and KO′K′ be normal to AD at O′. The critical angle for glass being 41° 45′ approximately,

the $\angle N'OO' \not> 41°45'$;

$\therefore$ the $\angle OO'K' \not< 48°15'$;

i.e. the $\angle OO'K' > 41°45'$;

$\therefore$ the ray OO′ will not emerge through AD, but will be reflected in the direction O′Q, and will emerge at Q in the direction QS.

W. A. Lindsay

Note on Fermat's Theorem.—The Theorem that $a^n - a$ is exactly divisible by n, if n be a prime number, may be established as follows from the Binomial Theorem.

We have

$$(a+b)^n = a^n + C_n{}^1 a^{n-1}b + C_n{}^2 a^{n-2}b^2 + \ldots + C_n{}^r a^{n-r}b^r + \ldots + b^n,$$

where $$C_n{}^r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{\underline{|r}};$$

$$\therefore\ (a+b)^n - a^n - b^n = C_n{}^1 a^{n-1}b + C_n{}^2 a^{n-2}b^2 + \ldots + C_n{}^r a^{n-r}b^r + \ldots + C_n{}^{n-1} ab^{n-1}. \quad (1)$$

Now since $C_n{}^r$ is an integer, the product

$$n(n-1)(n-2)\ldots(n-r+1)$$

must contain as a factor the product $1.2.3\ldots r$; but if n is a prime number it cannot contain as a factor any one of the integers $2, 3, 4, \ldots r$, each of which is $< n$;

$\therefore$ the product $\underline{|r}$ must be contained in $(n-1)(n-2)\ldots(n-r+1)$, and $\therefore$ n is a factor of $C_n{}^r$.

This is true for all values of r from 1 to $n-1$.

It follows from (1) that $(a+b)^n - a^n - b^n$ is exactly divisible by n.

Hence $(a+1)^n - a^n - 1$ is exactly divisible by n. (2)

i.e. $(a+1)^n - (a+1) - (a^n - a)$ is exactly divisible by n; which shows that if $a^n - a$ is exactly divisible by n so will $(a+1)^n - (a+1)$.

Now from (2) it follows that $2^n - 2$ is exactly divisible by n, (putting $a = 1$);

$\therefore$ $3^n - 3$ is exactly divisible by n,

$\therefore$ $4^n - 4$ is exactly divisible by n, and so on.

We might also reason as follows:—

To show that $a^n - a$ is exactly divisible by n, let $a = y + 1$.

From (2) above $(y+1)^n - y^n - 1$ is exactly divisible by n;
i.e. $(\overline{y-1+1}+1)^n - (\overline{y-1}+1)^n - 1$ is exactly divisible by n;
but $(\overline{y-1}+1)^n - \overline{y-1}^n - 1$ is exactly divisible by n (from (2));
$\therefore$ the sum of these two expressions is exactly divisible by n;
i.e. $(y-1+1+1)^n - (y-1)^n - 2$ is exactly divisible by n;
i.e. $(y-1+2)^n - (y-1)^n - 2$ is exactly divisible by n.

In the same way, by putting for $y-1$, $y-2+1$, we deduce that $(y-2+3)^n - (y-2)^n - 3$ is exactly divisible by n, and so on; deducing ultimately that $(y-\overline{y-1}+y)^n - (y-\overline{y-1})^n - y$,
i.e. $(1+y)^n - 1^n - y$,
i.e. $a^n - a$ is exactly divisible by n.

W. A. Lindsay

Fermat's Theorem deduced from the theory of circulating Radix Fractions:

$\dfrac{x^n-1}{n+1}$ is an integer, if x is an integer, and $n+1$ is a prime integer which is not a factor of x.

This is not a neat proof of Fermat's theorem, but, as far as I know, it is a new proof, and it may have some little interest, as it seems very possible that the theorem may have been suggested to Fermat in this way. Fermat published the theorem without any demonstration and without indicating what had suggested it, and any proofs, that I have seen, give no indication why one should look for such a theorem, and would very possibly never have been given if the theorem had not been already known.

It was only after I had written this proof that Professor Chrystal pointed out to me that it was a known theorem.

The conversion of recurring decimals into vulgar fractions led me to try to prove that $10^n - 1$ was divisible by $n+1$, $(n+1)$ being a prime integer to which 10 is prime.

e.g. $\frac{1}{7} = \cdot\dot{1}4285\dot{7} = \frac{142857}{999999}$.

$\therefore$ 999999 or $10^6 - 1$ is divisible by 7.

$\frac{1}{13} = \cdot\dot{0}7692\dot{3}$

$\therefore$ $10^6 - 1$ is divisible by 13.

$\therefore$ $10^{12} - 1$ is divisible by 13.

After proving this, I noticed that the theorem held more generally for any scale of notation x.