Ramanujan's constant [7]) is a near-integer to 12 decimal places, and this has inspired subsequent efforts to explain *why* this is the case. Yes, there could be something deeper, but if its near-integer status is purely coincidental then search for an 'explanation' can only yield another numerological chimera that is no more revealing than the existence of a line that connects two given points.

3. *Discussion*

One way to reduce susceptibility to mistakes of the kind discussed in this Note is to avoid using calculators that display spurious digits of precision. Educators can also emphasise to students that many modern tools (e.g., *WolframAlpha* / *Mathematica*) never display spurious digits, so if a displayed result deviates from an integer or other special number, then it truly is *not* equal to that special number. More generally, a full discussion of this topic with students is likely to be stimulating and provoke greater sensitivity to the possiblity that tantalizing features of a given solution may be purely accidental.

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107.30 Remark on Cauchy–Schwarz inequality

In the mathematical sciences, inequalities play an important role. There are many mathematical inequalities, some of which are the basis for constructing other inequalities. One of the fundamental inequalities is the well-known *Cauchy–Schwarz inequality*. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be real numbers. The Cauchy–Schwarz inequality states that (see, e.g., [1])

$$\sum_{i=1}^{n} u_i v_i \leqslant \sqrt{\sum_{i=1}^{n} u_i^2}\sqrt{\sum_{i=1}^{n} v_i^2}. \tag{1}$$

This inequality has been proved in many ways, and students may be aware of some proofs. Several different proofs of the Cauchy–Schwarz inequality can be found in [2, 3, 4, 5]. The Cauchy–Schwarz inequality is fairly basic and proving many inequalities requires using it; for example, it is used to prove the triangle inequality. Although, this inequality is closely related to many inequalities, but its relationship to *Jensen's inequality* has been less studied. The simplest form of Jensen's inequality is that if $f(x)$ is a concave function and $x_1, x_2, \dots, x_n$ are in its domain, then the value of the function $f(x)$ at a point equal to the arithmetic mean of $x_1, x_2, \dots, x_n$ is not less than the arithmetic mean of the numbers $f(x_i)$ for $i = 1, 2, \dots, n$. The function $f(x)$ can be replaced by a concave function of several variables, and the arithmetic mean can be replaced by any one of several other means. For example, two-dimensional Jensen's inequality states that if $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ is a concave function, for all real numbers $x_1, x_2, \dots, x_n$ and $y_1, y_2, \dots, y_n$, we have (see, e.g., [6, p. 770])

$$\frac{1}{n}\sum_{i=1}^{n} f(x_i, y_i) \leqslant f\left(\frac{1}{n}\sum_{i=1}^{n} x_i, \frac{1}{n}\sum_{i=1}^{n} y_i\right). \tag{2}$$

Here, we are interested in studying the relationship between the Cauchy–Schwarz inequality and Jensen's inequality. In fact, we show that the Cauchy–Schwarz inequality (1) is a special case of Jensen's inequality (2). We prove this fact below:

Let us consider the function $f(x, y) = \sqrt{x}\sqrt{y}$, which is concave for $x, y \geqslant 0$. By Jensen's inequality (2),

$$\sum_{i=1}^{n} \sqrt{x_i}\sqrt{y_i} \leqslant \sqrt{\sum_{i=1}^{n} x_i}\sqrt{\sum_{i=1}^{n} y_i},$$

where $x_i, y_i \geqslant 0$ for $i = 1, 2, \dots, n$. Setting $x_i = u_i^2$ and $y_i = v_i^2$ for $1 \leqslant i \leqslant n$, where u_i, v_i are real numbers. This gives the Cauchy–Schwarz inequality (1).

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107.31 Obtaining a more general result from a functional equation by not differentiating

Introduction

Relying too much on tools with which we are familiar is a human trait that can cause us to overlook details or features that might be interesting. This is captured in *Maslow's Law* or *The Law of the Instrument*:

To a person with a hammer, everything looks like a nail [1].

That occurred in our Theorem 8 of [2, p. 429]:

> A sufficient condition for the twice differentiable function $y(x)$ to be a quadratic polynomial (parabola) is that any three distinct points (x_i, y_i) $i = 1, 2, 3$, that satisfy $y = y(x)$ with $x_1 < x_2 < x_3$, form an inscribed non-degenerate triangle and the formula for the area of the triangle with vertices at the points is
> $$C(x_3 - x_2)(x_3 - x_1)(x_2 - x_1)$$
> for a single value of C for the curve.

The condition of twice differentiability is an unnecessary assumption that is instead a consequence of the conclusion. The *hammer* is differentiation and knowing how to solve a simple differential equation. The *nail* is the remainder of the theorem.

The requirement concerning the area of the inscribed triangle can be expressed as

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = C(x_3 - x_2)(x_3 - x_1)(x_2 - x_1). \tag{1}$$

Expanding the determinant on its first row and multiplying by 2 yields

$$x_1(y_2 - y_3) - y_1(x_2 - x_3) + (x_2y_3 - x_3y_2) = 2C(x_3 - x_2)(x_3 - x_1)(x_2 - x_1). \tag{2}$$