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On the Mathieu groups M_{22} and M_{11}

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In the list of known finite non-abelian simple groups there are infinitely many pairs of non-isomorphic simple groups which have the same order. It should therefore be of interest to investigate the following general problem:

"Are there any non-isomorphic non-abelian simple groups of the same order, other than the ones already known?"

Although this problem is very difficult in the general case some answers have been given for particular orders. For example, the Mathieu groups M_{24} and M_{12} (R.G. Stanton, Ph.D. thesis, University of Toronto 1948) and M_{23} (N. Bryce, Ph.D. thesis, Monash University, 1969) have been shown to be uniquely determined by their order and the assumption of simplicity. In this thesis we consider the final two simple groups of Mathieu, and prove

THEOREM A. *Let* G *be a finite non-abelian simple group of the order of* M_{22} *; that is, of order* 443,520 . *Then* G *is isomorphic to* M_{22} .

THEOREM B. *Let* G *be a finite non-abelian simple group of the order of* M_{11} *; that is, of order* 7920 . *Then* G *is isomorphic to* M_{11} .

We give below a brief outline of the proof of Theorem A.

Let G be a simple group of the order of M_{22} . The Sylow p-normalizers (p odd) of G are determined by applying certain results about ordinary group characters which are derived from the theory of modular representations. (Of particular importance are the results of R. Brauer on blocks of defect one and on groups divisible by a prime to the

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first power.) The proof is completed by determining the centralizer of a central involution and then using the characterization of M_{22} by D. Held (*J. Algebra* 8 (1968), 436-449).

The proof of Theorem B is very similar, except in this case we use the characterization of M_{11} by W.J. Wong (*J. Austral. Math. Soc.* 4 (1964), 90-112).