Universal Heliophysical Processes
Proceedings IAU Symposium No. 257, 2008
N. Gopalswamy & D.F. Webb, eds.

doi:10.1017/S174392130902986X

Alfvén waves in a gravitational field with flows

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Abstract. The gravitational stratification effect on magnetohydrodynamic waves at a single interface in the solar atmosphere has been studied in the penumbral region of the sunspot recently. The existence of slow and fast magneto acoustic gravity waves and their characteristics has been discussed. The effect of flows on magneto acoustic gravity surface waves leads to modes called flow modes or v-modes. The present geometry is that of a plasma slab moving with uniform velocity surrounded by a plasma of different density. As is applicable to the corona, we assume that the plasma β to be small. The dispersion characteristics change significantly with a change in the value of G (gravity) and uniform flow.

Keywords. Sun: Oscillations

1. Introduction

It is clear from the literature that gravity is one of the dominant forces in the solar atmosphere. It plays an important role in the dispersion characteristics of Alfven and magneto acoustic waves and several authors (Erdelyi *et al.*, 1999, Miles & Roberts 1989, Miles & Roberts 1992, Miles *et al.* 1992, Roberts 1991) have studied these waves in different contexts. Varga & Erdelyi (2001) have extended the work of Miles *et al.* (1992) by assuming the uniform flow of plasma over a field free medium and found that the flow causes some modes to appear and others to disappear. Satya Narayanan (2000) and Sengottuvel & Somasundaram (2001) have also discussed the effect of equilibrium flow on Magneto Acoustic Gravity Waves in the solar atmosphere and suggested the running penumbral waves from penumbra to be the gravity influenced magneto acoustic surface waves. Recently, McEvan & Diaz (2007) have investigated the effect of gravity on a horizontal coronal slab by extending the work of Edwin & Roberts (1982) and reported that the presence of gravity modifies the oscillatory frequencies of the slab and resulting in the possible transition between surface and body modes.

It is also well known that gravity waves play an important role in studying the coupling of lower and upper solar atmospheric regions and are therefore of tremendous inter- disciplinary interest. The gravity waves in the Sun may be divided into two types, namely, (i) the internal gravity waves, which are confined to the solar interior, and (ii) the atmospheric gravity waves, which are related to the photosphere and chromosphere, and may be further beyond. In general, the observation of gravity mode oscillations of the Sun would provide a wealth of information about the energy- generating regions, which is poorly probed by the p-mode oscillations. It has been suggested that the turbulent convection below the photosphere will generate the high order, non-radial g-mode oscillations (internal gravity waves). There are theoretical studies earlier on solar-atmospheric gravity waves by several authors (Lighthill (1967), Stein (1967), Schmieder (1977). Traces of internal gravity waves are present in the ω - k diagrams of Frazier (1968). One of the earliest observations of internal gravity waves from individual granules is from the work of Deubner (1974). Cram (1978) has investigated the evidence of low, but significant, power at frequencies relevant to internal gravity waves. He also concluded that from the studies of the phase lag between successive layers, there was an upward energy flux. In addition, Brown & Harrison (1980) observed indications of the possible existence of trapped gravity waves by analyzing the brightness fluctuations of the visible continuum. These internal gravity waves, by-products of the granulation, are expected to be fairly common and may not be negligible in the energy balance of the lower chromosphere. Deubner, (1981), has pointed out that gravity waves are extremely difficult to observe because these are local, small-scale features requiring very high spatial resolution observations. High temporal and spatial resolution data will reveal that these gravity waves are small-scale phenomena. There have been lots of claims on the detection of g-modes in the Sun, but, so far, there is not much of a convincing evidence from the observational side. However, Straus & Bonaccini (1997), have presented observationally, the strongest evidence of gravity wave present in the middle photosphere using the wavenumber and frequency resolved phase-difference spectra and horizontal propagation diagram. Some recent works on the effects of gravitational stratification on Alfven waves are found in (Mullan & Khabibrakhmanov 1999, McKenzie & Axford 2000, DeMoortel & Hood 2004). Rathinavelu *et al.* (2007) studied the effect of flow on Alfven Gravity Surface Waves in a plasma slab surrounded by plasma and neutral gas in the context of magnetosphere.

There are some observational investigations to show that there is a signature of atmospheric gravity waves at the chromospheric level using the time sequence of filtergrams and spectra obtained in CaII H & K and Mg b2 lines (Dame *et al.* 1984, Kneer & von Uexkull 1993, Rutten & Krijger 2003).

In the last few years, progress in the spatial and time resolution of solar coronal instruments have given us enough evidence on the presence of Magnetohydrodynamic activity in the corona (Nakariakov & Verwichte 2005). In uniform plasmas, it is well known that there are three types of waves, namely the Alfven wave, the slow and fast magnetosonic waves. In addition to the above, there are possibly additional modes such as the kink, sausage, longitudinal ones, since it is well known by now that the corona is highly structured due to magnetic fields. The magnetoacoustic modes have been identified in the corona. However, there is still no direct evidence of Alfven modes. Recently, Tomczyk *et al.* (2007), using the Coronal Multi-Channel Polarimeter (CoMP) have identified waves in the corona. The revelation that waves were observed in the corona is an important development which is certainly worth probing into. Also the observation has important implication in the context of coronal heating and coronal seismology. They interpreted their observations in terms of Alfven waves, based on the facts that 1) the observed phase speeds are much larger than the sound speed, 2) the waves propagate along the field lines, and 3) the waves were incompressible. However, in the work of Van Doorsselaere *et al.* (2007), the observations were interpreted in terms of fast kink waves rather than Alfven waves. A very interesting article on the existence of Alfven waves in the Solar Atmosphere is found in Erdelyi & Fedun (2007).

2. The dispersion relation

In the present study, we consider a very simple three layer model whose densities and magnetic fields are different, though uniform in each of them. This model may be thought of a simple extension to a current sheet that one comes across in many physical situations. We also assume that the thickness of the middle layer is not very large, while

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the layers above and below this layer is semi-infinite in extent. We are not introducing any rigid boundaries in this model. Also we assume that the magnetic fields are plane parallel in each of the layer with the strength differing. The middle layer is assumed to move with a uniform speed and gravity is the external force in addition to the magnetic field and pressure forces acting in the field. We invoke MHD approximation, which is valid to a large extent as most of the scales (length and time) are large compared to the mean free path of the particles. That is, we use the continuum hypothesis and also assume that the role of electric fields are negligible in the present scenario. The plasma β which is the ratio of the gas pressure to the magnetic pressure is assumed to be very small which is also a valid approximation, say in the solar corona.

Before discussing the dispersion relation of the waves that is under consideration, we give briefly below the standard dispersion relation for Alfven, Magneto Acoustic and Magneto Acoustic Gravity waves in an infinite fluid with constant magnetic field and compressibility.

It is well known that the effects of tension in an elastic string is to allow transverse waves to propagate along the string. In analogy, it is reasonable to expect the magnetic tension to produce transverse waves that propagate along the magnetic field B_0 with a speed

$$v_A = \frac{B_0}{(\mu\rho_0)^{1/2}} \tag{2.1}$$

In the case of shear Alfven waves, the dispersion relation is given by

$$\omega = kv_A cos\theta_B \tag{2.2}$$

These waves propagate at a certain angle to the direction of the magnetic field. When compressibility is taken into account, in addition to the Alfven mode, two additional modes appear, namely, the fast and slow magneto acoustic modes, with a dispersion relation given by

$$\omega^4 - \omega^2 k^2 (c_s^2 + v_A^2) + c_s^2 v_A^2 k^4 Cos^2\theta = 0 \tag{2.3}$$

When gravity is included, the dispersion relation is modified to yield

$$\omega^4 - \omega^2 (N^2 + k^{'2}) + N^2 sin^2\theta k^{'2} c_s^2 = 0 \tag{2.4}$$

Here N^2 is the Brunt-Vaisala frequency which is a very important parameter for the static stability of a stratified fluid. More details about the nature of these modes can be found in Priest (1982).

For the model under consideration, the basic equations of motion are linearized by assuming perturbations of the form

$$f(x,z,t) = f(x)exp(i(kz - \omega t)) \tag{2.5}$$

The approach is well known in the literature and we skip the details for the sake of brevity. The dispersion relation after algebraic substitution reduces to

$$[\epsilon_1\epsilon_2 + \epsilon_1\epsilon_g + g\epsilon_1\frac{(\rho_g - \rho_2)}{\omega}] + [\epsilon_1^2 + \epsilon_2\epsilon_g + \epsilon_2 g\frac{(\rho_g - \rho_1)}{\Omega} + \epsilon_g g\frac{(\rho_1 - \rho_2)}{\Omega}]tanh(2ka) = 0 \tag{2.6}$$

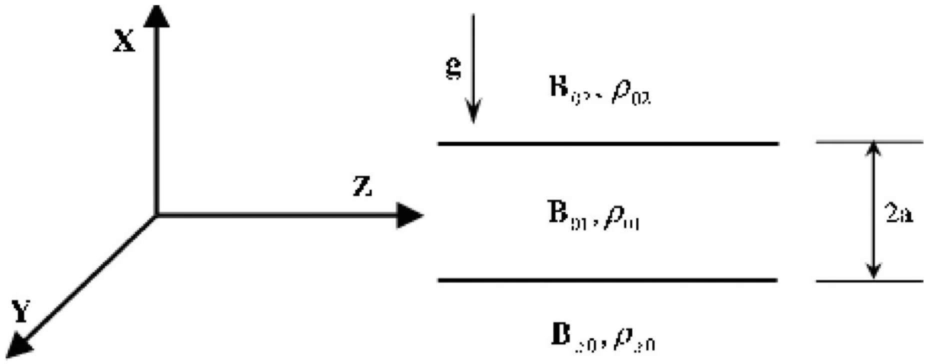

Figure 1. The Geometry.

The Epsilons and Omega are defined as

$$\Omega = \omega - kU_0 \tag{2.7}$$

$$\epsilon_1 = \frac{(\rho_1 \Omega)}{k(v_{ph} - \hat{U})^2}[1 - (v_{ph} - \hat{U})^2] \tag{2.8}$$

$$\epsilon_2 = \frac{(\rho_1 \omega)}{k v_{ph}^2}\left[\beta_1^2 - v_{ph}^2 \eta_1\right] \tag{2.9}$$

$$\epsilon_g = \frac{(\rho_1 g)}{k v_{ph}^2}\left[\beta^2 - v_{ph}^2 \eta\right] \tag{2.10}$$

Introducing the non-dimensional quantities

$v_{ph} = \frac{\omega}{k v_{A1}}$, $\hat{U} = \frac{U_0}{v_{A1}}$, $\beta = \frac{B_{0g}}{B_{01}}$, $\beta_1 = \frac{B_{02}}{B_{01}}$, $\eta = \frac{\rho_{0g}}{\rho_{01}}$, $\eta_1 = \frac{\rho_{02}}{\rho_{01}}$ $G = \frac{g}{k v_{A1}^2}$

The normalized dispersion relation can be simplified to yield

$$[1 - (v_{ph} - \hat{U})^2]\left[\beta_1^2 - v_{ph}^2 \eta_1\right) + \left(\beta^2 - v_{ph}^2 \eta\right) + G(\eta - \eta_1)\big]$$

$$\left[\frac{v_{ph}}{(v_{ph} - \hat{U})}(1 - (v_{ph} - \hat{U})^2)^2 + \frac{(v_{ph} - \hat{U})}{v_{ph}}\left(\beta_1^2 - v_{ph}^2 \eta_1\right)\left(\beta^2 - v_{ph}^2\right)\right.$$

$$\left. + \left(\beta_1^2 - v_{ph}^2 \eta_1\right) G(\eta - 1) + \left(\beta^2 - v_{ph}^2 \eta\right) G(1 - \eta_1)\right] tanh(2ka) = 0 \tag{2.11}$$

3. Discussion of the results

The normalized dispersion relation presented in equation (2.16) is much more complicated than the ones presented for the uniform fluid without density, magnetic field discontinuities, gravity and flows as is expected. The geometry presented in Figure 1, being a three layer model introduces more parameters, characterizing the geometry, say, the ratios of densities, magnetic fields, non-dimensional flow and gravity. The number of parameters characterizing the model is large and so in the present work, we have restricted our discussion for a chosen (rather arbitrary) set of values, in order to discuss the effects of flow and gravity.

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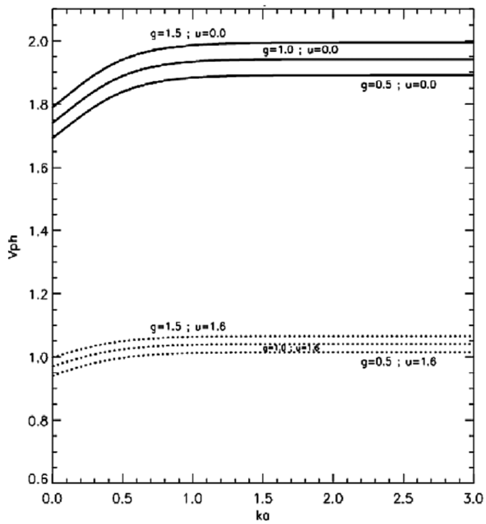

Figure 2. The Normalized Phase Velocity as a function of non-dimensional wavenumber.

The dispersion relation is solved numerically and the phase speed is plotted as a function of dimensionless wave number for various values of the interface parameters $\beta^2 = 0.5$, $\beta_1^2 = 1.5$, $\eta = 1.8$, $\eta_1 = 1.2$ and different values of non-dimensionalised g and u as shown in Figure 2. For the compressible case, we will have both the fast Alfven- gravity surface wave and slow Alfven-gravity surface wave. However, in the present study, since the plasma beta is small, the slow mode does not appear. This is a well known result in MHD. It is interesting to note that for increasing values of g, say 0.5, 1.0, 1.5, the normalized phase speed of the fast Alfven gravity surface mode increases as a function of ka. This is the case for $u = 0$. However, when flow is introduced ($u > 0$), it is interesting to note that the phase speed of the mode is significantly reduced. This means that the effect of flow has a dampening effect on the phase speed of the fast Alfven gravity surface mode. Another interesting observation from the above Figure is that the phase speed has significant variation only for $ka \approx 1$ while for $ka > 1$, the phase speed remains the same and asymptotically approaches the phase speed of the body wave.

Figure 3 presents the normalized phase speed as a function of ka as a three dimensional plot. It is evident that the dispersion is more pronounced for small values of ka only. More computations need to be done to get a full picture of the nature of these waves. The present model can have interesting applications in coronal streamers and solar wind. Computations for other sets of parameters are being worked out and applications to coronal streamers and solar wind is being considered. The detailed study will be reported later.

Acknowledgements

I wish to thank the members of the SOC of IAU 257, Dr. N. Gopalswamy, Dr. David Webb, Dr. Alexander Nindos for giving me an opportunity to participate in this meeting. I wish to thank the authorities of IAU for providing partial support to attend the meeting. I also thank Prof. S. S. Hasan, the Director of IIA, for encouraging me to attend the meeting and providing travel support.

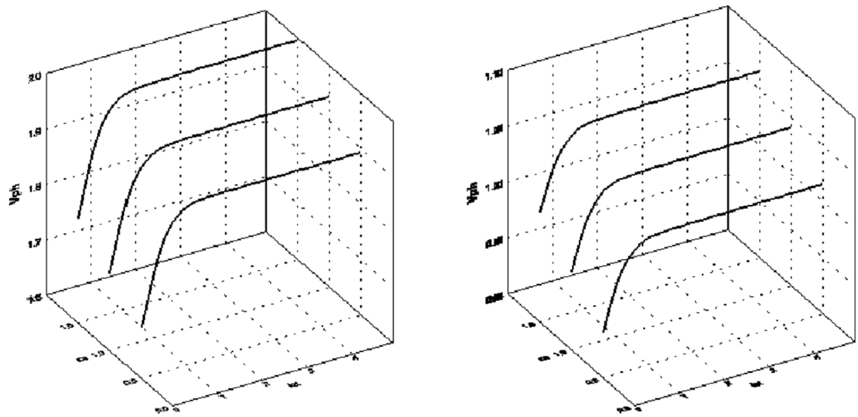

Figure 3. The Three Dimensional Plot as a function of ka

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