ROTATIONAL PERTURBATION OF A RADIAL OSCILLATION IN A GASEOUS STAR

R. SIMON

Institut d'Astrophysique de l'Université de Liège, Cointe-Sclessin, Belgium

The perturbation method has been applied to the problem of the oscillations of a gaseous star rotating around a fixed z-axis according to a general law of the type

$$\Omega = \Omega(r, \theta).$$

This rotation was assumed to be small and the analysis included all the effects of order Ω and Ω^2. Particularly, the rotational distortion of the star has been taken into account by means of a mapping between the spheroidal volume of the rotating star and the spherical volume of a neighboring nonrotating configuration. Care has also been taken, in the perturbation method, to account for the existence of the so-called 'trivial modes' of the sphere, i.e. those oscillations of vanishing frequencies in which the displacement is transversal and divergence-free.

A final and practical result has been obtained in the case of the perturbation of a radial mode (Simon, 1969).

In the case of the standard model (polytrope of index 3), for a rigid rotation, the following result was obtained numerically for the perturbation of the fundamental radial mode:

$$\sigma_R^2 = \tfrac{2}{3}\Omega^2,$$

for $\gamma = \frac{4}{3}$; and

$$\sigma_R^2 = \sigma^2 - 3.8576\,\Omega^2,$$

for $\gamma = \frac{5}{3}$, with

$$\sigma^2 = (0.0569)\,4\pi G\varrho_c,$$

where ϱ_c denotes the central density of the rotating star.

Finally, in the case of the homogeneous model, for a rigid rotation, the following result was obtained analytically for the perturbation of the fundamental radial mode, in the case of a constant γ:

$$\sigma_R^2 = \sigma^2 + (5 - 3\gamma)\,\tfrac{2}{3}\Omega^2,$$

with

$$\sigma^2 = (3\gamma - 4)\,\tfrac{4}{3}\pi G\varrho,$$

where ϱ is the density of the rotating star.

This latter result can easily be generalized to any radial mode; a point which was overlooked in the paper summarized here. We get in this way the following expression:

$$\sigma_R^2 = (1 - \Omega^2/2\pi G\varrho)\,\sigma^2 + \tfrac{2}{3}\Omega^2,$$

A. Slettebak (ed.), Stellar Rotation, 37–38. All Rights Reserved

in which ϱ is the density of the rotating star and σ the frequency of the radial oscillation of the nonrotating, spherical, homogeneous model which has the same density ϱ as the actual star. In the case of a constant γ:

$$\sigma^2 = [3\gamma - 4 + k(2k+5)\gamma]\, \tfrac{4}{3}\pi G\varrho ,$$

with $k=0$ for the fundamental mode, $k=1$ for the first harmonic, etc. Consequently:

$$\sigma_R^2 = \sigma^2 + [5 - 3\gamma - k(2k+5)\gamma]\, \tfrac{2}{3}\Omega^2 .$$

Reference

Simon, R.: 1969, *Astron. Astrophys.* **2**, 390.