Therefore, we get:

$$\left\{\begin{array}{l}(A^A)(A^A)\ \ldots\ \text{to } S_1 \text{ factors}\\ \times(B^B)(B^B)\ \ldots\ \text{to } S_2 \quad ,,\\ \times\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\\ \times\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\\ \times\ (N^N)(N^N)\ \ldots\ \text{to } S_n \text{ factors}\end{array}\right\} > \left\{\frac{\left\{\begin{array}{l}(A+A+\ldots\ \text{to } S_1 \text{ terms})\\ +(B+B+\ldots\ \text{to } S_2 \quad ,, \quad)\\ +\ .\ .\ .\ .\ .\ .\ .\ .\ .\\ +(N+N+\ldots\ \text{to } S_n \text{ terms})\end{array}\right\}}{S_1+S_2+\ \ldots\ +S_n}\right\}^{\begin{array}{l}(A+A+\text{to } S_1 \text{ terms})\\ +(B+B+\text{ to } S_2 \text{ terms})\\ +\ .\ .+(N+N+\text{to } S_n \text{ terms})\end{array}}$$

$$\therefore\ A^{S_1 A}\ B^{S_2 B}\ C^{S_3 C}\ldots N^{S_n N} > \left(\frac{S_1 A+S_2 B+\ldots+S_n N}{S_1+S_2+\ \ldots\ +S_n}\right)^{S_1 A+S_2 B+\ldots+S_n N}$$

I am, Sir,

Yours faithfully,

STEUART E. MACNAGHTEN.

18, *Lincoln's Inn Fields, W.C.*

4 *December* 1905.

ON THE USE OF $O^{[M]}$ SELECT PREMIUMS FOR VALUATION PURPOSES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Mr. King remarks, in his recent paper on the Valuation in groups of Whole-life Policies by Select Mortality Tables (§ 4), that the reserves by the $O^{[M]}$ Select Tables are greater than by any table or combination of tables hitherto used. It is interesting to notice that the great stringency of the $O^{[M]}$ Tables is due mainly to the net premiums employed. This is made clear if the reserves by the Select and Aggregate Tables for Model Office, No. 1, at the end of 50 years, are analyzed by the formula which is given by Mr. King.

Basis of Valuation	Value of Sums Assured	Value of Net Premiums	Actual Reserves
$O^{[M]}$, 3 %	1,450,683	750,263	700,420
O^{M}, 3 %	1,449,006	760,007	688,999
Difference	1,677	9,744	11,421

Of the whole difference in the reserves, 85 per-cent is due to the difference in the value of the net premiums.

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The result of substituting $O^{[M]}$ net premiums for net premiums by the O^{M} Table in the O^{M} 3 per-cent valuation can be very easily obtained by the aid of a formula given by Mr. King (*J.I.A.*, xxxvii, p. 465). The result by the combined basis, which may be described as $O^{[M]}$ and O^{M}, 3 per-cent, is as follows:

Basis of Valuation	Value of Sums Assured	Value of Net Premiums	Actual Reserves
$O^{[M]}$ and O^{M}, 3 %	1,449,006	750,324	698,682

Here the value of the net premiums is practically the same as in the Select valuation. The difference in the value of the sums assured remains, and on the whole the result is a fairly close approximation to the Select valuation.

$O^{[M]}$ net premiums can be employed in other combinations, and various results for Model Office, No. 1, at the end of 50 years, are given in the following table in comparison with valuations involving O^{M} net premiums. The results of using ultimate factors after five years, and throughout, have been obtained by the aid of Mr. Diver's table of the values of $\phi_{[x]+t}$ (Table I in his paper). The valuations are all on the basis of interest at 3 per-cent.

Basis for Net Premiums	Basis for Valuation Factors	Actual Reserve	Comparative Reserve O^{M}, 3 % = 10,000	Comparative Reserve $O^{[M]}$, 3 % = 10,000
O^{M}	O^{M}	688,999	10,000	9,837
O^{M}	O^{M} first five years . . . $O^{M(5)}$ after five years	693,884	10,071	9,907
O^{M}	$O^{M(5)}$ throughout . . .	698,214	10,134	9,969
$O^{[M]}$	O^{M}	698,682	10,141	9,975
$O^{[M]}$	$O^{[M]}$	700,420	10,166	10,000
$O^{[M]}$	$O^{[M]}$ first five years . . . Ultimate factors after five years	700,815	10,171	10,006
$O^{[M]}$	O^{M} first five years . . . $O^{M(5)}$ after five years	703,580	10,212	10,045
$O^{[M]}$	Ultimate factors throughout .	705,147	10,234	10,067
$O^{[M]}$	$O^{M(5)}$ throughout . . .	707,911	10,274	10,107

It may be remarked in passing that, if the valuation be made by Select tables, the difference made by employing ultimate factors instead of Select after five years is less than the error that may be introduced by using nearest ages at entry and at valuation in place of exact ages, and is in fact for practical purposes inappreciable.

The combination of $O^{[M]}$ net premiums with $O^{M(5)}$ valuation factors

throughout may appear to be excessively stringent; but it differs from the $O^{[M]}$ Select valuation less than the latter from the $H^{[M]}$; and in the case of an office whose mortality has the same relation to the $O^{[M]}$ Table that the latter has to the $H^{[M]}$ Table the combination $O^{[M]}$ and $O^{M(5)}$ throughout would therefore not be so severe as a Select net premium valuation based upon its own experience.

The conclusion may be suggested that in the case of valuations which aim at the highest standard, net premiums based upon Aggregate tables should be abandoned in favour of Select net premiums, whether the valuations are based on Select tables in other respects or not.

I am, Sir,

Yours faithfully,

DUNCAN C. FRASER.

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Liverpool.
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