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Determination of an Intermediate Perturbed Orbit using Multiple Observations

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Abstract. Two methods are briefly stated for finding the preliminary orbit of a small celestial body from its three or more pairs of angular measurements and the corresponding time instants. The methods are based on using the approach that we previously developed for constructing the intermediate orbit from a minimum number of observations. This intermediate orbit allows for most of the perturbations in the motion of the body under study. The methods proposed use the Herget's algorithmic scheme that makes it possible to involve additional observations as well. Using the determination of orbits of some asteroids as examples, we compare the results obtained by applying the Herget multiple-observation algorithm and the proposed methods. The comparison shows that the proposed methods are an efficient means for studying perturbed motion. They are especially advantageous if applied to high-precision observational data covering short orbital arcs.

Keywords. celestial mechanics, methods: analytical; methods: statistical; methods: data analysis; techniques: high angular resolution; ephemerides, asteroids

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With the advent of modern high-precision optoelectronic position measurements, which are one to three orders of magnitude more accurate than classical astrometric measurements, the problem of preliminary determination of perturbed orbit is particularly relevant. The methods proposed in (Shefer 2003, 2008) successfully resolve this problem from three and four positions of a small body on the celestial sphere. However, posing the problem of the determination of a reliable preliminary orbit from three or four positions only, we may face a number of challenges (Herget 1965; Marsden 1991). Because of this, it is desirable to have such an algorithm for constructing a preliminary perturbed orbit that would be able to use all the n ($n \geqslant 3$) available observations.

In this work, the approach developed by us (Shefer 2013) in respect to the problem of finding the intermediate perturbed orbit from all the available astrometric positions of the object, using Herget's idea (Herget 1965) of the introduction of additional observations, was applied. We used two methods for solving this problem: a two-parameter method and a six-parameter method (Shefer 2013). The methodical errors of orbit computation by the proposed methods are two orders of magnitude smaller than the corresponding errors of the commonly used approach based on the construction of an unperturbed Keplerian orbit. We studied the effectiveness of the methods in comparison with Herget's n-position method. Here, by Herget's method we understand a version of our two-parameter method, which ignore the action of perturbations. For short, the computational programs implementing the Herget's method, the two-parameter and six-parameter methods will be referred to as *H*, *S1*, and *S2*, respectively.

The objects whose orbits were used to compare the programs were the asteroids 1566 Icarus and 2010 TO48. The orbit of Icarus has an unusually large eccentricity for an asteroid, and the orbit of 2010 TO48 has a small inclination. In the motion of the asteroids, we took into account the perturbations from the eight major planets, Pluto, and the Moon based on the DE405/LE405 ephemerides.

We took the first seven observations of Icarus that were made at Palomar Mountain Observatory (California, USA) in 1949, from June 27 to July 24.

The asteroid 2010 TO48 was observed in one opposition only from September 4 to October 12, 2010. We took all the twelve observations of the asteroid that were acquired at the Steward Observatory (Kitt Peak-Spacewatch, Arizona, USA) and cover this time interval.

The basic observations in the *H* and *S1* programs were the first and last observations. The two reference position vectors used in the run of the *S2* program were constructed at times $t_a = t_1^{obs} - 0.01$ days and $t_b = t_n^{obs} + 0.01$ days.

Beyond the real observations, we used the model observations of the asteroids. For dates of the real observations, we constructed 10000 sets of positions on the celestial sphere whose deviations from the corresponding precise positions account for $0''.1$, $0''.01$, and $0''.001$.

For the selected real and model observations, the asteroid orbits were constructed by each of the three compared methods. We then performed the representation of all the observations in the set with full consideration of the perturbations. Among the 10000 representations obtained by each method for each of the errors, we selected the worst representation. Let σ be the mean-square error of the worst representation of a single observation.

In all examples under consideration, the methods proposed are more effective than Herget's method. In the case of Icarus, the *S1* and *S2* programs achieve maximum effectiveness at observation accuracies exceeding $0''.1$. The values of σ are less by a factor of about 10 than the corresponding values calculated using *H*. In the case of the asteroid 2010 TO48, *S1* and *S2* demonstrate high effectiveness for all the selected observation accuracies. Here the values of σ are less by a factor of about 10–40 than the corresponding values obtained using *H*. If the observational errors are less than $0''.01$, the proposed methods give almost identical results in terms of accuracy. When the observational errors are greater than $0''.001$, preference should be given to the *S2* program. The examples considered show that the use of the proposed methods can not only improve the representation of observations covering the reference time interval, but also provide more accurate forecast ephemeris, compared to the traditional methods. The higher the accuracy of the observations and the shorter the arc defined by these observations, the more accurate is the approximation of real motion with orbits constructed by the proposed methods. This is an extremely important advantage of the proposed methods over the algorithms of the traditional approach.

Thus, the numerical examples considered lead us to conclude that our methods are a highly effective research instrument allowing the determination of reliable parameters of perturbed motion already at the stage of preliminary orbit calculations.

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