} - 1)(C_{t-1})^{2\alpha_3} & \text{if } s = t. \end{cases}$$

The logarithm of the Gaussian likelihood function is

$$L(\alpha_1, \alpha_2, \alpha_3, \delta) = \sum_{t=1}^{T} [\log E_{t-1}(\eta_t^2) + \frac{(C_t - e^{\alpha_2} C_{t-1} - \frac{\alpha_1}{\alpha_2}(e^{\alpha_2} - 1))^2}{E(\eta_t^2)}],$$

where $E_{t-1}(\eta_t^2) = \frac{\delta^2}{2\alpha_2}(e^{\alpha_2} - 1)(C_{t-1})^{2\alpha_3}$.

Maximum likelihood estimation consists of solving

$$(\widehat{\alpha}_1, \widehat{\alpha}_2, \widehat{\alpha}_3, \widehat{\delta}) = \arg \max_{\alpha_1, \alpha_2, \alpha_3, \delta} L(\alpha_1, \alpha_2, \alpha_3, \delta). \tag{18}$$

It follows from equation (16) that

$$\frac{1}{c}\frac{1}{dt}E_t dc = \widehat{\alpha}_1 + \widehat{\alpha}_2 C(t). \tag{19}$$

3.2.2. Estimation of the latent factors related to the panel of Sentiment data

The latent factor analysis is used to estimate a small number of latent factors the panel of consumer sentiment indicators, spanning from February 1980 to December 2014. The panel of sentiment indicators form a group of Indexes of "Index Consumer Sentiment" and "Index of Consumer Expectations" by age groups, regions, income and education level, a total of 32 indicators.

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These data are downloaded from the University of Michigan website.[13] The latent factors are estimated using the optimization program (47)–(48) presented in Appendix A. Using the information criteria proposed by Bai and Ng (2002), one factor is selected, representing 76% of the total variation of the panel of sentiment indicators.

3.3. Description of data on total factor productivity and real capital stock

The optimal consumption path (14) incorporates the marginal product of capital at time t, which depends upon the TFP, $A(t)$, and the capital stock per capita, $K(t)$.

The capital stock is computed at constant 2005 national prices (in mil. 2005 US$). Capital stock is estimated based on accumulated and depreciated past investments. It includes Structures (residential and nonresidential), Transport equipment, Computers, Communication equipment, Software, and Other machinery and assets.[14]

The TFP is the output less the contribution of capital and labor. The TFP is adjusted for capacity utilization of the capital stock. The computation methodology related to the utilization-adjusted TFP is discussed by Fernald and Matoba (2009); Fernald (2014), and Basu et al. (2006).

We compute the monthly equivalent by using an interpolation technique for deriving a monthly series from annual data. A similar frequency conversion technique is used by the Federal Reserve Bank.[15] For more details about the disaggregation of low frequency data to higher frequency, we refer the reader to the following papers: Boot et al. (1967); Denton (1971); Chan (1993); Feijoo et al. (2003); and Feijoo et al. (2003) among others. In our context, the measurement error resulting from frequency conversion of the dependent variable has zero mean and and is uncorrelated with regressors. Thus, we can estimate consistently the parameters in this case. We argue that this is true in our case since the high frequency counterpart is obtained by pure statistical method not involving the regressors which testify that the measurement error is independent of regressors preserving the consistency of the estimator. Of course, the estimates will be less precise than with data without measurement error. The induced inflation in the variance of the estimator will impact the significance of the parameters via lower t-statistics. However, if all p-values are below the significant level then this variance inflation is unimportant (the t-statistics are very conservative toward the null hypothesis).

3.4. Database on the University of Michigan's Survey of Consumers' sentiment

Sentiment factors are derived from the monthly Survey of Consumers by the University of Michigan.[16] The survey on consumer expectations focuses on consumer's view prospects for their own financial situation, their prospects about the general economy over the near term, and their prospects about the economy over the long term. The survey contains consumer's prospects about personal finances, savings and retirement, economic conditions, unemployment, prices, government expectations, household goods buying conditions, vehicle buying conditions, and home buying and selling conditions.[17]

3.5. Calibrated parameters

Some parameters are calibrated to match stylized facts. Following a calibration methodology emphasized by Lucas (1980) and Kydland and Prescott (1982), the parameter of the production function ν is set at 0.36 in line with standard economic research. The subjective discount rate β is set at 4% per annum, which is equivalent to 0.33% per month. The capital depreciation rate, δ, is set at 2.54% per annum, which is equivalent to 0.25% per month.

3.6. Empirical results

The estimation of the parameters related to the optimal consumer path uses an econometric approach that incorporates the latent factor analysis on sentiment indicators. We used the

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Table 1. Estimation of preference parameters of the model equation (20)

	Coefficient	Std. error	*z*-Statistic	Prob.
γ	−1.323437	0.000106	−12524.99	0.0000
α	0.000056	2.00*E* − 08	2802.439	0.0000
θ	0.001778	0.000320	5.558132	0.0000
ϕ	18.18469	0.002778	6545.543	0.0000
σ (Standard error)	0.000135	6.16*E* − 09	21,983.56	0.0000

information criteria of Bai and Ng (2002) to select the optimal number of fundamental factors governing the sentiment indicators. The criterion selected one factor, denoted hereafter by F without subscript. More detail about the related econometric model is provided in Appendix B.

$$\frac{1}{c}\frac{1}{dt}E_t dc = \frac{1}{1-\gamma}\left[\left(-\beta(1+\alpha)+\alpha\frac{c^{\gamma}}{\gamma}\left(e^{(\theta+\phi F(t))}\right)^{\frac{1}{1+\alpha}}\right)+(\nu AK^{\nu-1}-\delta)+\underbrace{\frac{\alpha}{1+\alpha}(\phi F(t)+\theta)}_{\text{Sentiment}}\right]$$
$$+\frac{1}{2}\left\{\underbrace{(2-\gamma)\sigma_c^2(t)}_{\text{Short-run risk}}+\underbrace{\frac{\alpha}{(1+\alpha)^2(\gamma-1)}\phi^2F^2(t)\sigma_F^2(t)-\frac{2\alpha}{1+\alpha}\phi F(t)\sigma_{cF}(t),}_{\text{Long-run risk}}\right\} \tag{20}$$

Estimated coefficients of the structural model are reported in Table 1.

All the parameters are statistically different from zero, including the parameter α related to risk attitudes towards long-run uncertainty. A decomposition of consumption variations incorporates multiple component including sentiment, short-run risk, and long-run risk, as shown in Table 3. To contrast, with a time-additive expected utility, consumption variations are not impacted neither by sentiment component nor by long-run risks.

With a Schroder and Skiadas (1999) parametric recursive utility, the sign of the product of the parameters $\gamma\alpha$ is important for understanding consumers' attitudes towards long-run risk associated with future growth prospects. Occurrence of a negative sign for the product of the parameters $\gamma\alpha < 0$, would mean that consumers prefer an early resolution of uncertainty. And therefore, consumers are averse to long-run risks associated with future growth prospects. As shown in Table 7, the product of the estimated parameters $\gamma\alpha$ is negative and the parameters α and γ are statistically significant. This suggests that consumers prefer early resolution of uncertainty, and therefore are averse to long-run risk associated with uncertainty shocks to future growth prospects. This underscores the importance of the long-run risk channel for understanding the consumption path. Our results echo Bansal and Yaron (2004) and Sargent (2007) who emphasize that economic models incorporating long-run risks have the potential to provide additional channel for understanding dynamic consumers' behavior.

3.7. Descriptive statistics and correlation statistics between sentiment and risk factors

Using the components of the dynamic optimal consumption path (20), we computed the correlations between sentiment, short-run risks, and long-run risks. As shown in Table 2, there is a statistical significant negative correlation between sentiment and long risks.

This suggests that worsening sentiment about future prospects of the economy is linked to growing long-run uncertainty level about the economy. Long-run uncertainty is high when consumers are less confident in the future prospects of the economy. This suggests that ignoring sentiment about future prospects while analyzing long-run risk involved in consumption decisions would not provide the full picture for understanding forces that govern consumption decisions. The statistical significant correlation also suggests a linkage between sentiment

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Table 2. Correlation between sentiment about the future and risk factors

	Sentiment about the future	Short-run risk
Short-run risk	0.066639 (1.36547)	
Long-run risk	−0.632471 (−16.69398)	0.023053 (0.471453)

Note: The *t*-statistics are between brackets.

about the future and nonindifference towards the temporal resolution of future uncertainty. In Appendix C, Table 10, we have provided additional descriptive statistics related to sentiment and short-run and long-run risk factors.

4. Decomposition of the variance of the consumption growth rate: Impact of sentiment, short-run risk, and long-run risk

In what follows, we want to compute the contribution of sentiments about the future, short-run risks, and long-run risks in shaping the optimal consumption path. We use the democratic orthogonalization method that was originally developed by Löwdin (1970) in the Quantum Chemistry literature. The democratic orthogonalization method was recently introduced into the asset pricing and finance literature (Klein and Chow (2013); Bessler et al. (2015)). In this section, we use the democratic orthogonalization method to compute the relative importance of sentiments and risk factors in contributing to variations in consumption.

First, let us rewrite the optimal consumption path equation as follows:

$$\underbrace{\frac{1}{c}\frac{1}{dt}E_t dc}_{\text{Expected consumption growth}} = x_{1,t} + \underbrace{x_{2,t}}_{\text{Sentiment}} + \underbrace{x_{3,t}}_{\text{Long-run risk}} + \underbrace{x_{4,t}}_{\text{Short-run risk}} \tag{21}$$

where

$$x_{1,t} = \frac{1}{1-\gamma}\left[\left(-\beta(1+\alpha)+\alpha\frac{c^{\gamma}}{\gamma}\left(e^{(\theta+\phi F(t))}\right)^{\frac{1}{1+\alpha}}\right)+(\nu AK^{\nu-1}-\delta)\right];\quad t=1,\ldots,T; \tag{22}$$

$$x_{2,t} = \frac{1}{1-\gamma}\left[\frac{\alpha}{1+\alpha}\right](\phi F(t)+\theta);\quad t=1,\ldots,T; \tag{23}$$

$$x_{3,t} = -\frac{1}{2}\left\{\frac{\alpha}{1+\alpha}F(t)\sigma_{cF(t)} - \frac{\alpha}{(1+\alpha)^2(\gamma-1)}\phi^2F^2(t)\sigma_F^2(t)\right\};\quad t=1,\ldots,T; \tag{24}$$

$$x_{4,t} = \frac{1}{2}\left\{(2-\gamma)\sigma_c^2(t)\right\};\quad t=1,\ldots,T; \tag{25}$$

Equation (21) shows that the expected consumption growth is explained by the combined effects of the five factors that show up on the right-hand side. We need to isolate the specific contribution of each of these five factors on the expected consumption growth. If the factors explaining the targeted variable are correlated, the total variance cannot be allocated unambiguously among these explanatory variables, except when there is zero multicollinearity between them. While there are various orthogonalization techniques available, the optimal one that produces the appropriate orthogonal proxies of the original variables is the symmetric procedure of Löwdin (1970) and Schweinler and Wigner (1970). This approach is called democratic orthogonalization in the sense that it is symmetric and egalitarian instead of sequential and asymmetric.[18] The democratic approach minimizes the overall difference between the original explanatory variables and their orthogonal counterparts (Schweinler–Wigner basis) as highlighted by Löwdin (1970) and

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Srivastava (2000). Aiken et al. (1980) argue that the democratic orthogonalization is optimal with respect to all common norms (all the Schatten–von Neumann norms).[19] Moreover, Aiken et al. (1980) show that the minimum distance, in terms of least squares sense, between the original explanatory variables and their orthogonal counterparts is achieved only for democratic orthogonalization. In addition, if the eigenvalues of the correlation matrix are all distinct then these orthogonalized variables are unique.

4.1. Democratic decomposition of Löwdin (1970) applied to equation (21)

In a discrete time setting $t = 1, \ldots, T$, the right-hand side components of equation (21) are the columns of the following matrix

$$X = \begin{pmatrix} x_{1,1} & x_{2,1} & x_{3,1} & x_{4,1} \\ . & . & . & . \\ x_{1,t} & x_{2,t} & x_{3,t} & x_{4,t} \\ . & . & . & . \\ x_{1,T} & x_{2,T} & x_{3,T} & x_{4,T} \end{pmatrix} \tag{26}$$

If the columns of the matrix X are statistically independent or orthogonal, then the ratio of the variance of the ith column to the sum of all the variances of all columns would capture the contribution of that factor in explaining variations in expected consumption growth. But in general, the columns of the matrix X may not be statistically independent or orthogonal. In this case, it is possible to find an orthogonal equivalent of X whose columns are statistically independent or orthogonal, with the diagonal entries being exactly equal to the variances of the components of the columns of X_t. The democratic decomposition method developed by Löwdin (1970) is a statistical method that allows to transform a matrix of correlated variables into an information-equivalent matrix of variables that are noncorrelated. The democratic decomposition allows to isolate the specific contribution of each component, which is important in light of common variation, permitting a clear interpretation of the individual relationships.[20] Applied to our economic framework, the democratic decomposition extracts standalone orthogonal components of sentiment, short-run risk, and long-run risk while maintaining an optimal relationship with the underlining variables. The variances of orthogonal components of sentiment, short-run risk, and long-run risk emerging from the democratic decomposition are identical to those of the original variables. The democratic decomposition ensures that the orthogonal components of sentiment, short-run risk, and long-run risk best resemble the original variables. Using this procedure, the orthogonal components are used to compute the relative contribution of sentiment, short-run risk, and long-run risk in explaining variations in expected consumption growth.

In what follows we give a brief presentation of the democratic orthogonalization method developed by Löwdin (1970). Let us consider the general case where the components of the vector $X_t = (x_{1,t}, x_{2,t}, x_{3,t}, x_{4,t})'$ may display some correlation. In order to analyze the contribution of each component, we need to find an orthogonal equivalent, denoted by $Z_t = (z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t})'$ whose associated covariance matrix is diagonal and the diagonal entries are exactly the variances of the components of the vector X_t.

$$\mathcal{X} = \begin{pmatrix} x_{1,1} & x_{2,1} & x_{3,1} & x_{4,1} \\ . & . & . & . \\ x_{1,t} & x_{2,t} & x_{3,t} & x_{4,t} \\ . & . & . & . \\ x_{1,T} & x_{2,T} & x_{3,T} & x_{4,T} \end{pmatrix} - \begin{pmatrix} \bar{x}_1 & \bar{x}_2 & \bar{x}_3 & \bar{x}_4 \\ . & . & . & . \\ \bar{x}_1 & \bar{x}_2 & \bar{x}_3 & \bar{x}_4 \\ . & . & . & . \\ \bar{x}_1 & \bar{x}_2 & \bar{x}_3 & \bar{x}_4 \end{pmatrix} \tag{27}$$

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Denote by

$$\widehat{\Omega} = \mathrm{Cov}(\mathcal{X})$$

the sample covariance matrix of the components of $\mathcal{X}$. It can be factorized as

$$\widehat{\Omega} = F\Lambda F' \tag{28}$$

$$F'F = I \tag{29}$$

$$\Lambda \text{ is a diagonal matrix.} \tag{30}$$

As shown by Löwdin (1970), the democratically orthogonalized components are given by

$$\mathcal{Z} = Y\Upsilon. \tag{31}$$

where

$$Y = \mathcal{X}F\Lambda^{-\frac{1}{2}}F', \tag{32}$$

$$\Upsilon = \mathrm{Diag}(\widehat{\Omega}) \tag{33}$$

To be clear, the matrix $\Upsilon = \mathrm{Diag}(\widehat{\Omega})$ is the diagonal matrix whose main diagonal is equal to the main diagonal of $\widehat{\Omega}$. Note also that $Y'Y = I$.

The variances of the columns of the matrix (31) resulting from the democratic decomposition are identical to the variances of the columns of the initial matrix of components X [shown in equation (26)].

To estimate the cross-section variances $(s^2_{i,t})_{i=1,2,3,4\ t=1.,,,,T}$ related to the entries of $\mathcal{Z}$, represented by the matrix form as follows

$$s^2 = \begin{pmatrix} s^2_{1,1} & s^2_{2,1} & s^2_{3,1} & s^2_{4,1} \\ . & . & . & . \\ s^2_{1,t} & s^2_{2,t} & s^2_{3,t} & s^2_{4,t} \\ . & . & . & . \\ s^2_{1,T} & s^2_{2,T} & s^2_{3,T} & s^2_{4,T} \end{pmatrix} \tag{34}$$

we used 200 bootstraped $(\mathcal{Z})_{b=1,\ldots,200}$ samples of $\mathcal{Z}$. Using this procedure, the estimated cross-section variances of $\mathcal{Z}$ are used to compute the relative contribution of sentiment, short-run risk, and long-run risk in explaining variations in expected consumption growth as follows:

$$\text{Sentiment impact} = \frac{s^2_{2,t}}{s^2_{1,t} + s^2_{2,t} + s^2_{3,t} + s^2_{4,t}} \qquad t = 1, \ldots, T. \tag{35}$$

$$\text{Long-run risk impact} = \frac{s^2_{3,t}}{s^2_{1,t} + s^2_{2,t} + s^2_{3,t} + s^2_{4,t}} \qquad t = 1, \ldots, T. \tag{36}$$

$$\text{Short-run risk impact} = \frac{s^2_{4,t}}{s^2_{1,t} + s^2_{2,t} + s^2_{3,t} + s^2_{4,t}} \qquad t = 1, \ldots, T. \tag{37}$$

$$\text{Macro risk impact} = \frac{s^2_{3,t} + s^2_{4,t}}{s^2_{1,t} + s^2_{2,t} + s^2_{3,t} + s^2_{4,t}} \qquad t = 1, \ldots, T. \tag{38}$$

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Table 3. Descriptive statistics of the sentiment impact, the short-run risk impact, the long-run risk impact, and the macro-risk impact

Statistics	Sentiment impact	Short-run risk impact	Long-run risk impact	Macro risk impact
Mean	0.153310	0.168851	0.345091	0.513942
Median	0.147796	0.169330	0.343899	0.513645
Maximum	0.18222	0.209251	0.382310	0.546122
Minimum	0.133118	0.128863	0.015312	0.010748
Std. Dev.	0.013654	0.016760	0.0179	0.0113
Skewness	0.556317	−0.303801	0.446747	3.833378
Kurtosis	1.870104	3.190285	2.404871	3.833378

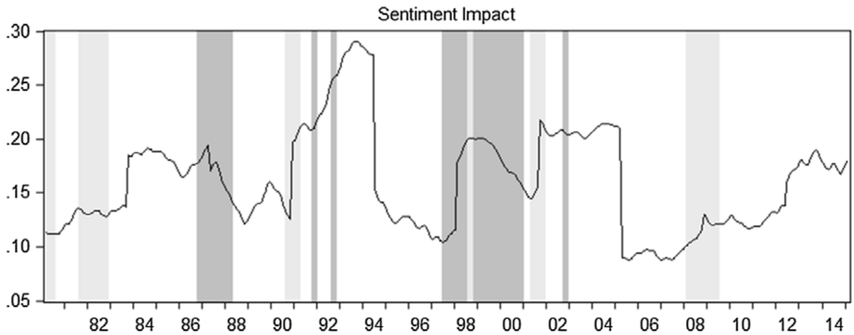

Figure 2. *Sentiment impacts* from 1980 to 2014.

4.2. Descriptive statistics of impact proportions

Table 3 displays the descriptive statistics of the sentiment impact, the short-run risk, the long-run risk, and the macro-risk impact, which is the sum of the short-run risk and long-run risk impacts. Table 3 shows that on average the sentiment component accounts for 15.33% of variations in consumption while macroeconomic risks component account for 51.39%, of which 16.89% pertains to the short-run risk and 34.51% pertains to the long-run risk.

Figure 2 displays the estimations of the contribution of the sentiment variations to changes in the variation of the expected consumption from 1980 to 2014.

The graphic analysis of Figure 2 shows that after economic crises (darker shaded areas) the sentiment impact experiences increases or bumps. The highest bump in sentiment impact occurs after early 1990s economic crisis in the USA while the highest bump in the macro risk occurs after the early 1980s recession. The sentiment impacts from financial crisis (light shaded areas) appear to be lower than the ones that occur during or after nonfinancial crises (dark-shaded areas). Sentiment contributions appear to be high during NBER recessions and low during financial crises.

Figure 3 displays the estimations of the *Macroeconomic risk impacts* from 1980 to 2014. The macroeconomic risk impact varies over time but stays more often greater than 48%. Some bumps in the impact of macroeconomic risk are observed after the early 1980s crises, east Asian crisis (1997), stock market downturn of 2002, and during of after 2007–2009 crises 2009.

Figure 4 displays the estimations of the *long-run risk impacts* from 1980 to 2014. The long-run risk is the dominant component of the macroeconomic risk impact in consumption, staying more often above 30%. The long-risk impact follows a similar pattern as the macroeconomic risk. One important fact to be hilighlighted from the graph is that the long-run risk impacts appear to be leading indicators of NBER recessions and financial crises. That is, the long-run impacts start increasing a few years before crises. Bumps in the long-run risk impacts are observed during

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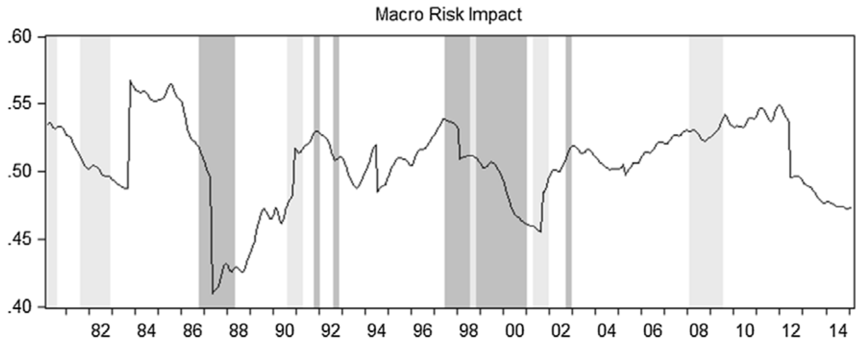

Figure 3. *Macroeconomic risk impacts* from 1980 to 2014.

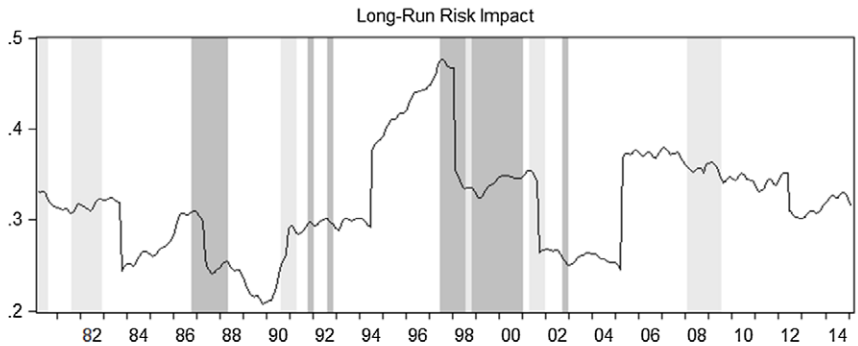

Figure 4 *Long-run risk impacts* from 1980 to 2014.

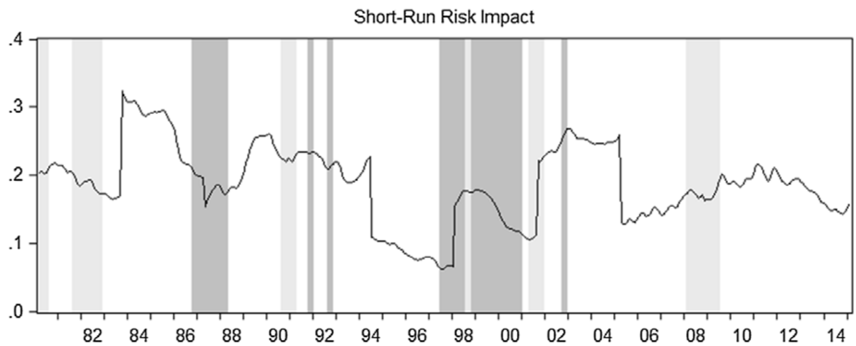

Figure 5. *Short-run risk impacts* from 1980 to 2014.

the following periods: early 1998s crisis, 1991–1992, financial crash of 1987, the east Asian crisis (1997)

Figure 5 below displays the estimations of the *short-run risk impacts* from 1980 to 2014. The short-run risk impact represents on average 15% of changes in consumption. The short-run risk impact displays bumps after most of the crises.

Figures 4 and 5 show that the long-run risk proportion is higher at the start of recession periods while the short-run risk proportion decreases at the start of recession periods. This fact is related to the consumer behavior, as consumer tends to be more pessimistic at the beginning of the recession periods and becomes more worried about the long-run impact of the crisis driving him/her to increase heavily his/her precautionary saving and reduce his/her consumption. At the

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end of the crisis, the conditions become more stable and the consumer becomes more optimistic and the short-run risk becomes more of a concern leading to a decrease in the long-run risk and an increase of the importance of the short-run risk.

4.3. Comparison with a time-additive expected utility model

This section provides an empirical comparison of the recursive utility model with sentiments and that of the time-additive utility without sentiments. Say another way, we address the question of how well the recursive utility model explains data relative to competing for time-additive utility specification. To rank the two competing models, we use the statistical criterion proposed by Akaike (1974), also known as the Akaike information criterion (AIC). The AIC is grounded in information theory. It quantifies the information loss when the true model of the data is not selected.

With a time additive expected utility, which corresponds to the aggregator $f(c, J) = U(c) - \beta J$, where $U(c) = \frac{c^\gamma}{\gamma}$, (which corresponds to the case $\alpha = 0$), the problem of the representative agent is written as

$$\max_{\{c(t):\, t\geq 0\}} E_0\Big[\int_0^\infty e^{-\beta t} U(c(t))dt\Big], \tag{39}$$

subject to:

$$dK(t) = \Big[F(K(t)) - c(t) - \delta K(t)\Big]dt + \sigma(K(t))dB(t), \tag{40}$$

$$c(t) \geq 0, \tag{41}$$

$$K(t) \geq 0, \tag{42}$$

$$K(0) = K_0 > 0. \tag{43}$$

and the optimal consumption path should follow the following rule

$$\frac{1}{c(t)}\frac{1}{dt}E_t dc(t) = \left(\frac{-c(t)u_{cc}(t)}{u_c(t)}\right)^{-1}\left[(F_K(t) - \delta) - \beta\right] - \frac{1}{2}\left[-\frac{c^2(t)u_{ccc}(t)}{c(t)u_{cc}(t)}\right]\sigma_c^2(t), \tag{44}$$

which reduces to

$$\frac{1}{c(t)}\frac{1}{dt}E_t dc(t) = (1-\gamma)^{-1}\left[F_K(t) - \delta - \beta\right] - \frac{1}{2}\left[1-\gamma\right]\sigma_c^2(t), \tag{45}$$

The parameter estimates and statistical tests of the time-additive model are reported in Table 4. For the time-additive utility and recursive models, the parameter of the production function ν is set at 0.36 in line with standard economic research. The subjective discount rate β is set at 4% per annum, which is equivalent to 0.33% per month. The capital depreciation rate, δ, is set at 2.54% per annum, which is equivalent to 0.25% per month. As shown in Table 4, the time-additive expected utility model yields an implausibly large value of $1 - \gamma = 4.09$ for the risk-aversion parameter. The latter issue is related to the so-called *Equity Premium Puzzle*, a term coined by Mehra and Prescott (1985) to describe the improbably high-risk aversion one must have, in the context of standard time additive expected utility, to own risk-free bonds given the immense equity return premium offered by equity markets.

Table 5 reports the measure of specification error given by the AIC information criterion for the recursive utility and the time-additive utility models discussed above. The AIC information criterion can be used to establish the quality of a statistical model for a given set of data (Akaike (1974)). The AIC information criterion then allows models to be compared, with the having the lowest value being preferable for a given set of data.

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Table 4. Parameters estimation with the standard time-additive expected utility model

	Coefficient	Std. error	z-Statistic	Prob.
γ	−3.093131	0.000123	−25212.48	0.0000
σ (Standard error)	0.000570	$1.71E-08$	33, 365.33	0.0000

Table 5. Akaike (1974) information criterion (AIC) for recursive utility and time-additive utiliy

Model	Recursive utility	Time-additive utility
AIC	8.6E + 06	1.56E + 08

We can see that the estimated recursive utility model always displays a smaller AIC value than the time-separable CRRA model. The AIC value for the recursive utility specification is $8.6E+06$, about 94% smaller than that of the time-separable CRRA model.

4.4. Robustness

In this section, we perform a robustness check (sensitivity analysis) by estimating all the parameters except the depreciation rate of capital which is calibrated to different values. The reason behind the calibration of the depreciation rate of capital instead of estimating it is that the discount rate and the depreciation rate of capital are not simultaneously identifiable as can be easily seen from equation (20). Therefore, we choose to calibrate the depreciation rate of capital. We followed the literature to calibrate this rate. This includes Kydland and Prescott (1982) and Greenwood et al. (1988) who used a depreciation rate of 0.1 while Dejong et al. (2000) used a depreciation rate ranging from 0.03 and 0.17 and Gomme and Rupert (2007) used a rate of 0.0391. For the sake of robustness in our analysis, we used the minimum and the maximum of the above rates as well as the median. Hence, the depreciation rate is set to 0.03, 0.17, and 0.1 per annum. Tables 6–8 below give the estimation of the other parameters for the three values of the depreciation rate. We notice from the three tables (Tables 6–8) that the estimates of the other parameters and their significance are barely sensitive to the depreciation rate changes.

4.4.1. Tests of the difference between the consumption growth rate implied by equation (20) (recursive utility) and the one implied by equation (45) (standard time-additive utility)

To test whether *the model with recursive utility is still quantitatively different from the one with time additive utility even if the parameter α looks small in the aggregator*, we computed the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*) and conducted the t-test for the equality of the means, the Wilcoxon/Mann–Whitney test for the equality of the medians and the F-test for equality of variances.

Table 9 shows that the three tests strongly reject the null hypothesis as the p-values are below any conventional significance level.

This result is enforced by the kernel nonparametric density estimation (Figure 6) of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*).

If there were no difference between the two models, the density would be degenerate at zero, with the entire mass concentrated at zero. On the contrary, the graph shows that the density is not degenerate and most of the mass is located away from zero. The results displayed in Appendix D (Tables 11 and 12) show that for two other calibrated values of the depreciation rate, $\delta = 0.1$ and $\delta = 0.17$, a similar feature is obtained.

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Table 6. Estimation of preference parameters of the model equation (20) when $\delta = 0.17$

$\delta = 0.17$	Coefficient	Std. error	*z*-Statistic	Prob.
γ	−1.5381	0.0002	−7277.6640	0.0000
θ	0.0291	0.0004	81.7542	0.0000
ϕ	17.8524	0.0029	6060.0360	0.0000
β	0.0035	0.0000	41,820.8200	0.0000
α	0.0001	0.0000	1238.8580	0.0000
ν	0.3200	0.0000	4,335,724.0000	0.0000
σ (Standard error)	0.0001	0.0000	11,989.9100	0.0000

Table 7. Estimation of preference parameters of the model equation (20) when $\delta = 0.1$

$\delta = 0.1$	Coefficient	Std. error	*z*-Statistic	Prob.
γ	−1.5344	0.0002	−7030.5710	0.0000
θ	0.0241	0.0004	66.1571	0.0000
ϕ	17.8227	0.0030	5895.4020	0.0000
β	0.0035	0.0000	40,073.8800	0.0000
α	0.0001	0.0000	1205.9700	0.0000
ν	0.3200	0.0000	4,403,892.0000	0.0000
σ (Standard error)	0.0001	0.0000	11,597.4700	0.0000

Table 8. Estimation of preference parameters of the model equation (20) when $\delta = 0.03$

$\delta = 0.03$	Coefficient	Std. error	*z*-Statistic	Prob.
γ	−1.5357	0.0002	−7499.3570	0.0000
θ	0.0277	0.0003	80.2423	0.0000
ϕ	17.8565	0.0029	6250.4420	0.0000
β	0.0035	0.0000	40,835.4500	0.0000
α	0.0001	0.0000	1279.3410	0.0000
ν	0.3200	0.0000	4,414,626.0000	0.0000
σ (Standard error)	0.0001	0.0000	12,363.3400	0.0000

Table 9. Tests of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*), when $\delta = 0.03$

$\delta = 0.03$	Test	Value
Mean equality test	Student-*t*	–42.7333
	p-Value	0.0000
Median equality test	Wilcoxon/Mann–Whitney	25.0247
	p-Value	0.0000
Variance equality test	Fisher-*F*	8.6188
	p-Value	0.0000

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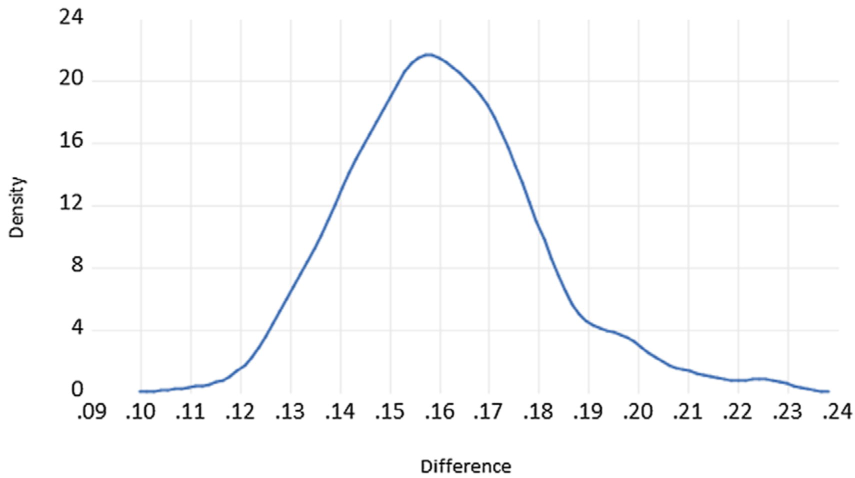

Figure 6. Density distribution of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*), when $\delta = 0.03$. For two other alternative values, see Figures 7 and 8.

5. Concluding remarks

In this paper, we have applied a latent factor method on a database of consumer sentiment and expectation indicators and a democratic decomposition technique to quantify the effects of both sentiment about the future and long-run risk in shaping optimal consumption decisions in a dynamic recursive utility framework in assessing investor strategies in a data-rich environment. The latent factor method seizes, in one common factor, the most relevant information on both deterministic and stochastic changes available in a large number of sentiment and expectations indicators. Therefore, the time series of one estimated latent factor are an approximative representation of current and future expected changes in the state of the economy. This economic factor contains useful information for the sentiment about the future and long-run risks components of the future consumption path. We find that sentiment about the future, short-run risk and long-run risk have a significant impact on optimal consumption decisions over time, whilst with a standard time additive utility neither sentiment nor long-run risk have an impact. In a recursive utility framework, we are able to find empirical support that endogenous consumption variations are driven by a multi-component mechanism, where on average the sentiment component accounts for 15.33%, the short-run risk accounts for 16.89%, and the long-run risk pertains to 34.51%. Estimation of risk preference parameters reveals that consumers prefer early resolution of uncertainty, and therefore are averse to long-run risks associated with uncertainty shocks to future growth prospects. This underscores the importance of the long-run risk channel for understanding the consumption path. Our results echo Bansal and Yaron (2004); Sargent (2007) who emphasize that economic models incorporating long-run risks have the potential to provide additional channel for understanding dynamic consumers' behavior. This evidence is encouraging and shows the usefulness and importance of using recursive utility frameworks for improving our understanding of the different channels that are underlying shapers of consumption decisions at the macroeconomics aggregate level.

The sentiment and risk decomposition of the consumption decisions and the empirical techniques used in this model more generally are potentially useful for other macroeconomic dynamic models with recursive utility function focusing on fiscal or monetary policy issues. In addition to long-run risks, accounting for sentiment about the future as a driver of economic decisions is consistent with forward-looking behavior (Acemoglu and Scott (1994)). Future developments of the

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role played by sentiments and long-run risk could include the situation of heterogeneous agent models with regards to both risk attitudes and sentiment about the future.

Notes

1 Other indicators include the Bank of England Systematic Risk Survey and The Bloomberg Consumer Comfort Index.
2 Other papers that have incorporated the role of sentiment in a recursive utility framework include Hansen (2010, 2012).
3 On a different note, there is an alternative strand of literature that looks at sentiments in frameworks that do not incorporate recursive preferences in optimal decisions or issues related to long-run risks (Bloom (2009); Angeletos and La'O (2013); Miao et al. (2015); Milani (2017)).
4 A decision maker may prefer a late resolution of uncertainty or an earlier resolution of uncertainty as a result of his/her attitudes towards the correlation of payoffs across periods—long-run uncertainty (Duffie and Epstein (1992)).
5 This statistical technique was first used in the Quantum Chemistry literature, and recently it has found application in asset pricing and finance (Klein and Chow (2013); Bessler et al. (2015)).
6 It is worth noting that the rate of time preference is constant, that is $-f_J(c, J) = \beta$.
7 When $\gamma = 0$, this aggregator becomes $f(x, J) = (1 + \alpha J)\left[log(x) - \frac{\beta}{\alpha} log(1 + \alpha J)\right]$.
8 There is a connection between preferences for the timing of resolution of uncertainty and preferences for information (Skiadas (1998)).
9 The estimated factors are mutually orthogonal by construction.
10 To be clear, we use a two-step method of estimation. In the first step, we estimate the factors (sentiment factors). In the second and final step, the latent factor variables are fixed at their estimated values from the first step, so that only the preferences parameters and value function of the structural model are estimated in the second step. In doing so the value function is endogenously estimated as a function of latent factors.
11 Data on real personal consumption expenditures of services and nondurable goods are from St. Louis Federal Reserve Bank website: https://fred.stlouisfed.org/series/PCESC96 https://fred.stlouisfed.org/series/PCENDC96 https://fred.stlouisfed.org/series/RKNANPUSA666NRUG
12 Data on civilian population are from St. Louis Federal Reserve Bank website: https://fred.stlouisfed.org/series/CNP16OV
13 https://data.sca.isr.umich.edu/subset/subset.php
14 See For more information, see Penn World Table 8.1 at https://www.rug.nl/ggdc/productivity/pwt/pwt-releases/pwt8.1?lang=en
15 https://www.dallasfed.org/institute/~/media/documents/institute/wpapers/2011/0099.pdf
16 The database is available at: https://data.sca.isr.umich.edu/subset/subset.php
17 In estimating the latent factors, we used an index instead of one subindex for sentiment because the latter will represent only a subgroup of elements affecting the consumer sentiment. In the survey, there are indicators related to consumer perceptions about his/her financial conditions and indicators related to consumer perceptions about the global business conditions in the country and the world. Furthermore, there are also perception indicators about the business conditions in the short and long run. All these indicators, when taken separately, affect sentiment differently. Considering a subindex may miss some essential features affecting sentiment about future prospects of the economy. Estimating a common factor that combines all these indicators is the only fundamental that better captures the actual state of the consumer sentiment.
18 The sequential approach assumes some (arbitrary) ordering of the explanatory variables where the first variable remains unchanged, and the remaining variables are selected sequentially. In contrast, the democratic orthogonalization approach treats all variables symmetrically.
19 The Frobenius norm is an example of these norms.
20 The democratic decomposition in contrasts to the sequential orthogonalization approach is independent of the ordering of the variables. It considers the entire set of vector data in one go.

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Appendix A: Sentiment latent factor optimization problem

The basic factor model can be expressed in a matrix form as

$$Z = F\Lambda' + E, \tag{A1}$$

where Z denotes the observed matrix of time series, F is a $T \times r$ matrix of r unobservable common factors, Λ is a $N \times r$ matrix of factor loadings and E is $T \times N$ matrix of idiosyncratic errors that are uncorrelated with the components of F.

We particularly focus on the case where a high-dimensional panel of N time series depends on a relatively small number of r common latent factors. The estimates of the latent factors are obtained by solving the following minimization program

$$\min_{F,\Lambda} \mathrm{Trace} \frac{(Z - F\Lambda')(Z - F\Lambda')'}{NT} \tag{A2}$$

subject to

$$\frac{F'F}{T} = I_r, \tag{A3}$$

where I_r is r a dimensional identity matrix. Because $F\Lambda' = FAA^{-1}\Lambda' = F^*\Lambda^{*\prime}$ for any invertible $r \times r$ matrix A, the factors and the factor loadings are not jointly identified. Thus, the normalization (48) is an identification constraint. It can be shown that each estimated factors $\widetilde{F}$ are the eigenvectors (multiplied by $\sqrt{T}$) associated with the largest eigenvalues of the matrix $\frac{ZZ'}{TN}$. A convergence result by Bai and Ng (2002) says that as both T and N grow to infinity, the estimated latent variables $(\widetilde{F})$ converge to their true scaled counterpart (F).

To select the number of factors, we use the following criterion (see Bai and Ng (2002))

$$IC_{p2}(r) = \ln\left(\frac{\mathrm{trace}\left(\widehat{E}\widehat{E}'\right)}{NT}\right) + r\frac{N+T}{NT}\ln\left(C_{NT}^2\right)$$

where $\hat{E}$ is the matrix of errors in the sentiment factor model and $C_{NT} = \min(\sqrt{N}, \sqrt{T})$ and $\widehat{E} = Z - \widehat{F}\widehat{\Lambda}'$.

Appendix B: The estimation of the model parameters

Our approach uses a latent factor approximation of the value function which takes the following functional form.

$$J(t) = e^{\left(\theta + \sum_{i=1}^{L} \phi_i F_i(t)\right)}. \tag{B1}$$

The econometric model for estimating equation (20) is given by

$$\begin{aligned}\frac{\Delta c_t}{c_t} = \frac{1}{1-\gamma}\Bigg[&\left(-\beta(1+\alpha) + \alpha\frac{c_{t-1}}{\gamma}\left(e^{\phi F_t + \theta}\right)^{\frac{1}{1+\alpha}}\right) + \left(\nu A_{t-1}K_{t-1}^{\nu-1} - \delta\right)\\ &+ \frac{\alpha}{1+\alpha}\left(\sum_{i=1}^{L}\phi_i F_{i,t} + \theta\right)\Bigg]\\ &+ \frac{1}{2}\left[(2+\gamma)\,\sigma_{c,t}^2 + \frac{\alpha}{(1+\alpha)^2(\gamma-1)}\sum_{i=1}^{L}\phi_i^2\sigma_{F_i,t}^2 - \frac{2\alpha}{1+\alpha}\sum_{i=1}^{L}\phi_i\sigma_{cF_i,t}\right] + \varepsilon_{c,t}\end{aligned} \tag{B2}$$

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where $\varepsilon_{c,t}$ is the regression error term or the unexpected consumption growth. We used the information criteria of Bai and Ng (2002) to select the optimal number of fundamental factors governing the sentiment indicators. The criterion selected one factor (see Appendix A for details).

The conditional mean of the factor F_t is modeled as an ARMA(p, q). That is

$$F_t = \tau + \sum_{j=1}^{p} \psi_j F_{t-j} + \sum_{k=1}^{q} \rho_k \varepsilon_{F,t-k} + \varepsilon_{F,t} \tag{B3}$$

where $\varepsilon_{F,t}$ is the regression error term or the unexpected part of the sentiment factor. Following the literature in time series, we optimaly selected the lags p and q using the Schwart criterion.

The variance–covariance matrix of the regression errors is given by

$$\Omega_t = \begin{pmatrix} \sigma^2_{c,t} & \sigma_{cF,t} \\ \sigma_{cF_i,t} & \sigma^2_{F,t} \end{pmatrix}$$

We follow Bollerslev et al. (1988) who proposed a general model for estimating the conditional covariance matrix Ω_t termed VEC model. The VEC(1,1) model is given by

$$h_t = \Gamma + A\xi_{t-1} + Bh_{t-1}$$

where

$$h_t = \text{vech}(\Omega_t) = \begin{pmatrix} \sigma^2_{c,t} \\ \sigma_{cF,t} \\ \sigma^2_{F,t} \end{pmatrix},$$

$$\xi_t = \text{vech}\left(\varepsilon_t \varepsilon_t'\right),$$

where $\varepsilon_t = (\varepsilon_{c,t}, \varepsilon_{F,t})'$, vech(.) is an operator that that stacks the lower triangular part of a $N \times N$ matrix, and the sizes of Γ, A and B are $\frac{N(N+1)}{2}$ dimensional constant vectors. The total number of parameters is $2\left(\frac{N(N+1)}{2}\right)^2 + \frac{N(N+1)}{2}$. In our case with $N = 2$, the total number of parameters is 21.

To reduce the number of parameters in the VEC model, Bollerslev et al. (1988) suggest the diagonal VEC (DVEC) model where A and B are restricted to be diagonal matrices. The resulting restricted model comprehend $\frac{3N(N+1)}{2}$ parameters. That is for $N = 2$, the total number of parameters is 9.

The DVEC model can be rewritten as

$$\Omega_t = \overline{\Gamma} + \overline{A} \odot \left(\varepsilon_{t-1}\varepsilon_{t-1}'\right) + \overline{B} \odot \Omega_{t-1}$$

where $\odot$ is the Hadamard product (element by element product) and $\overline{\Gamma}, \overline{A}$ and $\overline{B}$ are $N \times N$ matrices given by

$$\Gamma = \text{diag}(\text{vech}(\overline{\Gamma})),$$

$$A = \text{diag}(\text{vech}(\overline{A})),$$

$$B = \text{diag}(\text{vech}(\overline{B})),$$

where $\overline{A}$ and $\overline{B}$ are diagonal matrices.

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We estimate the model by quasi-maximum likelihood estimator. We use bootstrapped t-statistics and p-values for coefficients significance.

Appendix C: Descriptive statistics of sentiment and risk factors

Table 10. Descriptive statistics of sentiment and risk factors

	Sentiment	Short-run risk	Long-run risk
Mean	0.000171	0.000018	0.000213
Median	0.000176	0.000014	0.000210
Maximum	0.000228	0.000235	0.000343
Minimum	0.000101	0.000011	0.000154
Std. dev.	0.000027	0.000016	0.000040
Skewness	−0.281217	8.477768	0.797648
Kurtosis	2.519434	101.650500	3.208542

Appendix D: Robustness tests for calibrated depreciation rates: $\delta = 0.1$ OR $\delta = 0.17$

Table 11. Tests of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*), when $\delta = 0.1$

when $\delta = 0.1$	Test	Value
Mean equality test	Student-*t*	−42.3488
	p-Value	0.0000
Median equality test	Wilcoxon/Mann–Whitney	25.0249
	p-Value	0.0000
Variance equality test	Fisher-*F*	8.6219
	p-Value	0.0000

Table 12. Tests of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*), when $\delta = 0.17$

$\delta = 0.17$	Test	Value
Mean equality test	Student-*t*	−41.9644
	p-Value	0.0000
Median equality test	Wilcoxon/Mann–Whitney	25.0249
	p-Value	0.0000
Variance equality test	Fisher-*F*	8.6250
	p-Value	0.0000

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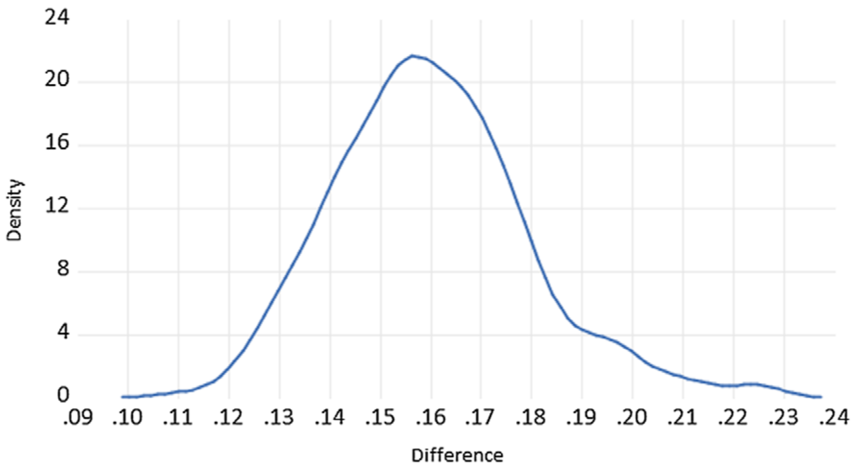

Figure 7. Density distribution of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*), when $\delta = 0.1$.

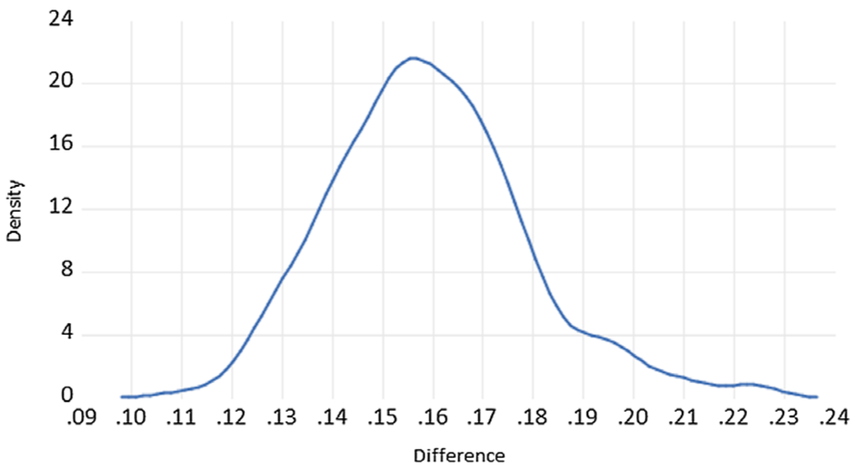

Figure 8. Density distribution of the difference between the consumption growth rate implied by equation (20) (*recursive utility*) and the one implied by equation (45) (*time additive utility*), when $\delta = 0.17$.

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