Take away the fixed part tet.hd. $OLMN$,

$\therefore$ tet.hd. $(PLMN) =$ constant volume.

But $\triangle LMN$, the base of the tetrahedron is fixed, so it follows that the locus of P is a unique (in virtue of volume of solid $OLMNP$) plane parallel to LMN.

Corollaries similar to those found in the case of the two dimensional equation can be deduced for the three dimensional one, and the length of the perpendicular from a point to a plane can be obtained by a method similar to (ii) of I.

JOHN MILNE.

A simple form of Integrometer.

Most continuously recording instruments are arranged so as to draw a graph exhibiting the quantity measured by means of cartesian coordinates. The record is on a roll of paper unwound from a drum. If the instrument be some form of meter, the value of the output, represented by $\int y\,dx$, may be obtained by measuring the area by a planimeter. The records, however, are somewhat bulky, and in this respect a polar graph is preferable. This is drawn by an indicator moving radially on a revolving disc. The peak or maximum of the curve is usually the most important, and this is the part which is represented best on such a polar diagram. Space may be further economised by having more than one convolution on one sheet of paper. Polar output records, however, are not freely used, perhaps because the planimeter does not give directly what is wanted, as it measures $\frac{1}{2}\int r^2\,d\theta$, instead of $\int r\,d\theta$.

The "Universal Planimeter" manufactured by A. Ott, and described in "Modern Mathematical Instruments" (G. Bell & Sons) may be used as a radial integrator, and a method of adapting the ordinary planimeter to this case is given by Russell and Powles in the *Engineer* of January 1896.

A different form of integrometer has been made by the writer.

Let O be the pole and P the point (r, θ), and let P trace a polar graph. Then if a rod OP turn about O, and if the point of contact (with the paper) of an integrating wheel, coaxal with and sliding along OP, could be made to trace the polar graph, we should have a type of integrometer, if the integrating wheel were geared with a counting wheel. It is, of course, difficult to make a satisfactory instrument under these conditions.

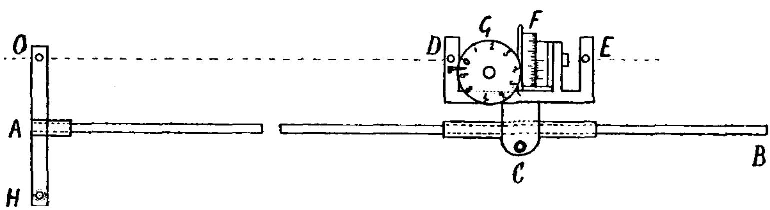
The construction adopted is shown diagrammatically. C is the carriage, which slides on the rod AB. The latter has a cross-piece at A and a support at H, and is free to turn about the pole O. The difficulty in construction, which would require the edge of the integrating wheel to trace the curve, is obviated by the use of two tracing points D and E. F is the integrating and G the counting wheel. If F_1 (not shown) be the point of contact of the edge of the wheel F with the paper, then O, D, F_1, and E are collinear and on a line parallel to AB, and $DF_1 = F_1E = a$.

To use the instrument, the tracing point D is taken round the arc, and the reading noted. The same is done with the tracing point E. The arithmetic mean of these two readings is taken, and this gives the value of $\int r\,d\theta$ between the required limits, since

$$\tfrac{1}{2}\left\{\int d\theta\,(r+a) + \int d\theta\,(r-a)\right\} = \int r\,d\theta.$$

It may be used as a radial averaging instrument, and also since $\int r\,d\theta$ may be written in the form $\iint \frac{r d\theta\, dr}{r}$, it may be employed to measure the potential of an area represented.

See also "Les Intégraphes," by Abdank-Abakanowicz, Gauthier Villars, Paris, 1886.



E. M. Horsburgh.