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THE FACULTY OF ACTUARIES
EXAMINATION PAPER, 15 OCTOBER 1925

(The time allowed was three hours.)

Board of Examiners

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PART I—Section A

- 1.* Calculate, with the aid of logarithms, the numerical value of

$$\frac{4036^{\frac{1}{2}} \times (38 \cdot 17^{-\frac{1}{2}} - 2 \cdot 021)^3}{86 \cdot 36^{\frac{2}{3}} + 0 \cdot 9162^{\frac{1}{2}}}$$

2. (a) Find, to two decimal places, the cube root of 436·218367.

(b) The uniformly growing grass on a certain meadow would be consumed by 150 cows in 15 days or by 120 cows in 20 days. How many cows are put in if 30 are removed after 8 days and the whole of the grass is consumed in 4 days more?

3. A function A varies *directly* as the square of a variable x : another function B varies *inversely* as the cube of x . If y varies *directly* as $A + B$, express y in terms of x .

If y has the values 20, 49 when x has the values 1, 2, determine the value of y when x has the value 4.

4. There are $2m$ things, m of which are alike and of one kind, and m alike of another kind: show that the total number of permutations of the $2m$ things, taken all together, is greater in this case than if there were $m-r$ alike of one kind and $m+r$ alike of another kind.

5. (a) Show that $(x+iy)(x-iy) = x^2 + y^2$ where $i = \sqrt{-1}$.

If $(a+ib)(a'+ib') = A+iB$, prove that

$$(a^2+b^2)(a'^2+b'^2) = A^2+B^2.$$

(b) What is the error in taking the sum to infinity as the sum of 1000 terms of the series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots ?$$

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

6. Establish the Binomial Theorem for a positive integral exponent.

Obtain approximations at the origin and at infinity to the curve

$$x^3 - y^3 = 3x^2.$$

Find the co-ordinates of the points of intersection of the curve with any lines peculiarly associated with it. Give a general idea of the form of the curve by means of a rough sketch.

7. Write down the values of the sine and tangent of the angles 0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$.

Exhibit *in one figure* the variations of $\sin \theta$ and $\tan \theta$. Briefly indicate the salient features of these curves.

Determine all the solutions of the equation

$$\sin \theta - \tan \theta = 0.$$

8. Express $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ in terms of trigonometric functions of θ .

Establish the following identities :

$$(i) \sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \operatorname{cosec}^2 \theta ;$$

$$(ii) \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta} = 1 + \frac{1}{2} \sin 2\theta ;$$

$$(iii) \cos \left(\frac{\pi}{3} + \theta \right) + \cos \left(\frac{\pi}{3} - \theta \right) = \cos \theta.$$

EXAMINATION PAPERS, 22 TO 26 APRIL 1926

(The time allowed for each paper was three hours.)

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PART I—Section A

1. The first of four vessels of equal capacity is filled with spirit to the extent of one-eighth, the second to one-sixth, the third to one-fourth, and the fourth to one-third. The first is then filled up with water, and from this mixture the second is filled up; again from the second mixture the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

2.* If $K = 1.057 \log_e \frac{t}{273.7} + 9 \times 10^{-7} (0.5t^2 - 503t) + 0.0902$ calculate the value of K when $t = 300.6$ to as high a degree of accuracy as can be obtained from your tables. (Assume $\log_e x = 2.303 \log_{10} x$.)

3. (a) Show that a recurring decimal is equivalent to an endless geometrical progression.

Sum to infinity

$$1 - .1 + .01 - .001 + \dots$$

(b) If s be the sum of a G.P. of n terms, p the product of all the terms of the series, and q the sum of the reciprocals of its terms, show that

$$p^2 q^n = s^n.$$

4. If $a = 2 + i$ where $i^2 = -1$ prove $2a^3 - 9a^2 + 14a - 5 = 0$.

Hence solve the equation

$$2x^3 - 9x^2 + 14x - 5 = 0.$$

Determine the modulus of $(2 + i)^2$, and show that it is the square of the modulus of $2 + i$.

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5. Resolve $\frac{2x^2}{(x-1)^2(x^2+1)}$ into partial fractions, and show that if this function be expanded in ascending powers of x the coefficient of x^n is n if n is even and $n + (-1)^{\frac{n+1}{2}}$ if n is odd.

6. Indicate what steps you would take in determining the graph of a rational function of x .

Draw the graph of $y = \frac{(x-1)(x-2)}{(x-3)(x-4)}$.

7. (a) Prove that

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

and determine for what values of θ the positive and negative signs will be applicable.

(b) If $A + B + C = \pi$ prove that

$$\sin^2 A + 2 \sin B \sin C \cos A = \sin^2 B + \sin^2 C.$$

8. Eliminate θ from the equations

$$x = 2a \sin \theta \sin 2\theta - a \cos \theta,$$

$$y = 2b \sin \theta \cos 2\theta + b \sin \theta.$$

PART I—Section B

FIRST PAPER

1. What is the probability that n persons, chosen at random, shall have the same birthday, supposing every fourth year to be a leap year?

2. A bag contains 8 sovereigns, another contains 8 shillings. A coin is taken out of each bag and placed in the other: this is done twice. Find the probable value of the contents of each bag thereafter.

3. A bag contains 3 balls, each of which is either black or white. A ball is taken out and replaced, this operation being performed 8 times, and it is found that on 5 occasions a white ball, and on 3 occasions a black ball, is drawn. What is the probability that the next drawing will give a white ball?

4. If an area A is increasing uniformly at the rate of 0.05 sq. ft. per sec., at what rate is $\sqrt{(1+2A)}$ increasing when $A = 4.5$?

Differentiate as regards x ,

$$(ax^2 + b)^n, \quad \frac{x - \sqrt{(x^2 - 1)}}{x + \sqrt{(x^2 - 1)}}, \quad \arctan \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}}, \quad \cos(\log \sqrt{x}).$$

5. Establish the Theorem of Leibnitz regarding the n th differential coefficient of w , where u and v are functions of x .

Calculate the sixth differential coefficient of $(x^2 - 2x + 1) \cos 2x$.

6. Explain how the turning values of $f(x)$ are determined. What are the conditions for maxima and minima respectively?

Find the maximum and minimum ordinates of

$$y = x^5 - 5x^4 + 5x^3 + 1.$$

7. Construct a difference table having given—

θ	$f(\theta)$
55° 40' 20"	9.75 122 249 297 287
55° 40' 40"	9.75 116 081 928 533
55° 41' 00"	9.75 109 913 275 461
55° 41' 20"	9.75 103 743 337 706
55° 41' 40"	9.75 097 572 114 905
55° 42' 00"	9.75 091 399 606 690

Use the Newton-Gauss formula of interpolation

$$(a+xw) = f(a) + x\Delta f(a) + \frac{x(x-1)}{2!} \Delta^2 f(a-w) + \frac{(x+1)x(x-1)}{3!} \Delta^3 f(a-w) + \dots$$

to compute the value of $f(55^\circ 41' 5'')$.

8. Use the method of divided differences to determine the value of p when $v=4$, given the following table of values:—

$p.$	91.6	20.55	6.874	1149.0	417.1	283.5
$v.$	5.5	10	15.5	2	3	3.5

SECOND PAPER

9. State any three methods whereby the convergency of a series may be tested.

Examine the convergency of the series whose n th terms are

(i) $\frac{x^n}{n!}$ (ii) $\left(\frac{n^2+1}{2n^2}\right)^n$ (iii) $\frac{\cos n\theta}{n}$.

10. (a) Given $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ deduce the formula

$$2 \log n - \log(n^2 - 1) = 2 \left\{ \frac{1}{2n^2 - 1} + \frac{1}{3(2n^2 - 1)^3} + \dots \right\}$$

and calculate $\log_e 11$, given

$$\log_e 2 = 0.69315$$

$$\log_e 3 = 1.09861$$

$$\log_e 5 = 1.60944$$

(b) Sum, to n terms, the series

(i) $1.7 + 2.8 + 3.9 + 4.10 + \dots$

(ii) $(1^2 + 1^3) + (2^2 + 2^3) + (3^2 + 3^3) + \dots$

11. Prove that $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ where n is a positive integer and $i = \sqrt{-1}$.

If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$ prove that, the product being continued to infinity, $x_1 x_2 x_3 x_4 \dots = -1$.

12. Evaluate

$$(i) \int_0^1 \frac{dx}{1+x^2}$$

$$(ii) \int_0^{\pi/2} x^2 \sin x \, dx$$

$$(iii) \int_0^1 x \sqrt{x^2+1} \, dx.$$

13. Find a formula of reduction for $\int \frac{dx}{(x^2-a^2)^m}$ and evaluate $\int \frac{dx}{(x^2-a^2)^2}$.

14. Defining the mean value of y , with respect to x , between $x=a$ and $x=b$ as $\frac{\int_a^b y \, dx}{\int_a^b dx}$ find the mean value of $a \sin^2 mx$ from $x=0$ to $x=\pi/m$, (i) with respect to x , (ii) with respect to $\cos mx$.

15. By use of the Euler-Maclaurin theorem

$$\int_a^{a+w} f(x) \, dx = \frac{1}{2}w \{f(a+w) + f(a)\} - \frac{B_2 w^2}{2!} \{f'(a+w) - f'(a)\} + \frac{B_4 w^4}{4!} \{f'''(a+w) - f'''(a)\} + \dots$$

obtain a formula for the sum of the p th powers of the first n integers. Hence show that the sum of the third powers of the first n integers is the square of the sum of the integers.

$$[B_1 = \frac{1}{2}, \quad B_2 = \frac{1}{6}, \quad B_3 = \frac{1}{42}, \quad B_4 = \frac{1}{30}.]$$

16. If u_x be a cubic function of x show that

$$\int_0^1 u_x \, dx = \frac{1}{24}(9u_0 + 19u_1 - 5u_2 + u_3)$$

and use this result to determine $\int_0^1 u_x \, dx$ when

$$u_0 = 13825, \quad u_1 = 15626, \quad u_2 = 17577, \quad u_3 = 19684.$$

PART II—Section A

1. Having given a table of the present values of annuities certain payable annually at the effective rate of interest i , how would you construct tables at the same effective rate if the annuities were to be payable p times a year?

2. Obtain a formula for the equal annual sum to be paid in order to redeem a debt at the end of n years, interest at the rate i to be realised on the whole debt and a sinking fund to be set up and accumulated at a lower rate j .

3. A company issues £500,000 of 5 per cent. Bonds, interest being payable half-yearly. Find the uniform sum to be set aside each half-year to pay the interest and to redeem the Bonds gradually by drawings at par in the course of 50 years. Given $\log 1.025 = .0107239$ and $\log 8.4647 = .92761$.

4. A Bond issue amounting to £1,000,000 was made in April 1922 bearing interest at $3\frac{1}{2}$ per cent. per annum payable half-yearly: it being a condition that in April 1923 and yearly thereafter a sum amounting to $4\frac{1}{2}$ per cent. of the original issue less the last half-year's interest be applied in redemption at par by drawings and the payment of the half-year's interest then due. Derive a formula to find when the last outstanding Bonds will be redeemed.

5.* Describe, giving formulae, three different methods of preparing a schedule showing the interest and principal repaid in an annuity certain payable p times a year.

Calculate the capital outstanding immediately after the sixth payment of an annuity certain of £200 payable yearly: the annuity having been calculated at 4 per cent. interest and the original term 10 years.

6.* A piece of land was let thirteen years ago for a payment of £50 each half-year, and it is provided that every nineteenth year with each half-yearly payment an additional payment of like amount shall be paid. The twenty-sixth half-yearly payment having just been made, find what uniform half-yearly payment should in future be received. The nominal rate of interest to be used is 5 per cent. convertible half-yearly.

7.* The owner of a Capital Redemption Policy for £100 payable 30 years hence and effected 10 years ago at an annual

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premium of £1, 0s. 3d. is offered its value to realise 4 per cent. to the purchaser, while the Company has offered to allow him to surrender it for 95 per cent. of the ten premiums paid accumulated at 3 per cent., or to allow him to convert it to a Paid-up Policy for £25. Show which offer is the most advantageous.

8.* A series of Bonds, totalling £300,000 bearing interest at 4 per cent. payable annually, is issued at $86\frac{1}{4}$, it being a condition of the issue that at the end of the twenty-first year £25,000 will be redeemed by drawings at par, at the end of the twenty second year £35,000 will be so redeemed, and at the end of the twenty-third year £45,000, and so on until the whole issue is redeemed. What rate of interest does the original purchaser secure?

9.* A sum of money accumulating under force of interest δ doubles itself in $14\frac{3}{4}$ years. How long would the money take to double itself at a nominal rate of interest numerically equal to δ and convertible twice a year?

PART II—Section B

FIRST PAPER

1. Obtain an expression for the single premium required for a yearly annuity to (x) to commence at the end of the year of death of (y), which in the event of (y) dying within n years will be payable to the end of the n years whether (x) survives or not, the premium being returnable in the event of no annuity payments falling to be made.

2.* Find by the H^m table in the case of two lives aged 20 and 30

(a) the probability that one at least will die before attaining age 50;

(b) the probability that the last survivor will die aged 50 last birthday.

3.* A is accepted for a thirty year endowment assurance, with profits, of £500 at an increased premium. He points out that he may pay more under the contract than the sum assured, and it is suggested that he should pay a premium decreasing throughout the term by a uniform amount each year, such that the present value of the decreasing premiums would be equal to the present

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value of the increased level premiums, the first premium being £30.

Calculate on an H^m 3% basis using age 35 the amount by which this premium must decrease annually so that the maximum payment he can make is £500, and find the increased level premium at which he was originally accepted.

1. (a) Explain the meaning of the symbols $Q_{x:\overline{y}|}^3$ and $Q_{x:\overline{y}|}^2$, and show that they are equal in value.
- (b) Show that $A_{x:\overline{y}|}^3$ is not, however, equal to $A_{x:\overline{y}|}^2$ and obtain an expression in terms of a compound annuity for the difference between them.

5.* Find on an H^m 3% basis the value of ${}_{|20}A_{31:40:50}$ by an exact method.

6. A clerical staff in a stationary condition is recruited throughout each year by 300 entrants at age 18. There are no withdrawals other than by death or by retiral on pension. Half of the survivors to age 60 then retire on pension and all surviving to age 65 retire at that age.

Find an expression for the total number of future years of active service that will be given by the existing staff.

7.* Find on an H^m 3% basis the cost of a yearly annuity of £100 to (40) to be entered upon on attainment of age 65 and to be continued for 10 years after the death of (40) or such lesser number of years as a life now aged 33 may survive.

8.* Given that $P_{20:\overline{45}|} = {}_{34}P_{20} = 1.681\%$ on an H^m 3% basis, prepare a rough graph of the difference between ${}_tV_{20:\overline{45}|}$ and ${}_{34}V_{20}$.

SECOND PAPER

9. (a) Prove that ${}_nQ_{x:x}^1 = \frac{1 - e^{-2\int_0^n \mu_x + t dt}}{2}$.

(b) Prove that $\mu_x = (e_x^2)^{-1} + \frac{d \log e_x^2}{dx}$.

10.* Calculate by the H^m (Text Book) Table the probability that the third death among four lives now aged 30, 35, 40 and 45 respectively will take place within 30 years.

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11.* The following is an extract from a Combined Marriage and Mortality Table :—

Age = x .	Number living at exact age x .		Decrements caused by :		
	Bachelors = $(bl)_x$.	Married = $(ml)_x$.	Bachelors marrying = $(bm)_x$.	Bachelors dying = $(bd)_x$.	Married men dying = $(md)_x$.
33	51,002	29,711	4335	376	207
34	46,291	33,839	4027	345	240
35	41,919	37,626	3773	320	273
36	37,826	41,126

Calculate: (i) the annual rate of mortality among bachelors aged 34 ;

(ii) the probability that a bachelor aged 33 will be alive and married two years later.

12. Obtain a convenient approximation for the net single premium, with interest at rate i , to secure a Whole Life Assurance of £1 on a life aged x , with compound reversionary bonuses at the rate of b per unit per annum, compounding yearly. Explain the extent and direction of the error involved.

13.* A life aged 50 is entitled to a temporary life annuity of £60, by payments of £20 due on 1st September, 1st January, and 1st May in each year, with proportion to date of death; the annuity ceasing after the payment due 1st May 1941.

Calculate, by the $O^{(NM)}$ Table with 3 per cent. interest, the value of the annuity on 1st June 1926.

14.* A Life Office issues only Whole Life Assurances without profits, payable on 31st December of the year in which death takes place. The premiums charged are the H^m (Text Book) 3 per cent. net premiums plus a loading for expenses, and are payable annually on 1st January: the valuation basis is also H^m (Text Book) 3 per cent. The following particulars are available in respect of year 1925 :—

Age on 1st Jan. 1925.	Sums Assured in force 1st Jan. 1925.	Reserves at 31st Dec. 1925 in respect of Policies still alive on that date.
41	£10,000	£1513
42	£15,000	£1753
43	£29,000	£3900

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Death was the only cause of exit during the year, and the following policies became claims:—

A, for £1000, effected 1st January 1909 at age 25.

B, for £300, effected 1st January 1919 at age 37.

No new policies were effected during the year. Calculate the profit or loss from mortality on the year's working.

15.* Calculate, by the H^m (Text Book) Table with 3 per cent. interest, the reserve before payment of due premium under a last survivor assurance, issued 15 years ago on two lives then aged 53 and 46 respectively, the elder of whom has since died.

16. Explain a continuous process by which a Table of Policy-values for Endowment Assurances by annual premium can be constructed, and illustrate your answer by setting out the working columns required.

THIRD PAPER

17. By application of the general formula

$$\frac{1}{m} \sum_{t=1}^{t=\infty} f\left(\frac{t}{m}\right) = \int_0^{\infty} f(h) dh - \frac{1}{2m} f(0) - \frac{1}{12m^2} \frac{df(0)}{dh} - \dots$$

show that
$$\ddot{a}_x^{(m)} = \ddot{a}_x \left(1 - \frac{\delta}{2m}\right) + \frac{\delta}{12m^2}.$$

18. An estate is held in perpetuity on successive lives on the condition that from the end of the year in which each successive life drops, a period of t years is to elapse before a new life is nominated, and a fine of 1 is to be paid at the beginning and at the end of each such period of t years.

Assuming that the life in possession is aged x , and that the succeeding lives will be nominated at age y , show that the present value of all the fines in perpetuity $= \frac{A_x(1+v^t)}{1-v^t A_y}$.

19.* On a net H^m 3 per cent. basis find the annual premium limited to 20 payments to secure an endowment assurance payable on (35) surviving 25 years, or at his death if previous, with the proviso that the death benefit is to be increased by 50 per cent. of the premiums paid and the survivance benefit by 50 per cent. of the original sum assured.

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20. An endowment fund provides two annuities of £50 each held at present by two lives aged 90 and 85. There is a waiting list of three who are entitled to succeed successively to an annuity on the death of a holder. The ages of those on the waiting list in their order are 80, 75, and 70.

Express in the form of annuities the present value of (70)'s interest in the fund.

21. Show by means of partial integration that

$$\int_0^n v^t p_{xy} \bar{A}_{x+t: \frac{1}{y+t}: \overline{n-t}} dt = \int_0^n t v^t p_{xy} \mu_{y+t} dt.$$

22.* A whole life policy for £1000 effected 25 years ago on a life then aged 25 has now £480 of reversionary bonus attaching to it.

On a net H^m 3 per cent. basis determine how much of the bonus must be surrendered in order to convert the remainder of the policy into an endowment assurance maturing at age 65—ignoring the question of bonus loading.

23.* Calculate the value of $\bar{A}_{56:66}^1$ on an $O^{(NM)}$ ultimate 3 per cent. basis by a double application of Simpson's approximate integration rule.

24. Assuming that a mortality table follow Makeham's Law, show how the values of l_x may be calculated at all ages, given values for $\text{colog } t p_x$, $\text{colog } t p_{x+t}$, and $\text{colog } t p_{x+2t}$.

25.* If in a certain mortality table $\mu_x = \cdot 02052$ and $p_x = \cdot 991$, find the value of p'_x in a table so related to the first that $\mu'_y - \mu_y = \cdot 002$ for all values of y , given $\frac{1}{\log_e 10} = \cdot 4343$.

PART III—Section A

FIRST PAPER

1. An investigation is required into the mortality experienced by a body of assured lives over a period of years. The data are supplied on cards giving the years of birth, entry, and exit, mode of exit, sex, name, and class of assurance. Explain how you would proceed, giving your exposed to risk formula, and state what portions of the experience you would investigate separately. Mention any points which would influence you in deciding whether to include all classes of assurance in one group, and whether to base the investigation on "policies" or "lives." Refer to any reasons why satisfactory "select" tables may not be obtainable from the

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data supplied, and indicate how you would arrive at a measure of the duration of "selection."

2. Give a brief description of the recent investigation into the rates of mortality experienced by life annuitants of British Offices. State the assumptions made regarding the future trend of mortality rates, and mention any others which might have been adopted.

3. It is desired to investigate the rates of mortality, marriage, and withdrawal experienced during a period of years by the bachelor members of a widows' fund in order to construct a triple decrement table showing the number of bachelors who survive to each age and the number who die, marry, and withdraw in each year of age out of a given number commencing at age 20. Show fully how you would construct the ungraduated table given the number of bachelors living at each age l.b.d. on 1st January of each year and the number who die, marry, and withdraw each year according to nearest age at exit. State, giving your reasons, whether you consider an aggregate table of this nature satisfactory.

4. Give a short survey of the development of the methods which have been employed in the United Kingdom in arriving at the rates of mortality in the general population, indicating the improvements which have been introduced from time to time. Illustrate your answer with references to existing tables.

5. Explain clearly why the *central* rate of sickness was employed in the M.U. 1893-1897 experience, and show how this differs from the function which had generally been dealt with in earlier experiences.

Obtain exposed to risk formulae for calculating (a) rates of withdrawal and (b) central rates of sickness from data supplied in the following form:—

Age x	Existing at 1/1/20 s_x	Entrants 1920-1925 n_x	Deaths 1920-1925 d_x	Withdrawals 1920-1925 w_x	Existing at 31/12/25 e_x	No. of weeks sickness in 1920-1925 between x and $x+1$
15	—	700	1	3	—	40
16	400	13,300	8	100	150	2050
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮

In the case of s_x and e_x x is the nearest age on 1/1/20 and 31/12/25 respectively, and in the case of n_x , d_x , w_x , and weeks of sickness, x is the nearest age at the beginning of the calendar year of entry, exit or sickness. It may be assumed that new entrants are not entitled to sickness benefit for the first four months of membership, and that those who withdraw cease to be entitled to sickness benefit three months before the date of withdrawal.

Employ your formulae to obtain the central rate of sickness and the rate of withdrawal at age 16, and mention any circumstances which would tend to vitiate your results at the initial ages.

6. (a) From the total registered deaths and the mean population at all ages in a decennial period the ratio of deaths to mean population is calculated separately in respect of two occupational groups. Explain why a comparison of the two ratios does not provide a reliable indication of the relative mortality rates of persons engaged in the two occupations examined. Indicate what steps you would take to obtain a more accurate comparison.

(b) If for any occupational group you have obtained rates of mortality at each age, state, giving your reasons, whether you consider that these rates may be employed to represent the future mortality of a body of lives engaged in the specified occupation.

SECOND PAPER

7. Construct a summation formula of graduation of which the operator is $[3] [3] [9]$ and compare its smoothing coefficient with that of Spencer's 21-term formula, viz. $\frac{1}{160}$.

8. The following is an extract from the early section of certain mortality data together with two graduations thereof:—

Age.	E_x	θ_x	Ungraduated q_x .	Graduated q_x .	
				(1)	(2)
41	251	2	·00797	·00794	·00961
42	297	4	·01347	·01096	·00969
43	334	3	·00898	·01332	·00978
44	388	5	·01289	·01508	·00988
45	448·5	8	·01784	·01609	·01000
46	531	9	·01695	·01616	·01015
47	608·5	16	·02629	·01535	·01034
48	689·5	5	·00725	·01387	·01056
49	780·5	7	·00897	·01234	·01081
50	918·5	9	·00980	·01130	·01121
51	1040	12	·01154	·01098	·01191
52	1211	12	·00991	·01138	·01273
53	1367	22	·01609	·01236	·01364
54	1533	21	·01370	·01375	·01461
55	1775	26	·01464	·01540	·01566

Test and discuss the graduations, and compare them on the assumption that for the higher ages the graduation is satisfactory in both cases.

9.* Calculate the mean deviation from the median and the standard deviation of the following:—

Death Rates per week per 10,000 living (in order of magnitude).
183
181
179
178
176
173
172
171
170
168
165

10. Discuss the applicability of Makeham's formula to modern mortality statistics illustrating your answer by reference to recent mortality investigations.

11.* Make a graphic graduation of the following data. The graduation is being made in order to obtain an indication of the prices at which Annuities can be sold to the class concerned:—

* A Short Collection of Actuarial Tables is supplied for use in answering these questions.

Age.	E_x	d_x	$100q_x$
35	723·0	2	·28
6	693·5	2	·29
7	681·0	4	·59
8	672·0	6	·89
9	682·0	7	1·03
40	661·0	7	1·06
1	638·5	4	·63
2	598·5	2	·33
3	557·0	3	·54
4	552·5	6	1·09
5	548·5	2	·36
6	535·5	8	1·49
7	511·0	2	·39
8	493·5	4	·81
9	472·0	5	1·06
50	437·0	3	·69
1	445·0	6	1·35
2	439·0	4	·91
3	436·5	8	1·88
4	402·0	7	1·74
5	382·0	7	1·83
6	361·5	7	1·94
7	329·5	9	2·73
8	290·5	10	3·44
9	272·5	6	2·20

Cross-ruled paper will be supplied for use.

PART III—Section B

FIRST PAPER

1. A Life Office decides to introduce an entirely new Children's Deferred Assurance scale. The unit is an annual premium of £10, and for each unit the following options are to be quoted:—

At the last policy anniversary preceding attainment of age 15

(1) Cancel policy and take Educational Annuity for four years.

At the last policy anniversary preceding attainment of age 21

(2) Cancel policy and take a cash payment.

(3) Cease premiums and take Paid-up Assurance without profits, Whole Life or Endowment Assurance.

(4) Continue payment of same premium for Whole Life or Endowment Assurance, with or without profits.

State the bases and formulae by which you would calculate the amounts per £10 premium for each of the above options, and mention what allowance you would recommend in event of the child's death, or of the policy being surrendered, before attainment of age 21.

Show also how you would calculate the additional annual payment required to make premiums until option date before age 21 depend on the life of a parent, and state how, if at all, this provision would affect the options and allowances.

2.* A, aged 50, has borrowed £1000 at 5 per cent. from B, and has undertaken repayment in 5 years by an annual payment of £231 to cover capital and interest. To secure himself against A's death, B desires to effect a temporary assurance on A's life, the sum assured in each year to be equal to the capital then outstanding.

Calculate, at O^{NM} 3 per cent. with a loading of 20 per cent., the office annual premiums required for the assurance. (Mention the basis you would use in practice, if different from the above.)

3. Discuss the effect of Build on the assessment of lives for assurance, in relation (a) to age of proposer, (b) to class of assurance, and (c) to family and personal history.

4. Describe briefly the geographical position and living conditions of each of the following eight South American towns:—

Bogota	Manaos	Rio de Janeiro
Guayaquil	Montevideo	Valparaiso.
Iquique	Para	

In each case, state in which of the following classes you would place the town, in considering a Whole Life Assurance on a first-class British life, of favourable occupation, resident therein. (Class I., no extra premium; Class II., extra of £1 per cent. or less; Class III., extra of over £1 per cent.)

5. (a) Wherein do the bases and formulae used for calculating premiums for "Industrial" life assurances differ from those used for "Ordinary" life assurances, and why?

(b) What special conditions affect premium-rates for Industrial life assurances on the lives of children?

6.* A Friendly Society issues a new Table, under which the following benefits are assured:—Sickness Benefits (ceasing at age 65): £1 per week for the first year of sickness, 10s. per week for the second year, 5s. per week thereafter; any sickness recurring within 12 months of the last attack being treated as a continuation thereof. Death Benefits: £10 at death of member and £5 at death of member's wife during his membership.

* *A Short Collection of Actuarial Tables* is supplied for use in answering these questions.

Annuity Benefit: 5s. per week commencing at age 65, or alternatively a Cash Endowment of £50 on the 65th birthday.

(a) Calculate by the Manchester Unity (Whole Society) Tables with 4 per cent. interest, the weekly contribution ceasing at age 65 for the above benefits, in respect of lives aged 16, 25 and 40 at entry—16 and 40 being the lower and upper limits of age at which new entrants are admitted.

(b) How would you reply to a suggestion that the contribution should, as under previous Tables issued by the Society, be made uniform for all members irrespective of age at entry?

SECOND PAPER

7. How would you arrive at the market value of the undernoted securities when a valuation of assets is being made:—

- (1) Securities quoted in the Daily Official List;
- (2) Dollar Securities which are quoted only in the United States;
- (3) Securities purchased in Great Britain which have no official quotation?

How would you deal with accrued interest in each case?

What information regarding Stock Exchange Securities must be given in the Third Schedule to the Assurance Companies Act 1909 and what Certificate must be appended thereto?

8. A Colonial Government made an issue of 5% Inscribed Stock at a price of 98% payable 5% on application and the balance in instalments. The first payment of interest, due 6 months after issue, was 2% representing interest on the various instalments as from the dates on which these fell due. A Company underwrote £10,000 of this issue at a commission of 1% and had to take up £4000 of the amount underwritten. When the allotment was received the remaining instalments due were paid under discount of £- : 7 :- for each £100 stock.

- (1) What is the net cost price % of the Stock to the Company?
- (2) Draft the various cash book, journal and ledger entries made up to and including the first interest payment.

9. Under what headings would the following items be included in the First Schedule of the Assurance Companies Act 1909? Mention how they would be dealt with.

- (1) Profits on Reversions which have fallen in;
- (2) Interest, or fines, charged on premiums paid when overdue;

- (3) Annual premiums a portion of which is allowed to stand as a debt on the policy and interest charged thereon. How do you suggest interest should be collected in this instance ?
- (4) Bonus surrendered in reduction of premiums ;
- (5) Commission received and allowed in respect of reassurances.

10. Draft a short memorandum for Directors on the advisability of investing funds in one of the following :—

- (1) Farm Loans in Canada ;
- (2) Foreign Government Securities issued in New York ;
- (3) Preference Shares of British Industrial Companies ;
- or (4) Loans granted over House Property coupled with a Life Assurance Policy under a House Purchase Scheme.

Mention the important points to be considered in each instance.

11. In November 1925 the Chancellor of the Exchequer made the following statement :—

‘The time has now come when we can take a further step towards the establishment of complete freedom and normal conditions in the money market. From this time forward no objection will be raised on general grounds by the responsible authorities to the issue of Dominion, Colonial, and foreign loans on the London market.’

Mention briefly what you know about the embargo to which the Chancellor referred and why it was considered expedient to remove this. What other steps in the direction indicated in the first sentence of the quotation were taken in 1925 ?

12.* What is the relationship between Indian Railway Annuities Class A and Class B ?

Assume that the market value of an Indian Railway Annuity Class B is £17, 10s., that the total amount of the B Annuities issued was £50,000, that the original term of the Annuity was 45 years of which 20 are still to run, that the Sinking Fund deduction (which was based on an ultimate repayment of £25 per £1 Annuity) is 5s. 9d. per £1 and that the investments presently held by the Sinking Fund are

Nominal Amount.	Security.	Total Cost Price.	Present Market Value %.
£150,000	5% War Loan 1929-47	£142,500	100
100,000	3½% Conversion Loan	74,250	76½
100,000	Indian Govt. 5½% Loan 1932	98,000	102
40,000	Cape of Good Hope 4% Stock 1916-36	38,000	91
12,000	of its own Class ‘B’ Annuities	222,000	17½ per £1
	Uninvested Cash Balance	3,973	

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

What would you estimate the gross yield to be at the present price? The annuity is payable yearly and no allowance need be made for expenses. Base your calculations on the assumption that during the remainder of the term the fund will accumulate at $3\frac{1}{2}\%$ free of Income Tax.

13. Criticise briefly as investments for an assurance company any two of the following foreign government obligations, mentioning as concisely as possible what you know of the financial position of the countries concerned. The securities mentioned are all sterling issues quoted in the Daily Official List.

Security.	Redeemable by purchase or drawings.	Yield % allowing for redemption.
Japanese 6% Loan of 1924	1929-59	£6 13 6
Czecho-Slovakian 8% Bonds of 1922	1925-52	7 16 6
Brazilian 5% Funding Loan 1914	1927-76	6 6 0
Siamese 6% Loan 1924	1930-64	6 0 0

PART IV

FIRST PAPER

1.* The following particulars are extracted from the Books of a Mutual Life Company which grants Whole of Life Policies, with Profits, by annual premiums payable yearly throughout life.

Business in force at 1st January 1925.

Nearest age on 1.1.1925.	Sum Assured.	Reversionary Bonus.	Office Premium.	H ⁿ 3% Net Premium.
61	8000	3000	210	185
62	5000	2000	140	120
63	6000	2500	135	115
64	4000	1800	95	80
Total	23,000	9300	580	500

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

New Policies effected during year 1925.

Date of Birth.	Sum Assured.	Office Premium.	Net Premium.
3.6.1863	250	19	16.6
5.12.1861	1500	121	110.1
8.9.1860	500	42	38.7

Cancelments and Alterations during year 1925.

Nearest Age on 1.1.1925.	Sum Assured.	Declared Bonus.	Interim Bonus.	Office Premium.	Net Premium.
61	500	200	...	14	10.8 ¹
63	400	190	6	10	7.8 ²
64	300	50	...	12	10.6 ³
61	500	150	...	17	15.0 ⁴

REMARKS.

- ¹ Death before payment of year's premium.
- ² Death after payment of year's premium.
- ³ Surrendered before payment of premium for £120.
- ⁴ Participating Paid-up Policy granted before payment of premium for Sum Assured of £300 with declared Bonus Additions of £90 and future participation.

<i>Funds at beginning of year</i>	£18,300
<i>Interest (less tax) received during year</i>	860
<i>Commission and Expenses paid</i>	100
<i>Profit on Investments</i>	11

From the information given draft out the Revenue Account for year 1925, value the business on an H^M 3% (Text Book) basis, analyse in detail the profits of the year and find the maximum rate of Simple Reversionary Bonus that can be declared, after retaining the same Carry Forward as at the end of the previous year.

Premiums are charged according to age next birthday and the following assumptions may be made:—

- (a) Claims are on the average paid in the middle of the year.
- (b) Premiums fall due on the average in the middle of the year.
- (c) Cancelments and alterations are effected on the renewal date.

All calculations should be worked out to one place of decimals.

2. State how you would value the following special class policies :—

- (a) Convertible Term Assurances before and after conversion.
- (b) Children's Deferred Assurances during the deferred period when the risk commences at age 21 and under which premiums up to age 21 are limited to the father's lifetime.

3. In the valuation of Industrial Assurance Policies state how policies on the lives of children under 10 years of age are generally treated and mention the reasons for such treatment.

SECOND PAPER

4. Describe the "collective" method of valuation of Widows' Funds, and discuss the relative merits of the "reversionary annuity" and "collective" methods.

Give full particulars of how you would allow in the valuation of an old-established Widows' Fund for the fact that the scale of contributions for future members is insufficient, on the valuation basis, to purchase the benefits.

5. The superannuation fund for the benefit of the employees of a certain large and old-established business house provides for pensions at retirement, which must take place not later than age 65, on the basis of a percentage (increasing in proportion to the number of complete years' contributions made before retirement) of the average salary during the whole term of contributing membership.

In addition, the following subsidiary benefits are granted :—

1. Withdrawals from service—return of member's contributions.
2. Death in service—return of both member's and Company's contributions, with interest at rate 3%.

The contributions are a fixed percentage of salary and are paid equally by member and firm.

Give full details of the particulars you would call for in making a valuation of the Fund, and mention the methods and sources of information you would use in deciding on your bases as to mortality and salary scale.

State in symbols the various tables which would require to be constructed (including all subsidiary tables) in order to value the liability in respect of prospective pensions to present contributors.

How would you estimate the effect on the Fund if salaries of all members were reduced by 20% on a fixed date?

6. What particulars are required to be furnished under Fourth Schedule of the Assurance Companies Act 1909 in respect of Employers' Liability business ?

7. In a Friendly Society consisting chiefly of members engaged in clerical occupations a very heavy sickness cost is exhibited. Mention any causes which would account for such a feature, and indicate how you would deal with it at a valuation.

8. Assuming secessions are to be allowed for in valuing a Friendly Society, how would you arrive at a basis of valuation ?

THIRD PAPER

9.* A, a female aged 65, has the life interest in, and B, a male aged 45, has, if he survive A, the reversion to a fund consisting of—

£10,000 5% War Loan 1929/47

£10,000 Union of South Africa 6% Stock 1930/40

and £10,000 New Zealand Govt. 3½% Stock 1940.

Value their respective interests for separate purchases assuming that A is insurable at an annual premium of £8, 5s. 0d.%, and that the annual premium for an assurance payable if B predecease A is £2, 3s. 0d.%. The market values of the securities mentioned may be taken as 100%, 103%, and 84% respectively.

10.* A, a female aged 60, has the life interest, provided she remain unmarried, in a fund consisting of—

£5000 Dorman Long & Co. 5½% 1st Mort. Deb. Stock

£5000 Grand Trunk Ry. of Canada Perp. 4% Cons. Deb. Stock
(G'teed Dominion of Canada)

£10,000 London & North Eastern Ry. 5% Preferred Ordinary
Stock

and £100 East Indian Ry. Annuity Class A (expiring in 1953).

The market value of these investments may be taken as 92%, 77%, 59%, and £12, 5s. per £1 respectively, and they are all authorised by the Trust Deed.

B has the absolute reversion to the fund and desires to sell his interest, retaining the option of repurchase within 5 years.

What sum would you be prepared to offer to B ?

Mention any special points to be considered.

* *A Short Collection of Actuarial Tables* is supplied for use in answering these questions. Calculations should be made at 3% interest with the H^m Mortality Table (Text Book Graduation), using actual ages, but candidates are required to state the rates of mortality and interest which they would employ in practice.

11. The Life Rent of an Estate valued at £100,000 is payable to A, a male aged 70, and B, a female aged 65, during their joint lives and the life of the survivor, subject, however, to the payment of an annuity of £500 to C, a widow aged 50, during her lifetime provided she does not remarry. On the death of the survivor of A and B a legacy of £10,000 is to be paid to D, a male aged 50, if he be then alive, and the balance of the fund, subject to the annuity to C, falls into the possession of E, a male aged 30, provided he be then alive.

E desires to sell his interest. State how you would deal with the matter, giving formulae, and mention the table of mortality and the rate of interest which you would employ. Full particulars of any policies of assurance required should be given.

12. A Trust Fund consists of—

- (1) Heritable Property in Scotland stated to consist mainly of farm lands, with the buildings thereon, bringing in a net income after allowing for all charges and deductions of £200 p.a.
- (2) Feu Duties for in all £300 p.a.
- (3) Mortgages for £10,000 secured over real estate and heritable property.
- (4) Leasehold Property in England.

Assuming that a proposal for the sale of the reversion to the fund be received by an office, mention briefly the points that would have to be considered when arriving at the value of the fund.

13.* Under the terms of a settlement a sum of £500 p.a. is payable to A, a female aged 70, during her lifetime, and after her death to B, a female aged 45, for the remainder of her lifetime.

B desires to borrow on security of her interest and is willing to assign a paid up policy for £300 on her life as collateral security. Her life is now insurable at an annual premium of £3, 5s. 0d. %.

What sum would you advise should be lent and what rate of interest would you charge?

14. Ten years ago A, a male then aged 35, and B, his wife, then aged 30, effected an assurance of £1000 with profits payable on the death of the survivor by an annual premium of £24, 10s. payable for 15 years. In lieu of an extra premium for A's occupation as a wine merchant a constant debt of £200 for 30 years was attached to the policy.

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question. Calculations should be made at 3% interest with the H^M Mortality Table (Text Book Graduation), using actual ages, but candidates are required to state the rates of mortality and interest which they would employ in practice.

Quotations are now asked for the surrender value of the policy and for the amount of paid up policy, free of debt, that can be granted in lieu thereof.

How would you arrive at the quotations desired? Mention the formulae which you would employ, and the table of mortality and the rate of interest which you would use.

FOURTH PAPER

15. A policy for £100 was effected many years ago by a man on his own life for the benefit of his wife AB and his children and payable to "The Trustee or Trustees for the time being entitled to give a receipt or discharge." The policy has become a claim by the death of the life assured, AB still survives and no notice has been received of the appointment of a Trustee. State the legal requirements and the modifications, if any, you would be disposed to make—

- (a) if the policy had been effected under the M.W.P.A. 1870;
- (b) if the policy had been effected under the M.W.P.A.(S.)A. 1880.

16. The assured under an Endowment Assurance Policy is now a certified lunatic. State under what circumstances the Company can allow a surrender of the policy and can make advances on its security. Under what circumstances other than lunacy and bankruptcy can a policyholder's interest be involuntarily alienated, and how in such circumstances is the title to deal with the policy completed?

17. State briefly your requirements in connection with a claim by death under a policy issued in England on the life of and in favour of AB described therein as of London: the title consisting of—

- (a) an absolute assignment in favour of CD;
- (b) a deed of arrangement executed a month ago by CD followed by
- (c) an assignment to EF by the Trustee, and
- (d) the death intestate of EF in Canada where he was domiciled.

18. State fully to what points you would give attention in determining whether the undernoted documents are sufficiently stamped—

- (a) Assignment by way of Mortgage as Collateral Security?
- (b) Marriage Settlement containing only policies of assurance and household effects?

- (c) An absolute Assignment executed in Canada endorsed on policy wherein the life assured was described as resident in Canada.

The policies affected may be considered to be issued by a Company within and having no branches without the United Kingdom. In each case the deed is the only one affecting the title.

19. An Insurance Company, registered under the Companies Acts as a Limited Company, proposes to acquire the business and share capital of a Proprietary Life Assurance Company incorporated by Act of Parliament. State the legal formalities to be gone through and which, if any, might be dispensed with. What action would you recommend should be taken to make the requirements less onerous?

20. In respect of year of assessment 1925/26 a Life Office having its Head Office within the United Kingdom has—

- (a) total funds (inclusive of foreign fund and annuity fund) amounting to £8,000,000 ;
- (b) a foreign fund amounting to £400,000 invested in 5% War Stock, the income from which has been re-invested in same security ;
- (c) an earned rate of interest of $5\frac{1}{2}\%$ on its funds ;
- (d) untaxed interests as per return to Inspector of Taxes of £120,000 (the balance of interest income having borne U. K. tax by deduction at source) ;
- (e) expense of management (less fines and fees) £80,000 ;
- (f) profit from Reversions £2000 ;
- (g) an interest income of £20,000 from its annuity fund ;
- (h) annuity payments during the year amounting to £40,000, all of which have been subjected to U. K. tax at the full standard rate of tax.

From the above particulars show how you arrive at the net liability of the office for U. K. Income Tax in respect of 1925/26. It may be assumed that no adjustment is necessary in respect of profits on the Annuity Fund.

All "Funds" may be taken as Mean Funds having been calculated by the formula $\frac{A+B}{2} - \frac{I}{2}$.