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ERRATA

VOLUME 20

Page 197, line 18. For « H. S. CALDWELL », read « S. H. CALDWELL ».
Page 321, column 2, line 10. For « H. S. », read « S. H. ».

VOLUME 22

Page 348, line 17. Delete the negation sign and for $= 0$, read $= 1$.

VOLUME 23

Page 134, line 5. For e , read \dot{e} .
Page 314, line 6 from the bottom. For R_e^a , read $R_{\dot{e}}^a$.

VOLUME 24

Page ii, line 5. For « Maart », read « March ».
Page 190, line 17. For « ŁUSZCZEWESKA », read « ŁUSZCZEWSKA ».
Page 287, the unnumbered footnote. For 1959, read 1958.
Page 347, column, next-to-last line. For « Semantics », read « semantics ».
Page 353. The first line at the top of column 2 should be indented.

VOLUME 25

Page ii, line 5. For $\delta\delta$, read 9.
Page 158, line 7 from the bottom. For \sim , read \sim .
Page 179, line 23. For XIII, read XXIII.
Page 182, line 14. For \sim , read \sim .
Page 250, line 9. For « "Función" », read « "Función" ».
Page 265, line 26. For « f. », read « ff. ».
Page 265, last line. For « f. », read « ff. ».
Page 266, line 17. For « f. », read « ff. ».
Page 269, line 1. Add a hyphen at the end of the line.
Page 284. Insert space between lines 7 and 6 from the bottom.
Page 297, line 4. For « *consequences* », read « *conséquences* ».
Page 330, line 6 from the bottom. For the first P_2 read P_0 .
Page 334, line 2. Delete parentheses, twice.
Page 354, line 24. For « *systeme* », read « *système* ».
Page 358, line 10 from the bottom. For], read [.
Page 360, line 3. For $(x/p)(y/q)$, read $\begin{pmatrix} x \\ p \end{pmatrix} \begin{pmatrix} y \\ q \end{pmatrix}$.
Page 361, line 22. For XXV 000(0), read XXV 356(1).
Page 361, line 27. For $a < b$, read $a <_{\Delta} b$.
Page 362, line 24. For « HILLARY », read « HILARY ».

ERRATA

- Page 370, last line. For « *for the synthesis* », read « *in synthesis* ».
Page 376, line 31 from the bottom. For « *pure* », read « "*pure*" ».
Page 377, line 19. For « ZIEMBINSKI », read « ZIEMBIŃSKI ».
Page 380, line 9. For XXV 000, read XXV 299(1).
Page 380, line 7 from the bottom. For « PAWŁOWSKI », read « PAWŁOWSKI ».
Page 381, line 3. For « Bochenski », read « Bocheński ».
Page 381. Insert space between lines 6 and 7.
Page 381, line 10. For XXV 271, read XXV 269.
Page 383, line 15. For « Ferruccio Rossi-Landi », read « FERRUCCIO ROSSI-LANDI ».

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- Page 16, line 11 and line 16. For †, read |.