Dynamics of Dust Particles in the Solar System

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This short communication is part of a thesis at the University of Heidelberg, FRG.
It deals with some dynamical aspects of small interplanetary particles, which are affected by non-gravitational forces like the Poynting - Robertson Effect.

The equations of motion of test particles are numerically integrated. The forces acting on these particles include the gravitational attraction of the Sun and the Planets as well as forces due to incident solar radiation.

The ephemerides of the Planets in the heliocentric rectangular 3-dimensional coordinate system used here are taken from the Development Ephemeris DE-102 (Newhall et al. 1983) . This restricts the time span of the integration to about 4412 years.

The strength of the non-gravitational forces in the equations of motion is given by a factor ß which can be varied for each test particle. ß is the ratio of the radiation force to the gravitational force of the sun. The Poynting - Robertson Effect and radiation pressure are introduced into the force function by

$$f = k^2 \beta \frac{M_\odot + m}{|\vec{r}|^2} \left[\frac{\vec{r}}{|\vec{r}|} - \frac{1}{c} \left\{ \dot{\vec{r}} + \frac{\vec{r}}{|\vec{r}|^2} \langle \vec{r} \cdot \dot{\vec{r}} \rangle \right\} \right] \qquad (1)$$

(k : Gaussian constant; $M_\odot$: mass of the Sun; m : mass, $\vec{r}$: position, $\dot{\vec{r}}$: velocity of the test particle; c : speed of light).

The procedure of numerically integrating test particles in a three - dimensional coordinate system with all the Planets over a time span of 4412 years consumes very much CPU - time (on an IBM 370-168). Some first tests showed no effects neither with small values of ß ($\beta < 0.1$), nor with

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particles starting their orbits far outside the orbit of Jupiter. So the problem was restricted to

i) a ß = 0.0
   b Values of ß between 0.1 and 0.5
ii) a not perturbed by planets
   b perturbed by the Planets from Mercury through Jupiter
iii) a Orbits similar to cometary orbits
   b Orbits similar to asteroidal orbits
   c Orbits appropriate to Zodiacal light particles

A total of about 340 orbits have been integrated which respond to one of the 12 combinations that can be built up.

If ß equals 0, we have the well-known two-body-problem (no perturbing planet), or the n - body - problem (with some perturbing planet).
If ß is not zero, but there are no perturbations by planets, then the particle spirals inward to the sun. Thereby the eccentricity decreases, and the angle of perihelion moves in the orbital plane.
The lifetime of such a grain (that is the time it takes from its initial orbit to reach e.g. an orbit with semi - major axis a = 0.1 AU) can be calculated by formulae given by Wyatt & Whipple (1950) . The constants in these equations that we use result from values adopted by the IAU. The computed lifetimes are in general agreement with the time span which the integration yielded for the respective grain to spiral from its initial orbit to an orbit with a = 0.1 AU.

The most general case that we discuss here is, if ß is not zero, and there are planets as perturbers. Some interesting things happen in this non - planar n - body - problem. The theoretical lifetimes are no longer in agreement with the corresponding times that the integration yield.
In the inner solar system, even with values of ß as high as 0.5, most of the grains are heavily perturbed by Venus and Earth. Their orbits are drastically altered, some of the grains come so close to planets that they may be captured as satellites. This has also been stated by Briggs (1962). Others leave such close encounters with high inclinations. Using lower values of ß rises the possibility that a particle comes very close to any planet.
As a result it can be said that such dust particles will not form a cloud which can be recognized as the Zodiacal dust cloud.

In the outer solar system, Jupiter dominates the motion of the test bodies. If ß is zero, resonances occur when the rotation periods of two bodies are in the ratio of small integers. If ß is non zero, the resonances are shifted, depending on the value of ß :

$$a_r = \left[\frac{M_0(1-ß)}{M_0 + m_p} \left(\frac{i}{j}\right)^2 \right]^{1/3} a_p \qquad (2)$$

(i/j is the ratio of the rotation periods, m_p , a_p is the mass and the semi - major axis of the planet in resonance). E.g. the Trojans which are in the 1:1 resonance with Jupiter, would have semi - major axes of about 4.1 AU, if they were dust - sized with ß = 0.5.

Some of the particles stay in resonances for a long time (Figs. 1 through 3 show orbits for different values of ß; the initial orbits for the Figs. 1 through 3 are identical). Some of them are rejected from their way spiralling inward (Fig. 4 shows an orbit that is similar to the orbit of comet Halley, with ß=0.5; this grain here is in resonance with Uranus : 1:1 at 15 AU; 2:3 at 20 AU; 1:2 at 24 AU). Some of the grains traverse very quickly the resonance zone (Fig. 5 : ß= 0.5 , 2:1 resonance with Jupiter at 2.6 AU).

Gonczi et al.(1982) mentioned in a planar treatment that the eccentricity always grows when a body goes through a resonance. Here we see that in three dimensions this is not always true. Fig. 6 shows that the eccentricity is decreased passing the resonance (here the resonance 3:1 with Jupiter at about 2 AU for ß=0.5).

References :

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Astrophys.Journal, Vol. 111, pp 134 - 141 (1950)

Briggs, R. E. : Astron. Journal, Vol.67,
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Gonczi, R., Froeschlé, Ch. and Cl. : Icarus,
Icarus, Vol.51, pp 633 - 654 (1982)

Figures : Plots of the variation of the semi - major axis and the eccentricity of a grain.

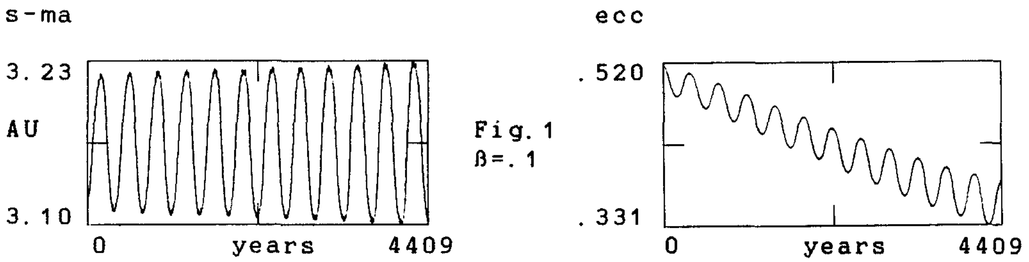

Fig. 1
β=.1

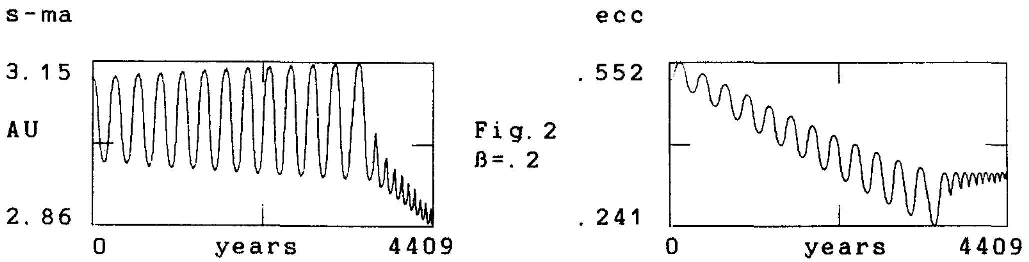

Fig. 2
β=.2

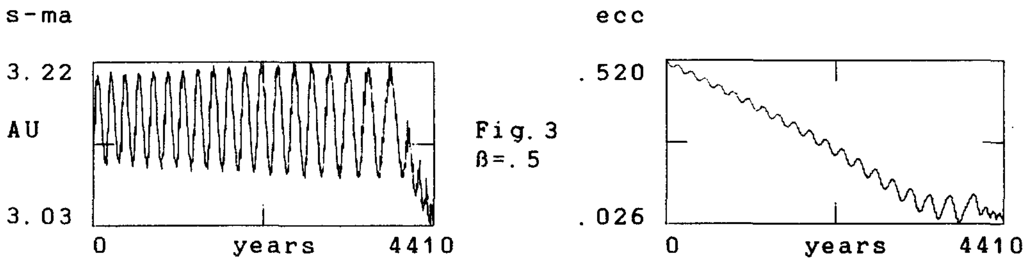

Fig. 3
β=.5

Figs. 1 - 3 : Plots of a grain which is initially in resonance with Jupiter. The orbital elements are the same for all three plots at the beginning.

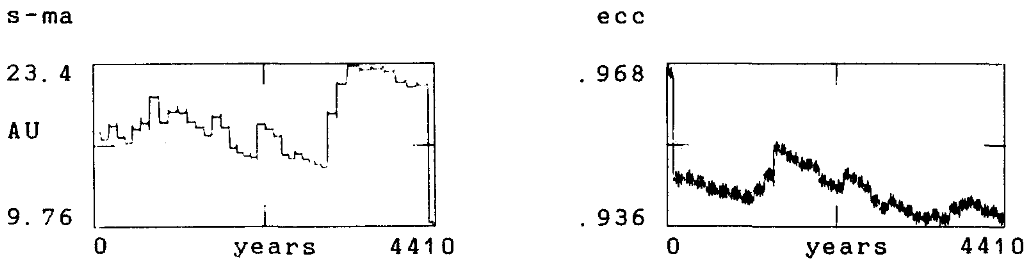

Fig. 4 : The orbit is perturbed by resonances with Uranus.
β = 0.5

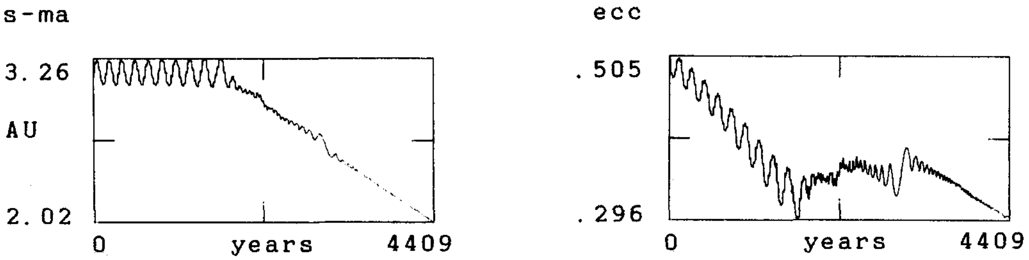

Fig. 5 : a grain traverses quickly a resonance with Jupiter (at about 2.6 AU). β =0.5

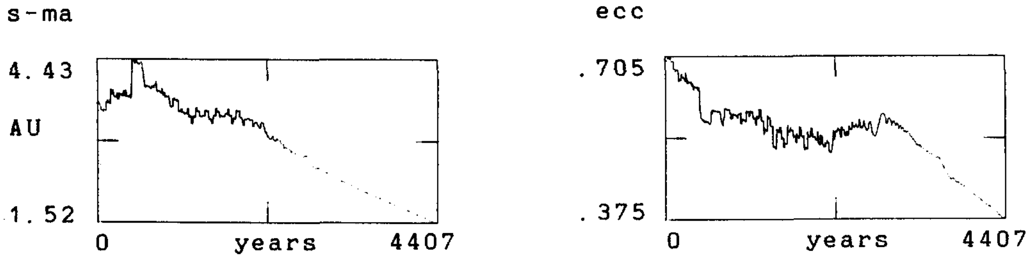

Fig. 6 : The eccentricity decreases when the grain passes the resonance with Jupiter (at about 2 AU). β =0.5

DISCUSSION :

Question by S. F. Singer :

How many of the orbits are stirred up by encounters?

G. Burkhardt : That depends on what you mean by 'stir up'. The inclination of an orbit is always altered when its plane has an inclination different from the gravitationally strongest planet at that time.

Question by J. D. Mulholland :

What percentage of the orbit calculations was halted due to gravitational singularities (close approaches) and what was the criterion; this relates to planetary accretion sweep-up.

G. Burkhardt : The criterion of the 'sphere of influence' is due to the stepsize of the integrator. It was chosen to be ten times the radius of the planet in question. Only fourteen (14) out of the about 340 integrations had to be halted because of this criterion.

Remark by J. D. Mulholland :

With that definition of the spheres of influence, the cross-section for accretion sweep-up by planets is several orders of magnitude smaller than the 5% rate of sphere of influence encounters. This supports the estimate that planetary sweep-up is a negligible influence on depletion of the dust cloud.