Publications of the Astronomical Society of Australia (PASA), Vol. 30, e030, 10 pages.

doi:10.1017/pas.2013.007

Luminosity Bias: From Haloes to Galaxies

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(Received December 23, 2012; Accepted February 12, 2013; Online Publication April 16, 2013)

Abstract

Large surveys of the local Universe have shown that galaxies with different intrinsic properties such as colour, luminosity and morphological type display a range of clustering amplitudes. Galaxies are therefore not faithful tracers of the underlying matter distribution. This modulation of galaxy clustering, called bias, contains information about the physics behind galaxy formation. It is also a systematic to be overcome before the large-scale structure of the Universe can be used as a cosmological probe. Two types of approaches have been developed to model the clustering of galaxies. The first class is empirical and filters or weights the distribution of dark matter to reproduce the measured clustering. In the second approach, an attempt is made to model the physics which governs the fate of baryons in order to predict the number of galaxies in dark matter haloes. I will review the development of both approaches and summarise what we have learnt about galaxy bias.

Keywords: dark energy – galaxies: formation – large-scale structure of Universe

1 INTRODUCTION

It has long been known that the distribution of galaxies on the sky is clumpy rather than random. Huge surveys of galaxies in the local Universe have further revealed that different types of galaxies are clustered in different ways. If galaxies are grouped into samples according to intrinsic properties such as their luminosity, colour, or morphology, then the measured clustering varies depending on the characteristics of the galaxies under consideration (Norberg et al. 2001, 2002; Zehavi et al. 2002, 2011). Figure 1 shows this for galaxies from the two-degree field galaxy redshift survey which have been divided into two classes according to their spectral type. Galaxies with 'early' or 'passive' spectral types trace out a different pattern of large-scale structure than the galaxies with 'late' or 'active' types. The early types delineate tighter filaments and the cores of clusters, whereas the late types sample the outer parts of these structures and appear more diffuse.

Such differences are driven by the variation in the processes which shape the formation and evolution of galaxies with environment and halo mass. The fact that the clustering patterns of different kinds of galaxies look different implies that measurements of galaxy clustering have the potential to tell us something useful about the nature and strength of these processes. To realise this, we need theoretical models which can describe the large-scale structure in the galaxy distribution and connect this to the underlying physics.

The large-scale structure of the galaxy distribution is also used to constrain the values of the basic cosmological parameters, including the equation of state of the dark energy. The distortion of the clustering signal due to the gravitationally induced peculiar motions of galaxies provides a measurement of the rate at which the structure is growing, which in turn depends on the cosmic expansion history (Guzzo et al. 2008; Wang 2008). The apparent location of baryonic acoustic oscillation (BAO) features in the power spectrum or correlation function provides a geometrical test, measuring the redshift–distance relation (Percival et al. 2007; Cabré & Gaztañaga 2009; Sánchez et al. 2009, 2012). The power of large-scale structure probes depends on how well we can model galaxy bias. For example, in BAO studies, the measured power spectrum is often divided by a featureless reference spectrum to remove the overall shape of the spectrum from the analysis. However, this shape contains further cosmological information if we can predict the form of the galaxy bias, so that we can infer the shape of the matter power spectrum. Galaxy bias is therefore a 'nuisance' parameter or systematic in large-scale structure probes. If we can model bias, we can enhance the scientific performance of wide-field galaxy surveys by marginalising over this parameter.

In this article, I will first review empirical approaches to modelling galaxy clustering, explaining how these developed

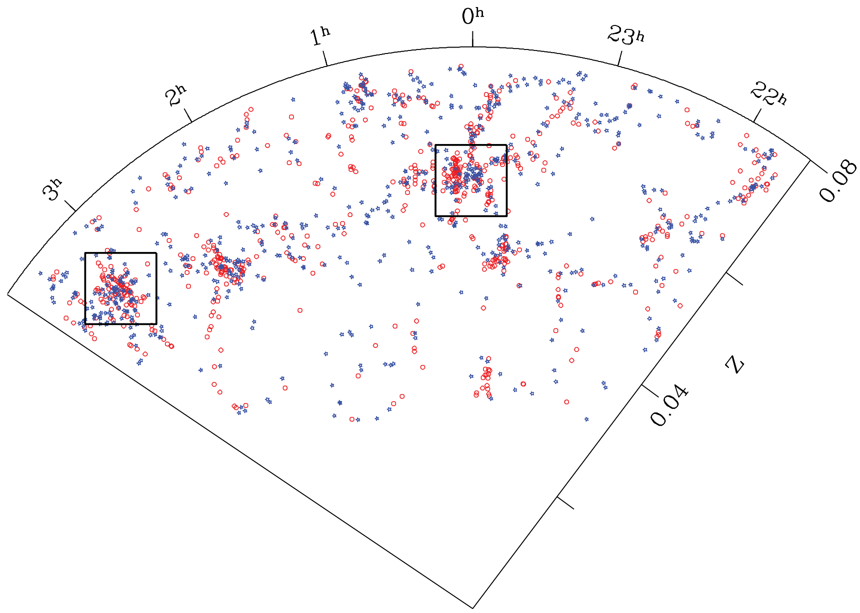

Figure 1. The distribution of galaxies with early (red points) and late spectral (blue points) types in a volume-limited sample (just faintwards of L_*), drawn from the two-degree field galaxy redshift survey. The early- and late-type galaxies trace out different features of the cosmic web. Adapted from Norberg et al. (2002).

as the quality of *N*-body simulations of hierarchical clustering of the dark matter improved. In the second half, I will discuss physical approaches to predicting galaxy bias and give an overview of what such models have told us.

2 EMPIRICAL MODELS OF GALAXY CLUSTERING

The central pillar of the paradigm for the large-scale structure of the universe is gravitational instability. Small perturbations in the matter density seeded during inflation are amplified by gravitational instability. The early stages of this process can be followed using perturbation theory (Bernardeau et al. 2002). Unless specialised assumptions are made, the latter, nonlinear stages of structure formation, can only be modelled through numerical simulation (Davis et al. 1985).

N-body simulations of the hierarchical growth of perturbations in the density of the Universe have played a central role in shaping the current cosmological model (Springel, Frenk, & White 2006). According to these calculations, the correlation function of the dark matter at the present day cannot be described by a simple power law. The correlation function of the mass today in a cold dark matter universe with a cosmological constant is plotted in Figure 2. The correlation function of galaxies in a flux-limited survey, roughly the clustering of L_* galaxies, is also shown for contrast (Baugh 1996). In this case, the correlation function is impressively close to a power law over more than three decades in pair separation. The effective galaxy bias, defined as the square root of the ratio of the galaxy and dark matter correlation functions, is therefore scale dependent.

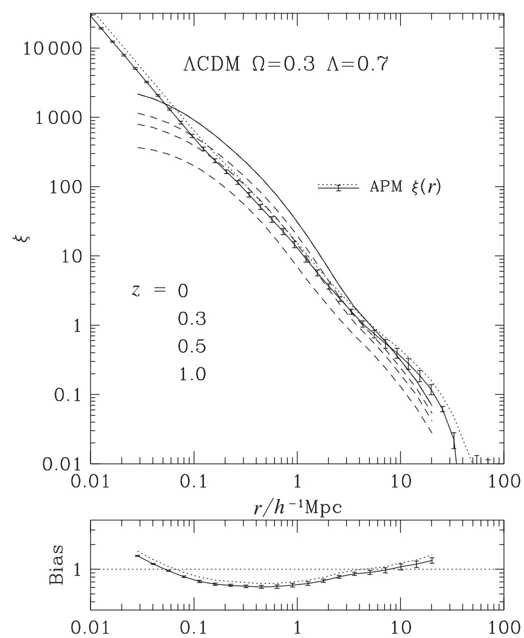

Figure 2. The clustering in the matter distribution, as quantified through the two-point correlation function. The lines show measurements from *N*-body simulations of a ΛCDM cosmology at different epochs, with the upper-most curve corresponding to the present day. The points show a measurement of the galaxy correlation function, which unlike the dark matter is well described by a power law in pair separation. The effective galaxy bias, the square root of the ratio of the galaxy and matter correlation functions, is shown in the lower panel and is scale dependent. Based on a figure from Jenkins et al. (1998).

Early *N*-body simulations lacked the resolution to reveal any irregularities in the structure of dark matter haloes. Large volume simulations suitable for following fluctuations on scales of tens of megaparsecs were only able to resolve haloes of group and cluster mass. Motivated by analytic calculations which explained the large correlation lengths of galaxy groups through the clustering of high peaks in a Gaussian density field (Kaiser 1984), the first attempts to model the spatial distribution of galaxies used the smoothed density field of the dark matter (Davis et al. 1985; White et al. 1987). Cole et al. (1997) assumed that the probability of finding a galaxy was some empirical function of the smoothed density field, with parameters tuned to reproduce the galaxy correlation function. This approach has continued to be developed, with the introduction of the idea of stochastic bias (Dekel & Lahav 1999) in which the overdensity in the galaxy distribution can be written as a nonlinear function of the overdensity in the matter distribution with a scatter. This framework has been further developed and applied to surveys by a number of authors (Sigad, Branchini, & Dekel 2000; Szapudi & Pan 2004; Marinoni et al. 2005; Kovač et al. 2011).

As codes became more efficient at calculating the gravitational forces between large numbers of particles and the processing speed of computers increased, it became possible to resolve haloes approaching galactic masses. The clustering of haloes depends, in the first approximation, on halo mass, with cluster-mass haloes being much more strongly clustered than haloes which might host the Milky Way (Cole & Kaiser 1989; Mo & White 1996). This led to models in which the form of the measured galaxy clustering could be obtained by applying a suitable weighting to haloes, which varies with halo mass (Jing, Mo, & Boerner 1998). This is the forerunner of today's halo occupation distribution models in which the weighting is expressed in terms of the mean number of galaxies per halo, as described later.

With further improvements to the simulations, it became possible to resolve structure inside dark matter haloes (Klypin et al. 1999; Moore et al. 1999). Haloes form through mergers and the accretion of mass. With sufficient resolution, the central regions of the accreted haloes can be preserved for many orbits, whilst the outer parts are stripped off. This prompted a new generation of modelling in which resolved subhaloes were associated with galaxies. In an early example of what today would be called 'subhalo abundance matching', Colín et al. (1999) were able to match the observed power-law clustering of galaxies by selecting all subhaloes above some threshold circular velocity (see Figure 3).

So how can the power-law galaxy correlation function be understood, given the shape of the dark matter correlation function? Benson et al. (2000) described the predictions of their galaxy formation model in these terms and argued that a power law could be obtained for the galaxy correlation function if the 'right' number of galaxy pairs was predicted in each halo. Models which were set up to reproduce the galaxy luminosity function were found to predict a power-law galaxy correlation function in a ΛCDM cosmology. Figure 4 shows

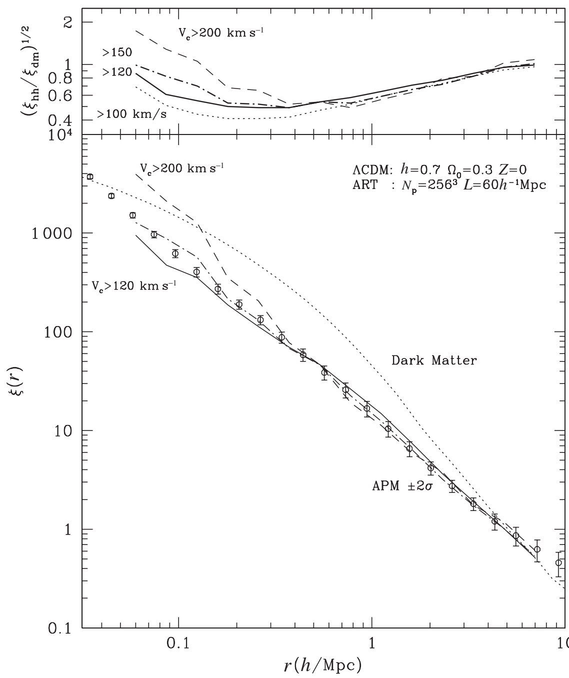

Figure 3. An attempt to reproduce the observed clustering of galaxies by associating galaxies with subhaloes with effective circular velocities above some threshold value (dashed, dot–dashed, and solid). The clustering of subhaloes is different from that of the overall dark matter (shown by the dotted line), and by tuning the circular velocity which defines the sample, a good match can be obtained with the observed galaxy clustering (shown by the points). Reproduced from Colín et al. (1999).

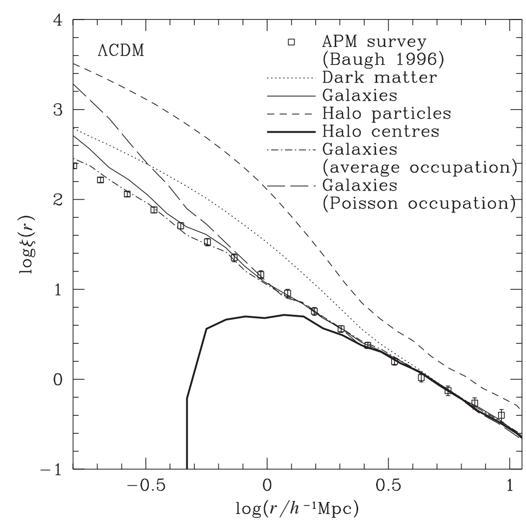

Figure 4. Reproducing the clustering of galaxies in ΛCDM. The correlation function of haloes which contain galaxies is shown by the heavy solid line. This curve turns over below $r \sim 0.5h^{-1}$ Mpc due to an exclusion effect which prevents haloes overlapping. The correlation function of the dark matter particles in these haloes is shown by the long-dashed line; this puts too many pairs in massive haloes and leads to an overprediction of the small-scale clustering. The number of galaxies predicted by a galaxy formation model set up to reproduce the luminosity function gives a reduced number of pairs by comparison with the particle case, and is in excellent agreement with the observed galaxy clustering. Based on a figure in Benson et al. (2000).

the components of the galaxy correlation function. The clustering of the haloes occupied by galaxies is shown by the heavy solid line. Each halo has unit weight in this example. The curve turns over at small pair separations due to an exclusion effect; if haloes got any closer to one another, they would be identified as a more massive halo by a percolation group finder. Considering only the dark matter particles contained within occupied dark matter haloes (long-dashed line) overpredicts the small-scale clustering. The number of galaxy pairs within a halo does not increase with halo mass in proportion to the number of particles, so a lower clustering amplitude is predicted on small scales (light solid line).

Today, these approaches have crystalised into two schemes: halo occupation distribution (HOD) modelling and subhalo abundance matching (SHAM).

HOD modelling has its roots in the clump model of Neyman & Scott (1952). In its modern form, HOD modelling took off around the start of the millennium, spurred on by the physical modelling described in the second part of this article. The HOD is a parametrisation of the mean number of galaxies per halo. The HOD is split into contributions from central galaxies and satellite galaxies (Zheng et al. 2005). Central galaxies are typically modelled using a softened step function, which encapsulates the transition from haloes which are not massive enough to host a galaxy which meets the observational selection, to the mass for which all central galaxies are included. The mean number of satellite galaxies per halo is described by a power law, which reaches unity at a higher halo mass than the central HOD (Cooray & Sheth 2002). The canonical form used to model the HOD of optically selected galaxy samples is shown by the fit in the left panel of Figure 8.

A limitation of the HOD approach is that it is descriptive rather than predictive. Given an observational measurement of the clustering of galaxies, the parameters of the HOD can be constrained to reproduce this clustering, returning an interpretation of the measurement in terms of the number of galaxies per halo. The basic HOD machinery cannot make a prediction for a new clustering measurement with, for example, a different galaxy selection or at a different redshift. However, refinements to the HOD model to include galaxy luminosity and colour have been devised (Skibba & Sheth 2009). As we will see later, the canonical form of the HOD outline above does not apply to all galaxy selections and there is no way to anticipate this without trying to implement a physical model of the galaxy population. Lastly, the basic assumption behind HOD modelling, that the clustering of dark matter haloes depends solely on halo mass, has recently been demonstrated to be inaccurate (Gao, Springel, & White 2005; Croton et al. 2006; Gao & White 2007; Angulo et al. 2009).

SHAM is an even simpler approach to realising a galaxy distribution in an *N*-body simulation. The fundamental assumption behind SHAM is that there is a monotonic relation between a galaxy property, e.g. stellar mass, and the mass of the subhalo which hosts the galaxy. This relation is also assumed to have zero scatter. The subhaloes from the simulation are then ranked in mass, breaking each halo into its component subhaloes. A volume-limited sample of galaxies, e.g. generated from a measurement of the galaxy luminosity function, is then also ranked by the galaxy property (in this example, luminosity) and the two lists are paired off, with the most luminous galaxy being matched up with the most massive subhalo until the end of the list is reached (Vale & Ostriker 2004; Conroy, Wechsler, & Kravtsov 2006). In the simulation, the mass estimated for subhaloes can be affected by stripping, so the mass of the subhalo at infall is used in the SHAM procedure.

SHAM seems to provide surprisingly good descriptions of observational samples (Conroy et al. 2006; Moster et al. 2010). This is all the more remarkable when one considers that no distinction is made regarding where the subhalo came from, that is, regardless of whether it was part of a cluster-mass halo or an isolated halo, there is assumed to be a connection to a galaxy property (Watson & Conroy 2013). One might imagine that environmental factors would change the nature of the connection between subhalo mass and galaxy property for a satellite galaxy in a cluster. In SHAM, the subhalo mass is frozen at infall into a larger structure. Subsequently, the satellite galaxy could continue to form stars using up any available reservoir of cold gas, which would appear to change the subhalo mass–galaxy property relation.

The SHAM approach has been extended to cope with the scatter in the galaxy property–halo mass relation (Moster et al. 2010; Rodríguez-Puebla, Drory, & Avila-Reese 2012). The assumption which underpins SHAM has been evaluated by Simha et al. (2012) using the output of a gas dynamic simulation. These authors found that the simulation produced relations between selected galaxy properties and subhalo mass which were monotonic, but with scatter. The scatter led to the clustering in a catalogue constructed by applying the SHAM hypothesis to differ somewhat from that in the original simulation output.

The connection between empirical models of galaxy clustering based on the smoothed distribution of matter and those which start from haloes has recently been made (Cacciato et al. 2012). In the next section, we discuss a more physical approach which does not rely upon existing clustering data being available.

3 PHYSICAL MODELLING OF GALAXY FORMATION

By itself, the cold dark matter model says nothing directly about galaxy formation. Inferences can be drawn about the sequence of galaxy formation, based on how structures grow in the dark matter. However, without an attempt at a physically motivated calculation of the fate of baryons in a cold dark matter universe, there is little hope of learning much about galaxy formation or of understanding the implications of observations of high-redshift galaxies for the cold dark matter cosmology (for reviews see Baugh 2006; Benson 2010).

White & Rees (1978) argued that galaxy formation is a two-phase process, with the bulk of the mass undergoing a dissipationless collapse which is responsible for building the gravitational potential wells or haloes in which galaxies form. The baryonic component of the universe is able to dissipate energy and therefore to collapse down to smaller scales, forming denser units, which retain their identity within the cluster. This model was able to explain the appearance of clusters of galaxies. However, without an additional process to reduce the efficiency of galaxy formation in shallow gravitational potential wells, the predicted luminosity function is much steeper than is observed at the faint end.

This pioneering work, along with a clutch of papers published around the same time looking at the radiative cooling of gas within gravitational potential wells, laid the groundwork for modern galaxy formation theories. The break in the galaxy luminosity function can be understood by comparing the time taken for gas to cool with the age of the universe. The time taken for all of the gas within a halo to cool radiatively increases with halo mass. This is because cooling is a two-body process (collisionally excited radiative transitions or bremsstrahlung) which depends on the square of the gas density. In hierarchical models, more massive haloes tend to form later when the density of the universe is lower. It is possible for the cooling time of the gas to exceed the Hubble time, thus limiting the supply of cold gas to form a galaxy (see the review of Fred Hoyle's contributions to galaxy formation theory by Efstathiou 2003).

The first papers to incorporate these ideas fully into the cold dark matter cosmology, introducing the semi analytical methodology, were published in 1991 (White & Frenk 1991; Cole 1991; Lacey & Silk 1991). This approach tries to follow a wide range of the processes which are thought to be important in determining the fate of the baryons. This is a daunting task. At the time, theories of star formation were rudimentary at best. There has been much progress in this area since 1991, but we are still a long way from having a reliable description of the process which underpins galaxy formation. The regulation of star formation efficiency comes from the stars themselves. Stars above ≈5–8 times the mass of the Sun end their life in a Type II supernova, which injects substantial amounts of energy and momentum into the interstellar medium (ISM). This alters the state of the gas in the ISM, perhaps leading to the ejection of gas from the galactic disc or even the dark matter halo. This process is known as supernova feedback and is critical to the success of any model of galaxy formation.

The absence of a precise description of a key process, such as star formation and supernova feedback, may lead one to consider giving up any hope of ever understanding galaxy formation. Instead, in semi-analytical modelling an attempt is made to write down the differential equation which gives the current best bet model of how the system behaves. As our understanding develops, or when new observations clarify how a process works, then the model can be improved. The differential equation may contain a free parameter. Often there is little guidance as to the appropriate range of values to take for the parameter. In such instances, the only approach is to be pragmatic and see what the model predicts for different parameter values. By comparing the model predictions to observations, the value of the parameter is set as the one which gives the most faithful reproduction of the data. This procedure is exactly what physicists undergo when attempting to describe complex phenomena: start off with a simple model, which can be adjusted or refined to improve the description of the observations. I will give an example of this principle in action in the next section.

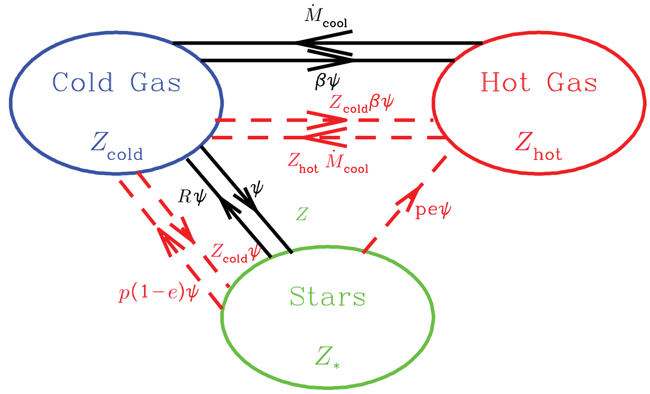

Figure 5. The flow of mass and metals between reservoirs of hot gas, cold gas, and stars. Semi-analytical models of galaxy formation solve the differential equations which describe the transfer of materials between these reservoirs. Reproduced from Cole et al. (2000).

The semi-analytical framework allows us to model a range of processes together, within the cosmological setting of the formation of structure in the dark matter. The ability to follow the interplay between processes is essential in studying galaxy formation. The models solve the set of differential equations which govern the flow of mass and metals between different reservoirs of baryons: hot gas, cold gas, and stars (Figure 5). The output of the models is the full star formation and chemical enrichment histories for a wide range of galaxies, including mergers between galaxies.

Semi-analytical modelling has some features which might be perceived as limitations or drawbacks. One example is the generality of the assumptions which are needed to be able to calculate the fate of the baryonic component. Another is the 'deterministic' way in which processes such as supernova feedback are modelled. In the semi-analytical model, the mass loading of the supernova-driven wind is specified by choosing model parameters, and precisely this amount of gas is ejected from the ISM. In a gas dynamics simulation in which the wind is fully coupled to the hydrodynamics equations (note this is not generally the case, with a semi-analytical model of feedback inserted into the simulation to describe feedback), the same number of supernovae could result in a very different mass of gas being ejected. The mass

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loading could be intricately linked to the resolution of the simulation.

Nevertheless, despite the progress made over the past 20 years, there is still widespread mistrust of semi-analytical modelling. This has led to a burgeoning reductionist movement in galaxy formation in which simplified models have been devised with the aim of elucidating how galaxies form. Examples include the 'bathtub' and 'reservoir' models (Bouché et al. 2010; Davé, Finlator, & Oppenheimer 2012). These calculations are inspired by models of supply and demand from economics, and track the inflow (sources) of gas into haloes and the 'sinks' of cold gas in star formation and supernova feedback. In their simplest form, the models follow one galaxy per halo and invoke ad hoc efficiency factors to specify the inflow of gas as a function of halo mass, without any attempt to calculate the rate at which gas can cool or to explain the form of the efficiency factor. Galaxy mergers are ignored. This class of calculation effectively takes one of the equations which has been considered within semi-analytical models for more than two decades and solves it in isolation.

The desire for a better grasp of how galaxy properties are shaped by different processes is understandable, but it is not clear that it can be usefully gained from such stripped-down approaches. The perceived 'complexity' of semi-analytical modelling is actually the great strength of the technique. The ability to model the interplay between processes is the key to building a realistic model of galaxy formation. By taking a more complete view of galaxy formation rather than a selective one, the consequences of the calculation—the predictions of the model—are more far reaching and therefore more tightly constrained by observations. If the model seems complex, then this is simply a reflection of the nature of the underlying processes such as star formation and heating by supernovae.

Semi-analytical modelling of galaxy formation is complementary to the approach of using a gas dynamics simulation, with the two techniques having many aspects in common. In general, gas dynamics simulations rely on fewer assumptions to follow some of the processes in galaxy formation. For example, the treatment of gas cooling in semi-analytical models assumes spherical symmetry, whereas this is not necessary in a hydrodynamics simulation. Nevertheless, in carefully controlled comparisons, the modelling of gas cooling in semi-analytical models can produce the same results as are obtained in the hydro-simulation (Yoshida et al. 2002; Helly et al. 2003; De Lucia et al. 2010). In other areas, the two methods are more similar than many people realise. A good illustration is star formation, which is firmly 'sub grid' in simulations which aim to follow more than one galaxy. The treatments of star formation in a gas dynamics simulation and in a semi-analytical model are very similar. Further discussion of how star formation is treated in semi-analytical models is given in the next section. A key limitation on the use of gas dynamics simulations to model galaxy clustering is their computational expense and the requirement for 'sufficient' resolution in mass and length scales (Governato et al. 2007). These considerations have tended to force gas simulators to use relatively small simulation boxes, typically measured in tens of megaparsecs. This in turns limits the predictions for the clustering to pair separations of a few magaparsecs. An alternative to trying to predict the galaxy correlation function is to focus instead on how haloes are populated with galaxies. If enough different environments can be sampled, e.g. by re-simulating patches from a larger volume at high resolution and with gas (Crain et al. 2009), then such a simulation could be used to predict the halo occupation distribution. One advantage of gas simulations over semi-analytics is that they can follow the redistribution of matter due to outflows of baryons. Calculations using the OverWhelmingly Large Simulations have shown that the physics of galaxy formation, particularly active galactic nucleus (AGN) feedback, has an impact on the distribution of matter which has implications for the interpretation of weak lensing measurements (Semboloni et al. 2011; van Daalen et al. 2011).

Nevertheless, to address clustering on scales of tens to hundreds of megaparsecs, the only viable technique is semi-analytics used in combination with large volume, high-resolution N-body simulations of the clustering of dark matter, which we focus on in the later sections of this review.

4 ILLUSTRATION: THE STAR FORMATION RATE IN GALAXIES

An illustration of how semi-analytical models work can be obtained by considering recent progress in how star formation is modelled within a galaxy.

The bulk of semi-analytical models attempts to predict the global star formation rate within a galaxy. The early modelling of the star formation rate was essentially based on dynamical arguments, with loose motivation coming from a comparison with the Kennicutt–Schmidt law (Bell et al. 2003). The star formation rate, ψ, is often parametrised as

$$\psi = \epsilon \frac{M_{\rm cold}}{\tau},$$

where $M_{\rm cold}$ is the total mass of cold gas in the galaxy and ε is an efficiency factor which controls the fraction of cold gas which is turned into stars in the timescale τ. The timescale for star formation is generally assumed to scale with the dynamical time within the galaxy:

$$\tau = t_{\rm dyn} f(v_{\rm disc}).$$

In some models, $f(v_{\rm disc}) = 1$; in the Cole et al. (2000) model, an explicit scaling of the star formation timescale with the circular velocity of the disc was implemented, to allow the model to produce a better match to the observed gas fraction luminosity relation for spiral galaxies: $f(v_{\rm disc}) = (v_{\rm disc}/200\,{\rm km\,s^{-1}})^{\alpha_*}$. Hence, in the most general case, two parameters are required to set the star formation rate: ϵ and α_*. These parameters are set by choosing values, running the model and then comparing the model predictions with observables. The key observables for constraining the

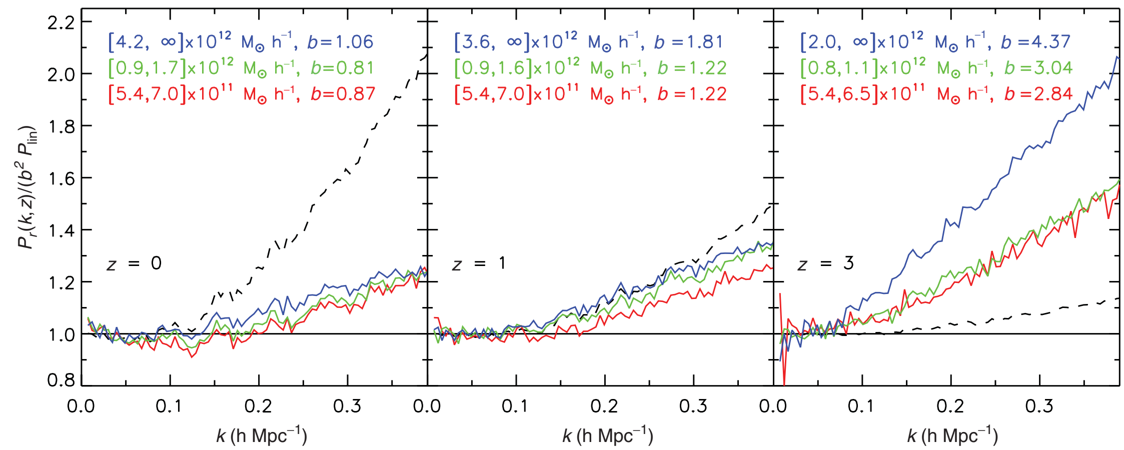

Figure 6. The scale-dependent bias of haloes of different mass, as measured from a very large volume *N*-body simulation. Each panel corresponds to a different redshift as labelled. The halo mass range and the measured asymptotic bias are given by the legend. If the asymptotic bias described the halo power spectrum, the ratio of the halo power spectrum divided by the linear power spectrum multiplied by the square of this bias would be unity. The clustering of haloes measured from the simulation deviates strongly from a ratio of unity, which indicates that the halo bias is scale dependent. Furthermore, the shape of these curves is different from that corresponding to the nonlinear matter power spectrum divided by the linear theory spectrum (shown by the dashed line). Reproduced from Angulo et al. (2008).

values of these star formation parameters are the gas fraction–luminosity relation, the galaxy luminosity function, and the colour–magnitude relation.

High-resolution imaging of galaxies at different wavelengths has revealed that star formation activity correlates better with the molecular hydrogen content of galaxies than with the overall cold gas mass. Lagos et al. (2011b) investigated more general star formation models in the GALFORM semi-analytical model, implementing different empirical and theoretically motivated star formation laws (see also Cook et al. 2010; Fu et al. 2010). The most successful of these was the empirical star formation law proposed by Blitz & Rosolowsky (2006), who suggested that the observational data could be explained if the ratio of molecular to atomic hydrogen is set by the pressure in the midplane of galactic discs; gas discs with higher pressure have a higher fraction of H_2.

This work illustrates the modularity of semi-analytical modelling and how it provides a framework in which new and improved descriptions of various processes can be readily implemented. The Blitz–Rosolowsky star formation law involves two observationally determined ‘parameters’. Although in the original parameterisation of the star formation rate there was little guidance about the range of parameter values which should be considered, there is now a much smaller volume of parameter space to search (at least once the Blitz–Rosolowsky law has been adopted). Furthermore, as the modelling of the star formation becomes more sophisticated, the predictions that can be made by the model expand. Rather than simply outputting the cold gas mass of galaxies, the atomic and molecular hydrogen contents are now predicted, meaning that the model should also be able to reproduce the mass functions of H I and H_2, their evolution and their relation to other galaxy properties (Lagos et al. 2011a). By combining GALFORM with the photon-dominated region model of Bell et al. (2006), it is also possible to predict the different carbon monoxide transitions and to make contact with observations from ALMA (Lagos et al. 2012).

Hence, by adopting the improved star formation model, the parameter space open to the model has shrunk in volume and the constraints on the model have increased through the capability to make new predictions which must match the available observations.

5 PREDICTIONS FOR GALAXY CLUSTERING

The combination of a semi-analytical model of galaxy formation with a cosmological *N*-body simulation extends the capability of the models to make predictions for the spatial distribution of galaxies (Kauffmann et al. 1999; Benson et al. 2000). The models follow the physics of the baryonic component of the universe to predict how many galaxies populate dark matter haloes as a function of their mass and formation history and tell us the properties of these galaxies. The semi-analytical model therefore predicts the mean number of galaxies per halo and it was the description of the model output in these terms which helped to stimulate the development of HOD modelling.

The form of galaxy bias can be understood by first looking at the clustering of dark matter haloes. The canonical model is that the clustering of haloes can be described by multiplying the matter power spectrum by the square of an asymptotic bias factor. Formally, the bias factor should be applied to the linear power spectrum of matter fluctuations. Angulo et al. (2008) investigated this hypothesis with a moderate resolution *N*-body simulation of a very large cosmological volume,

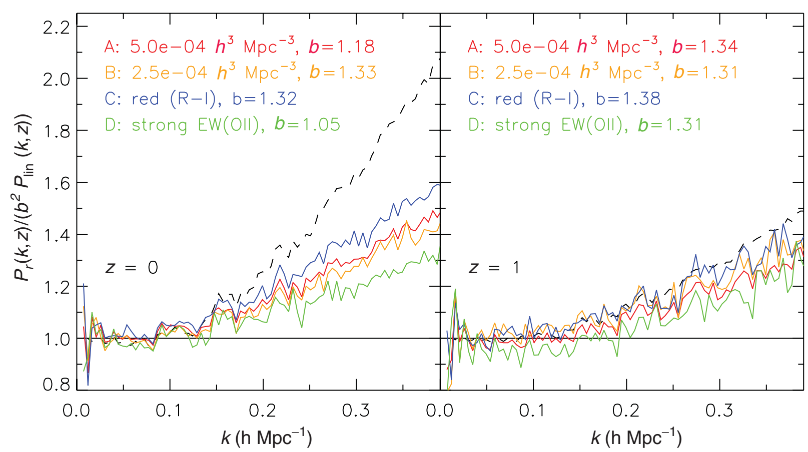

Figure 7. The predicted scale-dependent bias in the galaxy distribution. As in the previous figure, the power spectrum measured for different galaxy selections is divided by the linear theory power spectrum multiplied by the square of the asymptotic bias. Different colours correspond to different selections: red and orange show the predicted clustering for flux limited samples, the blue curves show the power spectrum for red galaxies, and the green curves show galaxies with strong emission lines. Reproduced from Angulo et al. (2008).

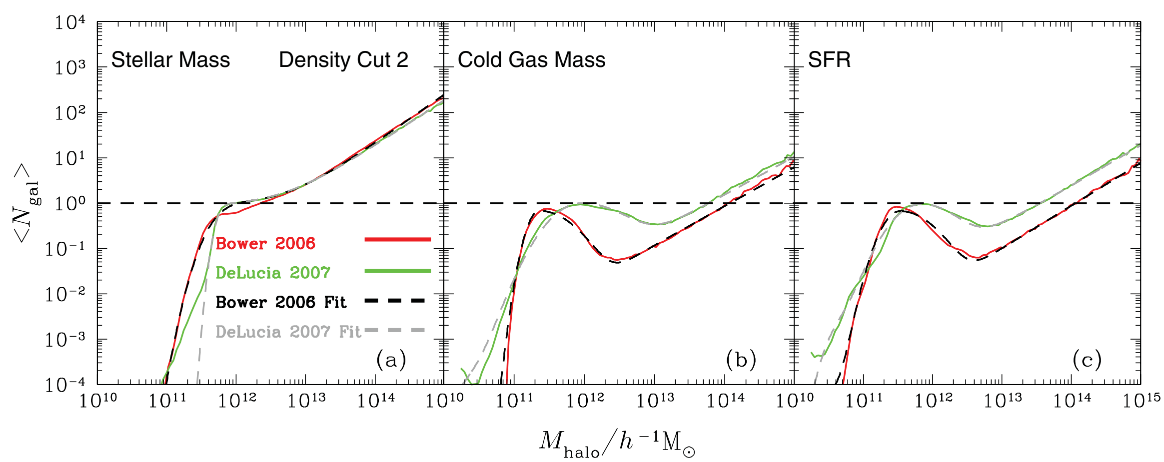

Figure 8. The form of the halo occupation distribution *predicted* by two different semi-analytic galaxy formation models, the models of Bower et al. (2006) and De Lucia & Blaizot (2007). The HOD for galaxies selected according to a different intrinsic property is shown in each panel: left, stellar mass; middle, cold gas mass; right, star formation rate. In all cases, the samples have been ranked in terms of the intrinsic property, and the same abundance of objects is considered. The form of the HOD predicted for the cases of cold gas and star formation rate selected samples is different from that for stellar mass selected samples, with a peaked HOD for central galaxies. The dashed curves show how well parametric equations for the HOD can reproduce the forms predicted in the models. For stellar mass samples, a five-parameter fit gives a good match to the model results. For cold gas or star formation rate samples, a nine-parameter HOD is needed. Reproduced from Contreras et al. (2013).

measuring $1340h^{-1}$ Mpc on a side. Figure 6 shows the ratio of the power spectrum measured for different samples of dark matter haloes divided by a scaled linear theory power spectrum. The scaling is the square of the asymptotic bias, which is measured from the simulation on very large scales (small k). This ratio deviates strongly from unity at quite large scales, typical of those used to fit BAO. This means that a simple bias squared times the linear theory spectrum is not a good way to describe halo clustering. If the linear power spectrum is replaced by the nonlinear matter power spectrum in the simulation, there is some improvement, but there are still substantial deviations, as shown by the discrepancy between the coloured curves and the dashed black line in Figure 6. This disagreement is particularly strong at high redshift, where the resolved haloes correspond to higher peaks in the density field than they do at lower redshifts.

The next step in the calculation is to combine the large volume N-body simulation with a semi-analytical model of galaxy formation. This is the only way to make predictions for galaxy clustering on scales of tens of megaparsecs and above. Current simulations which follow the hydrodynamics of the gas are restricted to volumes which are several thousand times smaller, and can only reliably predicted galaxy clustering out to pair separations of a few megaparsecs. Figure 7 reveals that both the asymptotic bias and the form of the scale dependence of the bias depend upon how galaxies are selected (Angulo et al. 2008). This in turns has implications for the apparent positions of the BAO when observed using different galaxy tracers.

Finally one might ask, given the uncertainty in the modelling of the processes behind galaxy formation, how far can we trust the predictions of semi-analytical models for galaxy clustering? The Millennium *N*-body simulation of Springel et al. (2005) provides an excellent test-bed on which different semi-analytical models can be run and compared. Contreras et al. (2013) compared the clustering predictions of the Durham and Munich models (Bower et al. 2006; De Lucia et al. 2006; Bertone, De Lucia, & Thomas 2007; Font et al. 2008; Guo et al. 2011). These groups have developed independent models which follow the same processes but with different implementations. These differences even extend to the first step in the galaxy formation code to extract merger histories for dark matter haloes from the simulation. A summary of the comparison is given in Figure 8. The different models give remarkably similar predictions for the HOD (an *output* of the models) for galaxy samples selected by stellar mass. The results are qualitatively similar for samples selected by the cold gas mass or star formation rate of the galaxies, but differ in detail. These differences can be traced to the way in which star formation is modelled by the different groups.

6 CONCLUSIONS

I have discussed empirical and physical methods for connecting dark matter haloes to galaxies. Empirical methods include: (1) applying a weighting scheme to the smoothed dark matter density field; (2) applying a weighting of dark haloes through the HOD which specifies the mean number of galaxy pairs as a function of halo mass; and (3) SHAM, in which galaxies and subhaloes are first ranked and then matched up. The physical approach is to carry out a calculation of the fate of baryons in a cold dark matter universe to predict which galaxies are in which haloes. Currently, this is only possible in cosmologically representative volumes by using a semi-analytical model of galaxy formation. I briefly reviewed how these models work and gave an illustration of the power of this approach by discussing recent work on improved models of the star formation rate in galaxies.

Much progress has been made in understanding the connection between haloes and galaxies and hence of galaxy bias. One clear conclusion so far is that galaxy bias is *scale dependent* and depends sensitively on the selection applied to construct the sample. This needs to be taken into account when analysing large-scale structure as a cosmological probe so that all of the data can be utilised. A comparison of the predictions from different models which aim to follow the same processes in galaxy formation gives some encouraging results (Contreras et al. 2013). The predictions for samples selected by stellar mass seem robust. However, there is more discrepancy between the predictions for other galaxy selections which are closer to what will be used in future galaxy surveys. This suggests that further theoretical work is needed if we are to maximise the potential of future surveys to tell us the values of the basic cosmological parameters and about the physics of galaxy formation.

ACKNOWLEDGMENTS

I would like to thank the organisers for inviting me to speak at such a stimulating meeting and for being indulgent in extending the deadline for this contribution.

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