Single premium for a similar assurance on two lives,

$$=1-d\frac{N_{m-1,m_1-1}-N_{m+t-1,m_1+t-1}}{D_{m,m_1}};$$

Annual premium for an endowment assurance on a single life,

$$=\frac{D_m}{N_{m-1}-N_{m+t-1}}-d;$$

Annual premium for a similar assurance on two lives,

$$=\frac{D_{m,m_1}}{N_{m-1,m_1-1}-N_{m+t-1,m_1+t-1}}-d.$$

Again, the present value of a perpetuity of £1 deferred t years, subject to the last v survivors being then alive, equals the present value of an annuity of £1 deferred t years to continue during the joint lives of the last v survivors + the present value of an assurance of £1 + $\frac{1}{i}$ deferred t years and payable on the failure of the joint existence of the last v survivors. Hence,

$$r^t p_{\overline{(m,m_1,m_2,\&c.)t}}^{\;v}\frac{1}{i}=(a)_{\rceil t}+(\mathrm{A})_{\rceil t}\left(1+\frac{1}{i}\right)$$

$$\therefore \mathrm{A}_{\rceil t}=r^{t+1}p_{\overline{(m,m_1,m_2,\&c.)t}}^{\;v}-d(a)_{\rceil t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and for the annual premium (7) has to be divided by $1+(a)_{\overline{t-1}|}$.

I remain, Sir,

Your obedient servant,

MARCUS N. ADLER.

London, 15*th March*, 1864.

ON THE PAYMENT OF $\frac{1}{m}$ YEARLY PREMIUMS.

To the Editor of the Assurance Magazine.

SIR,—I find in No. 54 of the *Assurance Magazine* Mr. Laundy's method of obtaining half-yearly and quarterly premiums from the annual premium. He derives it very skilfully from Mr. Orchard's expression given in the introduction of his valuable work, *Single and Annual Assurance Premiums.* I am much surprised to see Mr. Laundy availing himself of this expression, since another is nearer and more directly to be got at. Having, however, not found it till now in any work, it might prove useful to publish it here for general use.

Putting, for the age of x years,

π as the annual premium on a premium paid half-yearly,
p „ „ „ „ annually,
$a'^{\frac{2}{2}}$ as the annuity to be paid half-yearly in advance,
a' „ „ „ annually „

we find

$$\pi a'^{\frac{2}{2}}=pa' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

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Substituting $a'^{\frac{3}{4}}=a'-\frac{1}{4}$, (1) becomes

$$\pi(a'-\tfrac{1}{4})=pa' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2);$$

thus we get

$$\pi=\frac{pa'}{a'-\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Substituting $a'=1+a$, (3) becomes

$$\pi=p.\frac{1+a}{1+a-\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\pi=p\left(1+\frac{0{\cdot}25}{0{\cdot}75+a}\right) \quad . \quad . \quad . \quad . \quad (5).$$

This is Laundy's expression.

To find the annual premium on the payment of $\frac{1}{m}$ yearly premiums, we start from the approximate expression

$$a'^{\frac{m}{m}}=a'-\frac{m-1}{2m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Thus we have

$$\pi a'^{\frac{m}{m}}=pa',$$

$$\pi=p.\frac{a'}{a'-\frac{m-1}{2m}}$$

$$=\frac{1+a}{1+a-\frac{m-1}{2m}};$$

or, after a simple operation,

$$\pi=p\cdot\left(1+\frac{\frac{m-1}{2m}}{\frac{m+1}{2m}+a}\right) \quad . \quad . \quad . \quad . \quad . \quad (7).$$

This is Laundy's general expression.

I am, Sir,

Yours most obediently,

DR. AUGUST WIEGAND,

Director of Life Assurance Society, Iduna.

Halle, Germany, Prussia,
May 14, 1864.

ON THE SAME SUBJECT.

To the Editor of the Assurance Magazine.

SIR,—Having been allowed the privilege of perusing the preceding letter of Dr. August Wiegand, I can but express my gratification that a subject of such minor importance should have attracted the attention of your distinguished correspondent: and beg to thank him, through the