ON PRECESSION AND NUTATION VALUES

R. O. VICENTE
Dept. of Applied Mathematics
Faculty of Sciences
1294 Lisbon
Portugal

ABSTRACT. The results of research on the Earth's theory of rotation are presented, comparing observed and theoretical values specially for the luni-solar precession and nutation. Investigations of geodynamics show the strong dependence of the nutation values on the adopted model for the structure of the Earth, while the values referring to the precession are not affected as much. The new observational techniques are able to determine some of these values with greater precision than the classical optical techniques, but it is essential that the observations should be done in a systematic and regular way for several decades.

1. Introduction

The theory of the rotation of the Earth around its center of mass is fundamental for studies dealing with reference systems (terrestrial or celestial). For these purposes, the theories of luni-solar precession and nutation play a fundamental role. The definition of astronomical time is also closely related to these theories.

The first attempt to define a system of astronomical constants was done by Newcomb (1895), and the work is remarkable for its insight into the problems that appear when we try to define a consistent system of astronomical and geodynamical constants. The theories of precession and nutation developed during the last century were based on the hypothesis that the Earth was a solid and rigid body. The first observational evidence that the Earth behaved as an elastic body was obtained from the remarkable series of the variation of latitude showing that the period of the variation of latitude (free Eulerian nutation) was around 14 months, and not 10 months as predicted by Euler's theory. The qualitative explanation of the disagreement between theory and observation was given by Newcomb (1892) after a lengthy debate between observers and theoreticians.

The explanation of this disagreement between theory and observation is one of the known examples, in the history of science, of the lack of understanding of the limitations of any physical theory, in this case, the Earth's rotation theory. Any theory can only deal with a limited number of physical variables that will try to describe the phenomena in such a way that a model can be constructed. This model will be described by a set of equations of mathematical physics which can be integrated so that we can explain and, if possible, forecast the behavior of the Earth.

Woolard's (1953) rigid-body theory for the Earth was an improvement on previous theories, and it was incorporated in the IAU system of astronomical constants at that time. Another improvement, along the same lines, was done by Kinoshita (1977) and, more recently, by Kinoshita and Souchay (1990). The advent of technologies, employing artificial satellites and quasars, gives us the possibility of determining astronomical coordinates with greater precision than the classical optical techniques. If we are aiming at precision of 1 mas (0".001), we have to be extremely careful about our definitions of models, constants, and reference systems.

2. Observed values of the nutations

The values adopted for the main term of the nutation in obliquity, the so-called constant of nutation , during the last hundred years, were based on the observed values while the following terms (semiannual and fortnightly) were based on theoretical values. This approach is, of course, inconsistent from a logical point of view but it was justified for two main reasons:

1. The main term, due to its importance, was derived from observations because in any transformation of coordinates we should have the best possible value available in order to minimize the errors; the observed value, corresponding to the real structure of the Earth, satisfied that requirement because the theoretical value corresponds to an imperfect theory.
2. The semiannual and fortnightly nutations were derived from theory because their magnitudes were so small that it was very difficult to determine reliable values from the classical optical observations.

Let us describe briefly the history of this subject to see not only the limitations imposed by the theory of forced precession and nutation, but also to avoid the mistakes made in the past. Some of the observed values of the main term of the nutation in obliquity (N), determined from classical optical observations, are shown in Table I.

TABLE I

AUTHOR	INTERVAL OF THE OBSERVATIONS	N
Newcomb(1895)	old observations of greater confidence	9".210±0".008
Spencer Jones (1939)	1911-1936	9".2066±0".0055
Morgan(1943)	1903-1925	9".206±0".007
Fedorov(1958)	1900-1934	9".1980±0".0018

The great difficulty in determining this nutation from the observations stems from the fact that it has a long period of about 19 years, and, therefore, the observations should cover several periods in agreement with the best principles of statistical analysis. Unfortunately, the great majority of observations, so far done, do not satisfy this fundamental requirement.

Let us comment briefly on the values of N listed in Table I. The value determined by Newcomb was based on observations made during several periods employing different observational techniques, but it has the inconvenience common to all observations made more than a century ago, that is, the low precision of the techniques employed at that time. The values determined by Morgan (1943) and Spencer Jones (1939) correspond to 22 and 25 years of observations and, therefore, are not as reliable. The value obtained by Fedorov (1958) covers a period of 34 years, which is therefore better than any other values but, again, not reliable enough from a proper statistical point of view.

The short period nutations, semiannual and fortnightly, have been difficult to determine from the classical optical observations because of their smaller amplitudes. All these values suffered from the fact that the instruments were localized at only one observatory or that they were determined from the International Latitude Service (ILS) chain of instruments situated at the same latitude. Nevertheless, the results were remarkable and we must remember that, in those days, we had scanty knowledge about geodynamics, namely, the behavior of the core and plate tectonics.

The advent of the modern technique of Very Long Baseline Interferometry (VLBI) has opened the possibility to determine the values of nutation from this type of observation. Here we have to distinguish the case of the short period nutations from the main nutation. So far, the VLBI observations (Herring et al. 1986) suffer from the same difficulties we have already pointed out for the classical observations, that is, very few instruments and the localizations of the observatories are not the best from the point of view of global tectonics and the stability of the sites.

We know very well the need for regular and systematic observations and, therefore, the more reliable VLBI observations are the ones corresponding to the International Radio Interferometric Surveying (IRIS) network which started around 1984. This short interval of observations conditions immediately the determination of the values of the two types of nutation.

1. It rules out the possibility of determining the main nutation, with a period of about 19 years, or of any other long period nutation. This is in agreement with the proper use of statistical techniques. A number of research papers have been published, dealing with the determination of the main nutation term, but the results cannot be trusted for the above mentioned reasons.
2. The case of the short period nutations, namely, the semiannual and fortnightly, is slightly better because the observations of the IRIS network already cover several periods. The difficulty with these nutations is that their amplitudes are very small and, therefore, the need for a well-distributed network of observatories is very important. This last condition is not yet satisfied by the IRIS network and, therefore, the results so far obtained should be considered with great care.

3. Comparison of theoretical and observed values of the nutations

Let us consider briefly the history of the comparison between theoretical and observed values, and the relationships between luni-solar precession and nutation. The research of Hill (1893) gave for the theoretical expressions of the constants of luni-solar precession P and nutation N the following :

$$P = \frac{C - A}{C}\left(\alpha_1 + \alpha_2 \frac{\mu}{1+\mu}\right), \qquad N = \frac{C - A}{C}\,\alpha_3 \,\frac{\mu}{1+\mu} \qquad (1)$$

where A and C are the principal moments of inertia of the Earth, μ is the ratio of the mass of the Moon to that of the Earth, and α_1, α_2, and α_3 are accurately known constants for a certain epoch. These expressions show the close relationships between precession and nutation which is logical because they correspond to complementary parts of the theory of the Earth's rotation.

The rigid body theory developed by Woolard (1953) employed Newcomb's value of the constant of nutation (9".2100) for the determination of the amount of the other nutations, for instance, the semiannual (0".5522) and fortnightly (0".0884) in obliquity.

The several comparisons of the observed and theoretical values of the nutations, done, for instance, between 1930 and 1950, showed that the theoretical values were always different from the observed ones. One of the first scientists to point out this discrepancy was Jackson (1930). The explanation of this disagreement was one of the outstanding difficulties of the system of astronomical constants. The approach of Herring (1988) to derive observational corrections to the principal nutations from VLBI observations is not valid for all the reasons already mentioned. Incidentally, We should notice that most VLBI results are derived employing the same computer program, for instance "CALC", and, therefore, the final values are similar.

Another comparison has been obtained by McCarthy and Luzum (1991) employing VLBI data and Lunar Laser Ranging (LLR) data. In spite of the fact that the VLBI series is not yet of sufficient duration, the combination with the longer LLR series improves the results. The authors use the correct procedure of employing the adopted IAU theory of nutation (Seidelmann 1982) and determining corrections for the principal nutation terms. In this way we can improve the knowledge of the nutation coefficients from the observations which represent the behavior of the real earth.

Recent research on the comparison of catalogues of extragalactic radio sources employed in VLBI observations (Walter 1991), point out that the nutation models applied to the reduction for the original measurements have not been used in a consistent way. This is obviously an additional cause of concern.

4. Influence of the Earth's structure on precession and nutation

The theory of the Earth's rotation up to the time of Woolard (1953) was based on models of the Earth considered as a solid and rigid body. One of the reasons for admitting this hypothesis was the lack of detailed knowledge about the internal structure of our planet. The advances made in internal geophysics, especially based on seismological research, permitted the determination of the main parameters for all the layers which were considered significant for defining the internal structure (Bullen 1963).

One of the main features of the structure of the Earth is the existence of a core divided in two regions - outer and inner core. The outer core is considered as fluid while the inner core is considered as solid; the expressions fluid and solid for the core have seismological meaning, referring to the behavior of the core when seismic waves propagate through it. We do not yet have the possibility of reproducing in our laboratories the conditions of pressure and temperature which exist inside the core. The importance of a fluid core on studies dealing with precession and nutation of rotating bodies have been pointed out in some classical papers (Hough 1895, Poincaré 1910). Unfortunately, all these studies, when applied to the case of the Earth, only considered the case of a rigid Earth, and, therefore, they were not very adequate models because they ignored the existence of elasticity. The research of Jeffreys' (1949, 1950) emphasized the importance of the liquid core.

The theory of nutation that first considered the existence of a liquid core and the elasticity of the mantle was developed by Jeffreys and Vicente (1957). Another important feature of this theory refers to the relevance of the ellipticity of the core for the values of certain types of nutation. This theory also pointed out that the external forces (due to the Sun and Moon) which cause the forced nutation also give rise to tidal attractions, deforming the Earth, and corresponding to the tides of the solid Earth. For instance, the forced nutations correspond to diurnal tides. This theory shows that, because of the ellipticity of the core, there is a great difference between the displacements that alter the direction of the principal axis and other displacements that do not affect its position. All diurnal tides tend to alter the position of the axis of rotation and, therefore, apply to the forced nutation.

We must remember that in the dynamics of rotating bodies we only have nutation, that is, oscillations of the axis of the body around a main axis considered as fixed. The fact that some of the nutations present special features led to their classification as precession. In the case of the Earth, the luni-solar precession corresponds to a nutation with a period of about 26,000 years and that implies certain features for this motion.

The solution of the equations of motion corresponding to the theory of the Earth's rotation, adopting a Lagrangian formulation, that is, in terms of displacements, reveals that the roots are grouped in pairs near certain values. This is an important feature in this theory, producing a phenomenon which was called "double resonance" by Poincaré (1910). We have to consider two cases.

1. If there is only resonance, that is, only one root very near to the period of one of the free oscillations of the system, the system behaves nearly as a solid body and the fluid core has no influence on the period of the oscillation.
2. If there is double resonance, that is, a pair of roots very near to the period of one of the free oscillations of the system, the influence of the fluid core becomes important and the amplitude of the nutation will be different in comparison with a solid body.

Case 1 applies to the luni-solar precession and, therefore, the existence of the fluid core does not affect the values obtained for the precession because the Earth behaves as a solid body. Case 2 applies to the forced nutations, namely, having periods of 6798.4, 182.6 and 13.7 days. The existence of elasticity does not alter the conclusions referring to double resonance. The fundamental difference between Case 1 (resonance - valid for the luni-solar precession) and Case 2 (double resonance - valid for the luni-solar nutation) shows that it is far more difficult to consider suitable models of the Earth's structure for the theory of nutation than for the theory of precession (Vicente, 1964).

The more recent theory of nutation was developed by Wahr (1981) and adopted by the International Earth Rotation Service (IERS) Standards. The

Earth's model adopted is not the best one available at the moment but the differences will not probably be significant if another model were adopted. For the question of consistency, it would be convenient to consider the Preliminary Reference Earth Model (PREM) which has been widely adopted by the international community (Dziewonski and Anderson, 1981). The adoption of the PREM model will not introduce very significant differences in agreement with Déhant (1990, Table 4) and the same happens considering the mantle inelasticity (Déhant, 1990, Table 3) when we consider precision of 1 mas (0".001).

There are a number of studies (Wahr and Bergen, 1986; Déhant 1988) dealing with different aspects of the behavior of the core and the mantle; they are interesting studies in mathematical physics but do not offer much improvement for the theory of nutation. On the other hand, a number of geophysical studies are based on Wahr's computer programs and, therefore, the results have to be similar.

We must remember that the equations of fluid dynamics and elasticity are partial differential equations and we do not know exact solutions. For this reason, all the solutions presented for different Earth models are approximations and, sometimes, it is very difficult to compare their results, especially when we are aiming at precisions of 1 mas. One very important point is the treatment of the boundary conditions for elliptical, rotating Earth models which has not been emphasized in most studies, especially in view of the critical importance of the ellipticity of the core.

There are attempts to combine new nutation series derived from rigid body dynamics with geophysical data. One of them (Zhu et al., 1990) tries to incorporate the very short series of VLBI observations and, therefore, is not reliable for the reasons already stated (Zhu et al., 1990, Fig 5 and 6). The comparisons of nutation corrections, considering different geophysical hypotheses, as was done by Zhu et al. (1990, Table XIV) justifies the criticism already mentioned - the different behavior of the principal nutation terms in longitude and obliquity, and the unreliable values of the compared results for precisions of 1 mas.

5. Theoretical and observed values of the luni-solar precession

We have shown that the Earth's structure does not have much influence on this type of precession and, therefore, the theory of precession does not need to consider such complex models as the theory of nutation. There is general agreement about the theory, and the expressions for precession have been published for several systems of astronomical constants (Lieske et al., 1976). Due to the fact that this precession has a very long period, it is necessary to accumulate observational data covering an interval of time as long as possible. This is one of the great difficulties in its determination, and the subject has been studied during the last decades (Fricke 1967, 1971). The relationship between the luni-solar precession and the general precession in longitude shows the importance of this precession and, therefore, for the determinations of the equinox and equator for any given epoch (Fricke 1982).

In any case, the values derived from the observations correspond to a better representation of the phenomenon than the theory where a number of approximations had to be made in order to be amenable to a numerical solution of the system of equations. A recent estimation of the principal precession and nutation terms has been done by Charlot *et al*. (1991) based on two decades of LLR and one decade of VLBI data. The improvement in the values obtained for the precession and the longer period nutations is noticeable, especially in the combined solution due to the longer series of LLR data. Nevertheless, we have to remember the past difficulties of determining precession from a short series of observations and so we have to be careful about these results. We must remember that the Deep Space Network (DSN) and the LLR network have very few stations, and we know already the implications of this fact for determining the short period nutation especially.

6. Conclusions

We have seen that the theory of luni-solar precession does not depend so much on the existence of the fluid core and its ellipticity and, therefore, it is easier to obtain agreement between theory and observation. The reasons given before show the need for a suitable model of the Earth's structure in order to define an adequate theory of the nutation. Wahr's theory is adequate for the time being and it is not convenient, for the purposes of a system of astronomical constants, to change fundamental values very often.

In any case, the theories of precession and nutation can only give us approximations to the real behavior of the Earth, because the models adopted do not include all the variables needed to describe such phenomena. For this reason, it is necessary that the values of the principal terms in precession and nutation be derived from observations. The observational series have to be done in a most careful and well planned way, covering adequate intervals of time and applying proper statistical analysis. The modern techniques of LLR and VLBI do not yet have series of observations that satisfy all these important requirements, including a well distributed network of observing stations, especially satisfying the stringent requirements of geophysical stability needed when we try to attain a precision of 1 mas.

REFERENCES

Bullen, K. E.: 1963, Introductions to the Theory of Seismology, C. U. P.

Charlot, P., Sovers, O. J., Williams, J. G., Newhall, X. X.: 1991, This proceeding

Déhant, V.: 1988 in "*Earth Rotation and Reference Frames for Geodesy and Geodynamics*, eds. A. Babcock and G. A. Wilkins, Reidel, 323.

Déhant, V.: 1990, *Geophys*, *J*. *Int*. **100**, 477.

Dziewonski, A. M., Anderson, D. L.: 1981 **Phys. Earth Planet**. Int. *25*, 297.

Fedorov, E. P.: 1958 *Nutation and the Forced Motion of the Earth's Pole*, Acad. Sci. Ukraine, Kiev (translated by B. Jeffreys, 1961, Pergamon Press).

Fricke, W.: 1967, *Astron*. *J*. 72, 1368.

Fricke, W.: 1971, Astron. and Astrophys. **13**, 298.
Fricke, W.: 1982, Astron. and Astrophys. **107**, 113.
Herring, T. A., Gwinn, C. R., Shapiro, I.I.: 1986 J. Geophys. Res 91, 4745.
Herring, T. A.: 1988 Annual Report B I H for 1987, p. D-106.
Hill, G. W.: 1893, Astr. J. 13, 1.
Hough, S. S.: 1895, Phil. Trans. A, **186**, 469.
Jackson, J.: 1930, Month. Not. R. A. S. **90**, 733.
Jeffreys, H.: 1949, Month. Not. R. A. S. **109**, 670.
Jeffreys, H.: 1950, Month. Not. R. A. S. **110**, 460.
Jeffreys, H., Vicente, R. O.: 1957, Month. Not. R. A. S. **117**, 142.
Kinoshita, H.: 1977, Cel. Mech. **15**, 277.
Kinoshita, H., Souchay, J.: 1990, Cel. Mech. (to be published).
Lieske, J. H., Lederle, T., Fricke, W., Morando, B.: 1977 Astron. Astrophys. **58**, p. 1.
McCarthy, D. D., Luzum, B. J.: 1991 Astron. J. (to be published).
Morgan, H. R.: 1943, Astr. J. **50**, 125.
Newcomb, S.: 1892, Month. Not. R. A. S., **52**, 336.
Newcomb, S.: 1895, Astronomical Constants, Washington, Gov. Printing Office.
Poincaré, H.: 1910, Bull. Astronomique, **27**, 321.
Seidelmann, P. K.: 1982, Cel. Mech., **27**, 79.
Spencer Jones, H.: 1939, Month. Not. R. A. S., **99**, 211.
Vicente, R. O.: 1964, Vistas in Astronomy, **5**, p. 1.
Wahr, J. M.: 1981, Geophys. J., **64**, 705.
Wahr, J. M., Bergen, Z.: 1986, Geophys. J., **87**, 633.
Walter, H. G.: 1991, This proceeding.
Woolard, E. W.: 1953, Astr. Pap. Amer. Ephem. Wash., 15, Part 1.
Zhu, S. Y., Groten, E., Reigber, Ch.: 1990, Astron. J., **99**, 1024.