COSMOLOGICAL INTERPRETATION OF REDSHIFT DATA ON QUASARS THROUGH THE V/V_{max} TEST

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INTRODUCTION

The ultimate aim of statistical studies of redshift, magnitudes and flux densities of quasars is to derive the general luminosity function $\Phi(z, F_{opt}, F_{rad}, \alpha, \ldots)$ which describes the space density as a function of redshift, intrinsic optical luminosity, intrinsic radio luminosity, spectral index, etc. We assume throughout that the emission-line redshifts z of quasars are cosmological. The function Φ contains information that will be pertinent for any theory of the formation and evolution of quasars.

We use samples of quasars with known redshift z that are complete above given limits of magnitude m_{lim} and flux density S_{lim} over a certain area of the sky. The derivation of the function Φ from the observed distribution function n(z, m, S) for a complete sample is the subject of the present paper.

USEFUL OBSERVATIONAL STATISTICS

Before proceeding, it is of interest to inquire whether there are other distribution functions of the observables z, m and S for quasars that carry quantitative information about the function Φ. We list in Table 1 the six possible distribution functions of the observables. Consider the distribution function n(z, S) which represents the observed distribution of z and S regardless of magnitude m (i.e., summed over all magnitudes m). At this stage it is important to remember that quasar redshifts can only be detected at optical wavelengths, hence require $m < m_{lim}$. Also confirmation of a suspected quasar candidate is achieved at optical wavelengths only. As a consequence any practical distribution function should include m if it contains z. Hence n(z, S) is not a practical distribution function of the observables.

D. L. Jauncey (ed.), Radio Astronomy and Cosmology, 259-268.

TABLE 1

Distribution Function	Comments
n(z, m, S)	quasi-stellar radio sources
n(z, S)	impractical (z implies m-selection)
n(z, m)	quasi-stellar objects
n(S)	impractical (no confirmation, no distance scale)
n(m)	impractical (no distance scale)
n(z)	impractical

The observed distributions n(S) and n(m), usually called counts, do not contain z and hence cannot yield a distance scale. In the case of a uniform space distribution in Euclidean space, the slope of the source counts $d \log n(S)/d \log S = -2.5$, independent of the luminosity function. However, quasars are observed at redshifts where deviations from Euclidean space are large, hence this special case is of limited interest. No quantitative statements concerning Φ are possible solely with counts n(S) or n(m).

Only the observational statistics of quasi-stellar radio sources n(z, m, S), and those of optically selected quasars n(z, m) appear to be practical and useful in deriving the function Φ. We shall discuss the application of the V/V_{max} method on the observed distribution n(z, m, S) in the next section.

THE V/V_{MAX} METHOD

The V/V_{max} method (Schmidt 1968) tests whether the objects in the complete sample n(z, m, S) can have been drawn from a population with a uniform distribution in space. We can derive for each object in the sample the maximum redshift z_{max} at which the object would just still belong to the sample (i.e., $m \leqq m_{lim}$ and $S \geqq S_{lim}$). It is worth noting that for a complete sample the relevant distribution function is $n(z, m, S, z_{max})$, containing four independent variables. If V(z) is the volume in the Universe out to redshift z, and $V(z_{max}) = V_{max}$, then V/V_{max} is a measure of the position of the object within the volume V_{max} that is available to it within the sample limits. If the objects are drawn from a uniformly distributed population, then V/V_{max} should have a uniform distribution between 0 and 1 and

$\langle V/V_{max}\rangle$ should be 0.5, except for statistical errors.

The V/V_{max} test is the only one available that can be applied to objects limited in two or more observables (i.e., m, S, etc). It allows accurate allowance for the geometry of any assumed cosmological model of the Universe. If the density distribution is found to be non-uniform, a given density law $\rho(z)$ can be tested by checking whether V'/V'_{max} has a uniform distribution between 0 and 1, where $V' = \int \rho \, d V$. A method to determine $\rho(z)$ from the V/V_{max} values of the objects in the complete sample has been discussed by Lynden-Bell (1971). Once $\rho(z)$ has been satisfactorily estimated, or determined by Lynden-Bell's method, the multi-variate luminosity function is determined as $\Psi(F_{opt}, F_{rad}, \alpha, \ldots) = \Sigma(V'_{max})^{-1}$, where the summation is over all objects in the complete sample.

The reader is referred to a critique of the V/V_{max} method by Longair and Scheuer (1970) and a retort by Rees and Schmidt (1971) which highlights the main features of the method. Individual values of V/V_{max} are not particularly sensitive to the adopted shape of the optical energy distribution, contrary to a claim by Setti and Woltjer (1973).

RESULTS OF THE V/V_{MAX} METHOD

Table 2 lists $\langle V/V_{max}\rangle$ values derived for six samples of quasi-stellar radio sources. The second sample is that of Lynds and Wills (1972) based on a part of the 4C catalogue, while the third sample is mostly based on identifications by Olsen (1970) of 4C sources with redshifts (Schmidt 1975) in the declination range 20° - 40°. In both cases I used the entire 3CR sample with appropriate weight, rather than the 3CR sources in the area sampled. The last sample of Table 2 is based on parts of the 5 GHz NRAO survey (Schmidt 1976).

TABLE 2

COMPLETE SAMPLES OF QUASARS IN RADIO SURVEYS

Sample	n	$\langle V/V_{max}\rangle$
3CR	44	0.64
4C(LW1)	24+3CR	0.66
4C(20°-40°)	51+3CR	0.64
4C(LW2) (Wills 1974)	80	0.67
PKS(± 4°) (Wills 1974)	65	0.65
6cm(S1, S2, I)	51	0.61

The average V/V_{max} values are in the range 0.64 - 0.67, except for the low value of 0.61 for the 6-cm sample. Since this high-frequency sample contains the largest fraction of sources with flat radio spectra, it is of interest to see whether or not $\langle V/V_{max} \rangle$ appears to depend on the radio spectral index $\alpha = d \log S / d \log \nu$. On the basis of available samples I find (see Table 3) that steep-spectrum quasars have $\langle V/V_{max} \rangle = 0.67 \pm 0.02$, and flat-spectrum quasars $\langle V/V_{max} \rangle = 0.52 \pm 0.05$ (Schmidt 1976). The transition from steep spectrum to flat spectrum was taken to be at $\alpha = -0.2$.

TABLE 3

QUASARS WITH DIFFERENT RADIO SPECTRA

	Steep Spectrum	Flat Spectrum
α	≤ -0.20	> -0.20
n	158	28
$\langle V/V_{max} \rangle$	0.67 ± 0.02	0.52 ± 0.05

The effect on V/V_{max} of errors and variability in magnitude and flux density of quasars is of interest. A preliminary discussion suggests that for 3CR and 4C quasars the corresponding error in $\langle V/V_{max} \rangle$ might be around +0.02. Quasars selected at 6 cm exhibit more variability and the error in $\langle V/V_{max} \rangle$ is estimated to be -0.01 or -0.02. It seems most unlikely, then, that the difference in V/V_{max} of steep-spectrum and flat-spectrum quasars is caused by these errors.

The first evidence for a low V/V_{max} of flat-spectrum quasars was noticed in the 3CR sample by Schmidt (1968, page 406) and discussed by Kinman (private communication), Rowan-Robinson (1973), Van der Kruit (1973) and Setti and Woltjer (1973). Quasars from the 4C sample of Lynds and Wills (1972) did not show the effect (Van der Kruit 1973) but those from the Olsen 20^{o} - 40^{o} declination zone did (Schmidt 1974a). The addition of substantial numbers of flat-spectrum sources from the high-frequency NRAO survey has now resulted in a V/V_{max} of sufficient precision to establish the effect.

Qualitative confirmation of the low V/V_{max} for flat-spectrum quasars can be obtained from high-frequency source counts, since a very large fraction of the flat-spectrum objects among these are identified with quasars according to Brandie and Bridle (1974) and Peterson and Bolton (Wall 1975). Condon and Jauncey (1974) derived a slope of -0.85 for the integral counts of flat-spectrum sources in the Parkes 2700 MHz survey. I find from a list of 8 GHz sources by Brandie and Bridle (1974) a slope of -1.1 for the flat-spectrum sources. Such a low slope is consistent with a uniform space distribution ($V/V_{max} = 0.5$) if the redshift distribution of these quasars is similar to that of the NRAO 6-cm quasars.

Finally, we consider in Table 4 the optically selected quasars, regardless of their radio flux density. These are usually selected on the basis of their U-B color. The completeness of the Braccesi sample has been discussed by Schmidt (1974b). The average V/V_{max} of 0.70 ± 0.065 deviates significantly from that expected for a uniform distribution. The Sandage-Luyten blue objects for which I have determined redshifts (Schmidt 1974c) are probably less suitable since they were mostly selected according to U-V (Setti and Woltjer 1973). Nonetheless, their $\langle V/V_{max} \rangle$ is close to that for the Braccesi sample. Our tentative conclusion is that optically selected quasars have a $\langle V/V_{max} \rangle$ close to or identical to that of radio selected steep-spectrum quasars.

TABLE 4

COMPLETE SAMPLES OF OPTICALLY SELECTED QUASARS

Sample	n	$\langle V/V_m \rangle$
Braccesi (13^h)	20	0.70
Sandage-Luyten ($0^h, 1^h, 8^h$)	34	0.66
Low-luminosity quasars	6	0.40

Samples of optically selected quasars contain a small number of quasars with optical absolute luminosities lower than those of any quasi-stellar radio sources (Schmidt 1972). Their average V/V_{max} of 0.40 ± 0.12 is lower than those for other quasars, but the difference is hardly significant.

In conclusion the V/V_{max} method has yielded the following results. First, both optically selected quasars and steep-spectrum radio quasars exhibit a $\langle V/V_{max} \rangle$ of around 0.67. Trial density laws in a $q_o = 0$ cosmology that correspond to such a $\langle V/V_{max} \rangle$ value are $\rho = (1+z)^5$ and $\rho = \exp(-10(t-t_o)/t_o)$ where t is cosmic epoch and t_o the present epoch. Flat spectrum radio quasars may have a uniform distribution in space. However, it is not clear at all why the number of presumably short-lived phenomena such as quasars should be the same at all times. It is quite possible that the evolution of flat-spectrum quasars is much more complex and that their $\langle V/V_{max} \rangle$ is accidentally close to 0.5. Perhaps the main significance of the V/V_{max} value of flat-spectrum quasars is, then, that it is significantly different from that of other quasars.

FURTHER EVIDENCE ON THE GENERAL LUMINOSITY FUNCTION

The V/V_{max} test extracts information about the distribution of sources in depth, i.e., about the z-dependent part $\rho(z)$ of the general luminosity function $\Phi(z, F_{opt}, F_{rad}, \alpha, ...)$. We have already seen that ρ depends on the spectral index α, hence

$$\Phi(z, F_{opt}, F_{rad}, \alpha, ...) = \rho(z, \alpha)\ \Psi(F_{opt}, F_{rad}, \alpha, ...)$$

The density law ρ may, in fact, also depend on the optical absolute luminosity F_{opt}. The observed $n(z, m)$ distribution of optically selected quasars should, in principle, allow a derivation of the dependence of ρ on F_{opt} (Schmidt 1972). However, the available samples contain mostly objects near $\simeq$ 18. A survey with the Palomar 18-inch Schmidt telescope undertaken by R. Green over an area of 10,000 square degrees should yield a sufficient number of brighter quasars, near $m \simeq 15$. This large project will take several years for completion.

Quasars with redshifts larger than 2.5 are difficult to find optically, because Lyman - α emission enters the B-band, so the ultraviolet excess disappears. Osmer and Smith have succeeded in detecting quasars with $z = 2.5 - 3.1$ by means of an objective-prism Schmidt survey, a technique that requires strong Lyman - α emission but no ultraviolet excess. It appears that the numbers found (Osmer 1976, Smith 1976) may well agree with the prediction based on the density law $\rho \sim \exp(-10\ t/t_o)$, but further systematic work is needed.

An attempt to factorise the multi-variate luminosity function $\Psi(F_{opt}, F_{rad}, \alpha)$ has been made by Schmidt (1970). The fact that the observed redshift distribution of (steep-spectrum) radio quasars and optically selected quasars at optical magnitude 18 appears identical, led to the hypothesis that F_{rad} appears in the luminosity function only in its ratio to F_{opt}, i.e.,

$$\Psi(F_{opt}, F_{rad}, \alpha) = \phi(F_{opt})\ \psi(R)\ f(\alpha)$$

where $R = F_{rad}/F_{opt}$. It is easy to show that, in this case,

$$n(>f_{rad}^{lim}, f_{opt}) = G(>R^{lim})\ n(f_{opt}),$$

where the left hand side represents the optical magnitude distribution of radio quasars stronger than f_{rad}^{lim}, while $n(f_{opt})$ is the optical magnitude distribution of optically selected quasars. For derivations and details about the meaning of f_{rad} and f_{opt}, see Schmidt (1970). The function $G(>R)$ has been determined from this relation. As an example, optical counts of optically selected quasars near 17-th magnitude increase by a factor of 6 per magnitude, while those of 3CR quasars increase by a factor of only 2 per magnitude. Hence $G(>R)$ decreases by a factor of 3 per optical magnitude at the corresponding value of R.

Another advantage of this introduction of $\psi(R)$ and $G(>R)$ seemed to be that the peak in the optical magnitude distribution of 3CR quasars at 18.5 magnitude was easily represented by the proper choice of $G(>R)$. Since it decreases with optical magnitude while $n(f_{opt})$ increases, a maximum for the product is reasonable. Since $R^{lim} = f^{lim}_{rad}/f_{opt}$, we would then expect the peak in the magnitude distribution to be at an optical flux level proportional to that of the limiting radio flux density of the radio catalog. However, most reports at this Symposium on the magnitude distribution of identified radio quasars mention a peak around 19-th or 20-th magnitude. Since the limiting flux density for, say, the Westerbork identifications reported here by Dr. J. Katgert-Merkelijn is more than 100 times lower than the 3CR limit, we would have expected a peak beyond magnitude 23. The reason for this discrepancy is unclear at present.

As mentioned earlier, the flat-spectrum radio quasars appear to have an approximately uniform space distribution, in contrast to both steep-spectrum radio quasars and optically selected quasars. The relationship of their luminosity function to that of the other quasars has not been investigated yet.

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DISCUSSION

Jauncey: The n(S) and uniform space distribution of the flat, short-lived QSO's looks just like the "birth" of matter in a steady-state universe!

Longair: Is the redshift distribution for steep-spectrum radio quasars the same as that for flat-spectrum quasars?

Schmidt: At 18^m both seem to have a similar redshift distribution. A larger sample of flat-spectrum quasars is required to check this.

Rowan-Robinson: It is a prediction of the new scheme I outlined in an earlier session that there should only be weak evolution for compact, flat-spectrum sources, reflecting the normal evolution of gas and stars in a galaxy. The steep-spectrum, extended sources can arise from the interaction of a beam with an intergalactic medium whose density changes with time. I feel that we should get away from the use of arbitrary mathematical forms for evolution functions, and think about how a galactic nucleus knows what the epoch is.

R. Fanti: Could you comment on the fact that the average V/V_m for optically selected quasars is the same as for radio quasars with steep spectra?

Schmidt: I can only say that it apparently indicates that the density distribution of the two types is the same. It is too early to claim any understanding of why this is the case.

Tinsley: I would like to know what cosmological model is used in your exponential density law, since the relation between time and redshift depends strongly on q_o at large redshifts.

Schmidt: I used $q_o = 0$, so $t/t_o = (1+z)^{-1}$.

Menon: I think that the sharp distinction between the flat-spectrum quasars and the steep-spectrum quasars is due to selection effects. I am sure that if we observe most low-frequency sources and study their high-frequency spectra, most of them turn out to have flat high-frequency indices. There is already some evidence to that effect from my observations of a sample of weak Ooty occultation sources with the NRAO interferometer at 8 GHz. A high percentage of them which have normal spectra until about 2 GHz turn out to have strong central components at 8 GHz and would appear as flat spectra if observed only at high frequencies. Since the flat-spectrum sources have presumably shorter lifetimes, in considering their distribution we probably should also introduce a birth function.

Schmidt: The spectral index I used applies to frequencies below 5 GHz, and should not be dominated by the central component. Hence I do not agree at all that the effects found are due to selection effects.

Sanitt: Since there are redshift-dependent selection effects which affect the optically selected quasars, how can one be sure that the redshift distributions of optically and radio selected quasars are identical?

Schmidt: I believe that the selection for ultraviolet excess has a negligible effect on the redshift distribution for $z < 2.5$. It is the approximate identity of the redshift distribution below $z=2.5$ for optically selected and radio selected quasars that leads to the introduction of the radio luminosity function in the form $\psi(R)$.

Rees: You said that V/V_m was anomalously low for "low luminosity" quasars. Surely you would expect to obtain ~ 0.5 for nearby objects unless the evolution is extraordinarily steep at epochs close to the present?

Schmidt: It does depend on the density law, indeed. For the exponential density law $\exp(-10\ t/t_o)$ the density decreases rapidly, and even at relatively small redshifts one might still expect an average V/V_m well above 0.5.

Peterson: The mean V/V_m of flat-spectrum Parkes QSO's observed by Peterson, Jauncey, Wright and Condon (Astrophys. J. (Letters), 207, L5 (1976)) gives $<V/V_m> = 0.52$ for ten QSO's brighter than 18^m with $S_{11\ cm} > 0.3$ Jy. This value of $<V/V_m>$ for flat spectrum QSO's in the Parkes 11cm Survey is expected because Wall and Condon and Jauncey have shown that the slope of log N - log S for all the flat-spectrum Parkes sources is less than 1.5 and that nearly 80% of all these sources have QSO identifications. Thus both $<V/V_m>$ and log N - log S indicate a uniform distribution in flat space-time for the flat-spectrum QSO's.

Because the colour balance of U,B plates taken for QSO identifications is magnitude-dependent, in the sense that faint QSO's appear too red and may therefore not be identified, it is important to have photometric calibrations in your survey for optical QSO's.

Schmidt: The Bright Quasar Survey films are scanned by a PDS microphotometer and photometrically reduced on the basis of photometric standard stars observed in each field. We should be able to handle any colour imbalance in the U,B exposures without difficulty.

Longair: Exponential models of the cosmological evolution of radio sources provide a good fit to the data from the V/V_m test for quasars because of the observed fact that in the 3CR sample $<V/V_m>$ is about 0.67 independent of redshift; i.e. you get just as much evolution from $z = 0$ to 0.5 as you do from $z = 1$ to 2. Exponential evolution functions of cosmic time mimic this behaviour rather closely and are much more satisfactory than power-law models, say $(1+z)^\beta$, where all the strong evolution is pushed to large redshifts.

When you look at a much larger sample of quasars, is it still true that V/V_m is more-or-less independent of redshift?

Schmidt: I have not looked into that, but I might generalize your remark by saying that there are methods (such as Lynden-Bell's C^- method) that attempt to derive the density law rather than just assuming it and seeing whether it fits. In the past I have naively split the whole sample into the nearer half and the farther half, to see whether one law fits better than the other; that has always remained rather inconclusive and I cannot answer your question.

Bolton: I think the interesting thing to look at next is the log n - log s or V/V_{max} for the optically selected quasars as a function of their continuum spectral index. If the result is the same for the radio quasars, its interpretation is going to present some difficulties.