Appendix 7

The coefficients $\mathscr{C}^{lm;l'm'}_{l_1m_1;l'_1m'_1}$

The coefficients involved in the additional invariance constraints on the dynamical reaction parameters for a spin-s particle (subsection 5.3.1(v)) are real and are given in terms of vector addition coefficients as follows.

(i) $\mathscr{C}^{lm;l'm'}_{l_1m_1;l'_1m'_1} = 0$ unless *all* the following conditions are satisfied:

$$\begin{gathered} l + l' + l_1 + l'_1 \quad \text{is even} \\ m_1 + m'_1 = m + m' \\ |m - m_1| \le 2s \qquad |m - m'_1| \le 2s \\ |m' - m'_1| \le 2s \qquad |m' - m_1| \le 2s. \end{gathered}$$

(ii) If the above are satisfied, then

$$\begin{aligned} \mathscr{C}^{lm;l'm'}_{l_1m_1;l'_1m'_1} = (-1)^{m_1 - m} & \sqrt{\frac{(2l_1+1)(2l'_1+1)}{(2l+1)(2l'+1)}} \\ & \times \sum_{\mu} \langle l, m | s, -\mu; s, m+\mu \rangle \langle l', m' | s, m' - m'_1 - \mu; s, \mu + m'_1 \rangle \\ & \times \langle l_1, m_1 | s, m' - m'_1 - \mu; s, \mu + m \rangle \langle l'_1, m'_1 | s, -\mu; s, \mu + m'_1 \rangle . \end{aligned}$$

The coefficients satisfy the following symmetry properties:

$$\begin{aligned} \mathscr{C}^{lm;l'm'}_{l_1m_1;l'_1m'_1} &= \mathscr{C}^{l'm';lm}_{l'_1m'_1;l_1m_1} \\ (2l+1)(2l'+1)\mathscr{C}^{lm;l'm'}_{l_1m_1;l'_1m'_1} &= (2l_1+1)(2l'_1+1)\mathscr{C}^{l_1m_1;l'_1m'_1}_{lm;l'm'} \\ \mathscr{C}^{l-m;l'-m'}_{l_1-m_1;l'_1-m'_1} &= \mathscr{C}^{lm;l'm'}_{l_1m_1;l'_1m'_1}. \end{aligned}$$

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We list the non-zero independent coefficients for a spin 1/2 particle:

$$\mathscr{C}^{11;11}_{11;11} = 1$$

$$\mathscr{C}^{11;10}_{11;10} = \tfrac{1}{2} \qquad \mathscr{C}^{11;10}_{11;00} = \tfrac{1}{6}$$

$$\mathscr{C}^{11;1-1}_{00;00} = \tfrac{1}{6} \qquad \mathscr{C}^{11;1-1}_{10;10} = -\tfrac{1}{2} \qquad \mathscr{C}^{11;1-1}_{10;00} = -\frac{1}{2\sqrt{3}}$$

$$\mathscr{C}^{11;00}_{11;00} = \tfrac{1}{2}$$

$$\mathscr{C}^{10;10}_{10;10} = \tfrac{1}{2} \qquad \mathscr{C}^{10;10}_{00;00} = \tfrac{1}{6}$$

$$\mathscr{C}^{10;00}_{00;10} = \tfrac{1}{2}$$

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