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Natural logarithms

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Marusic *et al.* (*J. Fluid Mech.*, vol. 716, 2013, R3) show the first clear evidence of *universal* logarithmic scaling emerging naturally (and simultaneously) in the mean velocity and the intensity of the streamwise velocity fluctuations about that mean in canonical turbulent flows near walls. These observations represent a significant advance in understanding of the behaviour of wall turbulence at high Reynolds number, but perhaps the most exciting implication of the experimental results lies in the agreement with the predictions of such scaling from a model introduced by Townsend (*J. Fluid Mech.*, vol. 11, 1961, pp. 97–120), commonly termed the attached eddy hypothesis. The elegantly simple, yet powerful, study by Marusic *et al.* should spark further investigation of the behaviour of all fluctuating velocity components at high Reynolds numbers and the outstanding predictions of the attached eddy hypothesis.

Key words: pipe flow boundary layer, turbulent boundary layers

1. Introduction

The majority of flows of importance to engineering applications in industry and transportation involve turbulent flow over a surface. In such a flow, be it through the inside of a transcontinental natural gas pipeline or in the external boundary layer over the wing of a commercial airliner, viscosity is responsible for the generation of a wall-normal gradient of the mean velocity because of the no-slip condition at the wall, while the resulting inhomogeneity provides a source of energy to sustain turbulence. In a large parameter space incorporating the full complexities of real flows, researchers have devoted significant time to investigating the so-called canonical turbulent flows of incompressible Newtonian fluids, namely cylindrical pipe and high-aspect-ratio channel flows and an external boundary layer developing under zero pressure gradient. One hope is to uncover similarity relationships between spatial variables and measures of the flow field that can be exploited for modelling, prediction and, ultimately, control of the intricate and complex mechanisms sustaining wall turbulence.

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Presented with the intractability of the strongly inhomogeneous, nonlinear, multiscale mechanics of wall turbulence, the seminal attached eddy hypothesis (AEH) of Townsend (1961) appealed to a self-similar hierarchy of flow structures populating different wall-normal regions of the flow to regenerate statistical properties of the velocity field. The attached eddies, or vortices with legs reaching down to (or attached to) the wall, have features in good agreement with the statistical properties of vortical structures that are commonly documented to populate this wall-normal region (Adrian 2007). Of particular relevance to the present work, the AEH predicts a logarithmic dependence of both mean and fluctuating wall-parallel velocity components on distance from the wall in the so-called inertial region, where the appropriate scaling length is the distance from the wall. The satisfying connection between observations of coherent vortical structure and velocity statistics offered through the AEH both demonstrates its importance to the field and underscores the importance of potential experimental evidence in support of its predictive power. While the (in)famous logarithmic variation of mean velocity, commonly termed 'the log law', has been derived in other ways, and has been well-debated and the subject of numerous challenges, as outlined by Marusic *et al.* (2013), confirmation of logarithmic behaviour in the turbulence intensities has been somewhat elusive. Recently, however, Hultmark *et al.* (2012) demonstrated simultaneous appearance of the log law in the mean flow and turbulence in pipe flow, while Hultmark (2012) used scaling analysis to derive such a relationship without requiring the details of the AEH.

2. Overview

The study by Marusic *et al.* (2013), which is unique in its Reynolds number reach and fidelity of the measurements, validates the universal nature of the logarithmic scalings by extraordinarily careful measurements using different diagnostic techniques and in different facilities. Their findings can be summarized in the determination from figure 1 (also their figure 1) that there exists a common region in wall-normal coordinate, z, where logarithmic variations of the mean, U, and (mean square of) the fluctuating streamwise velocities (or turbulent intensity), $\overline{u^2}$, occur simultaneously (under suitable non-dimensionalization, in this figure with the so-called inner or viscous scales). Importantly, the same logarithmic scaling is obtained in different canonical flow configurations, in different fluids and by different researchers in a range of specialized facilities, thus validating the use of the term 'universal scaling'. That these facilities range in physical (flow) dimension from around 10 cm (Superpipe) to 100 m (SLTEST) highlights the effectiveness of the dynamical similarity arguments that permit generation of similar flow conditions through matching of the Reynolds number by increasing either the numerator through physical scale or decreasing the denominator through viscosity (all other non-dimensional parameters kept constant).

Given the elapsed time between the predictions of Townsend and the developments made by Perry & Chong (1982), it is perhaps surprising that it has taken until now to obtain decisive information on the log scaling/validity of the model. What has taken so long? Marusic *et al.* (2013) make clear that specialized facilities capable of reaching high Reynolds numbers are required to escape the far-reaching direct influence of viscosity close to the wall. Some recent attempts have been made to identify simultaneous changes in both the statistical moments and power spectra of high-Reynolds-number wall turbulence, e.g. McKeon & Morrison (2007).

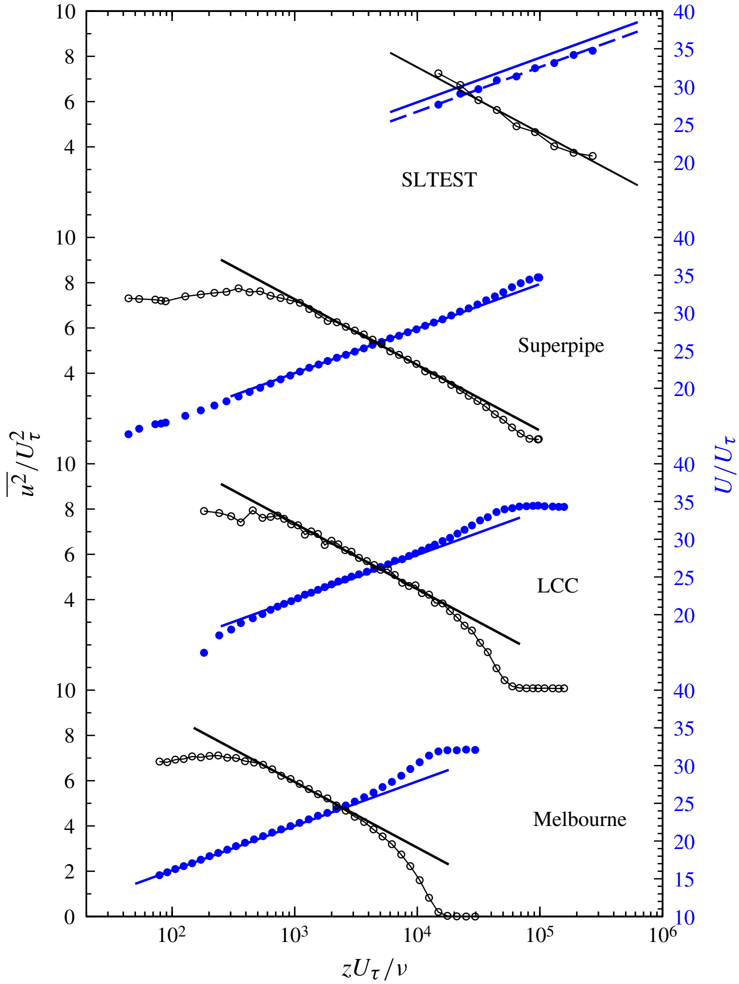

FIGURE 1. Simultaneous occurrence of logarithmic scaling in mean velocity, U, and streamwise turbulent intensity, $\overline{u^2}$ (normalized with viscous scales), with respect to the wall-normal coordinate, z, in all four experimental facilities described in Marusic *et al.* (2013), over an approximately 30-fold increase in Reynolds number (bottom to top: $Re_\tau \approx 18\,010$–$628\,000$).

Exquisite attention has been paid by Marusic *et al.* (2013) to the measurement resolution concerns which beset investigations of high-Reynolds-number flows, as even the smallest probes become large relative to the smallest turbulent scales. A notable strength of the work is the use of a single, appropriately selected diagnostic technique in each facility (to the extent of employing a unique nano-scale thermal anemometry probe in the Superpipe, where the spatial resolution constraints are the most severe). The results are a triumph of careful experimental and measurement system design and should serve as a benchmark to experimentalists who wish to be able to draw similarly robust conclusions concerning the behaviour of high-Reynolds-number wall turbulence.

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3. Future

The observations of Marusic *et al.* (2013) provide a dramatic confirmation of the *simultaneous* existence of two logarithms in the profiles of mean and fluctuating streamwise velocity intensity in truly *high-Reynolds-number* wall turbulence, and in some sense a return to the universality between flows intended when the scaling constant in the mean velocity log law was named the von Kármán *constant*. They also provide encouraging support for Townsend's attached eddy hypothesis, an important link between observed scaling of the velocity statistics and a realistic assembly of vortical structures that could sustain them. The importance of a theoretical underpinning of scaling results can hardly be understated for modelling in the complex world of wall turbulence. The work is sure to invigorate investigations of the full stress tensor (including potential agreement with AEH predictions) and scalar fields, both of which come with even more stringent measurement difficulties than those overcome in this study and hitherto have been rather under-explored by comparison.

In this vein, the mean velocity has long been known to be the least sensitive (yet most studied) profile in most turbulent flows, and thus the more robust proxy for determination of the location and characteristics of the inertial region outlined by Marusic *et al.* (2013) will be welcomed by fluid mechanicians. As the authors note, deviations from logarithmic behaviour in the mean velocity are slow and small, a fact that has underpinned extensive earlier debate on the true nature of this scaling (they leave the exact wall-normal range of observation of the log laws and values of the accompanying constants for subsequent definitive determination, but note that they likely challenge at least some of the recent reports in the literature). The more rapid deviation of the streamwise turbulent intensity from logarithmic scaling may serve as a useful diagnostic for the true extent of a logarithmic scaling demarking the inertial region. While, perhaps ironically, the results reported by Marusic *et al.* (2013) emphasize the emergence of the elegantly simple logarithmic scaling at large Reynolds numbers in specialized facilities, significant insight into the sustaining mechanisms of turbulence may also be obtained through new investigations into deviations from the AEH predictions at the more accessible Reynolds numbers typical of most laboratory facilities and direct numerical simulations. Will the results of the range of studies likely to be sparked by Marusic *et al.* (2013) agree with the predictions of the attached eddy hypothesis? Confirmation (finally) of the outstanding predictions of a fifty-year-old model that draws together aspects of vortical structure and velocity statistics remains an intriguing, powerful possibility for the turbulence world.

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