I

Convection and turbulence as the basis of magnetic activity

- Nature of the turbulence in solar and stellar convection zone
- Rotational influence on turbulence
- Simulations vs. mean-field approach
- Mixing length theory, global convection, turbulence models

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The Confrontation of Mean-Field Theories with Numerical Simulations

Fritz Krause

Astrophysikalisches Observatorium Potsdam, O-1560 Potsdam, Fed. Rep. of Germany

Abstract: The investigation of magnetic phenomena on the Sun is confronted with the problem of turbulently moving electrically conducting media in rotating objects. One way of attacking this complicated problem, which was at first successful, was the development of mean-field magnetohydrodynamics, where the cooperative action of the small-scale turbulence was taken into account by certain average effects. The development of more and more powerful computers now offers possibilities of calculating the small-scale phenomena in a direct way. This paper is an attempt to compare these approaches, at least as far as there are comparable results.

1. Introduction

Dynamos operating in real cosmical objects are closely connected with turbulent motions, which are of highly complicated structure both temporally and spatially. Thus it can clearly be seen that a detailed description of the dynamo process demands an unmanageable set of data. The way out of this dilemma was discovered long ago at the beginning of the theory of turbulence: equations for the mean-fields are derived and the fluctuating fields enter via averaging processes into the equations. Reynolds stresses and eddy viscosity are examples. Analogously mean-field magnetohydrodynamics has been developed. The mean cross-product of the fluctuating velocity field with the fluctuating magnetic field, the turbulent electromotive force $\overline{\mathbf{u}' \times \mathbf{B}'}$, appears as the analogue of the Reynolds stresses. The turbulent magnetic diffusivity and the α-effect, which proved to be the key for the solution of the dynamo problem, are the best known effects from the concerted action of the small scale fields. With physically reasonable assumptions dynamo models for the solar magnetic field and its cycle have been constructed, which describe surprisingly well the basic features of this phenomenon.

Parallel to this line of investigation there have been attempts undertaken to attack the dynamo problem of the Sun, i.e. the problem of magneto-convection in rotating objects, by direct numerical simulation. However, the high degree of

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complexity of the problem makes it necessary to include parametrization of the collective phenomena here also, but at a lower level than in mean-field theories. It should be noted that a sufficiently good modelling of the basic features of the solar magnetic field has not yet been achieved.

The continually growing power of computers makes parametrizations more and more superfluous. However, an appraisal of the results, represented by a large set of data, needs eventually the calculation of certain averages, i.e. parameters like eddy viscosity, α-effect, etc. must be calculated by averaging operations over the data sets. There is now the possibility of comparing this new information concerning the mean-field parameters derived by mean-field methods (mainly analytical) with the values derived from numerical simulations. Thus we have the possibility of confirming such basic relations as that between α and the helicity. However, a general comparison will meet with difficulties, since basic assumptions for mean-field theories do not necessarily coincide with those of the numerical simulations.

2. Magneto-convection in spherical shells

That the magnetic phenomena on the Sun are closely related to the turbulent motions in the convection zone is, for example, clearly to be seen from the enhanced decay of sunspots: if the decay time of the magnetic field is calculated according to $t_{decay} = \mu\sigma d^2$, where d is the diameter of the sunspot, $\mu = \mu_o$ the vacuum permeability, and σ is the electrical conductivity, we find a time scale of some thousand years, whereas in reality we see a decay time of some weeks.

This destructive action of turbulence is immediately accepted; however it has to be suspected that turbulence may also have some constructive features: Cowling's theorem states that axisymmetric (or, more general, simple-structured) motions cannot provide dynamo excitation. For example, the differential rotation cannot generate and maintain the solar magnetic field. Consequently, if the solar magnetic field is dynamo generated, the turbulent convective motions must play a key role.

First hints at the constructive action of turbulence were found in the fifties: in a paper from 1951 L. Biermann showed that an anisotropic turbulence in a rotating body must generate a differential rotation. Thus a large scale motion is generated from small-scale turbulence. A few years later E.N. Parker (1955) claimed that the turbulence of an electrically conducting media in a rotating body excites a dynamo: a large scale magnetic field is generated by this "cyclonic turbulence".

The investigation of the magnetic phenomena on the Sun is thus confronted with the problem of turbulently moving electrically conducting media in a rotating object. The basic equations are given by the conservation laws of mass, momentum, and energy

$$\frac{\partial \ln \varrho}{\partial t} + (\mathbf{u}\cdot\nabla)\ \ln\varrho + \mathrm{div}\ \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{p}{\varrho}\nabla\ \ln p + \mathbf{g} - 2\Omega\times\mathbf{u} + \frac{1}{\varrho}\mathbf{j}\times\mathbf{B} + \frac{1}{\varrho}\,\mathrm{Div}\ \tau, \tag{2}$$

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$$\frac{\partial e}{\partial t} + (\mathbf{u} \cdot \nabla) e = -\frac{p}{\varrho} \operatorname{div} \mathbf{u} + \frac{\kappa}{\varrho} \Delta e + Q_{visc} + Q_{joul}, \tag{3}$$

together with the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \operatorname{curl}(\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu\sigma} \Delta \mathbf{B}. \tag{4}$$

In addition, an equation of state and boundary conditions have to be taken into account. Here denotes ϱ the mass density, $\mathbf{u}$ the velocity, p the pressure, $\mathbf{g}$ the gravity, Ω the angular velocity of the overall rotation, $\mathbf{B}$ the magnetic field, $\mathbf{j}$ the current density, e the specific internal energy, κ the heat conduction and τ the viscosity tensor defined by

$$\tau_{ij} = \nu\varrho(u_{i,j} + u_{j,i} - \tfrac{2}{3}\delta_{ij} \operatorname{div} \mathbf{u}), \tag{5}$$

with the kinematic viscosity ν. Finally define Q_{visc} and Q_{joule} the heat production by viscosity and electrical resistivity. These quantities are quadratic expressions in the velocity gradient and the current density, respectively.

In particular, the boundary conditions specify that heat energy flows into the spherical shell under consideration at the bottom and leaves it at the surface, for example by the emission of radiation. The equations (1)–(4) have the simple solution

$$\mathbf{u} = \mathbf{B} = \mathbf{j} = 0, \frac{\partial \varrho}{\partial t} = 0, \tag{6}$$

with e determined by

$$\frac{\partial e}{\partial t} = \frac{\kappa}{\varrho} \Delta e, \tag{7}$$

together with the appropriate boundary conditions. In this case the transport of energy is simply carried on by molecular conduction.

However, this solution generally proves to be unstable. First the velocity field $\mathbf{u} = 0$ becomes unstable, and energy transport occurs by convective motions – hot material rises towards the surface, cold material sinks down to the bottom. In addition, since the medium in the solar convection zone is electrically conducting, the magnetic field $\mathbf{B} = 0$ also becomes unstable and dynamo excitation of the solar magnetic field sets in.

The scheme represented in Fig. 1 illustrates the physical situation. Transport processes of this kind can show two basic bifurcations: firstly the thermal instability at the critical Rayleigh number and secondly the dynamo instability at the critical value of the dynamo number. The system evolves from a stove via a heat engine into a power station.

It is now clearly to be seen that the considered problem requires the solution of the full set (1)–(4) of nonlinear differential equations, i.e. a problem of unmanageably high complexity.

In order to attack this problem methods already developed in the theory of turbulence have been applied, namely the derivation of equations controlling the

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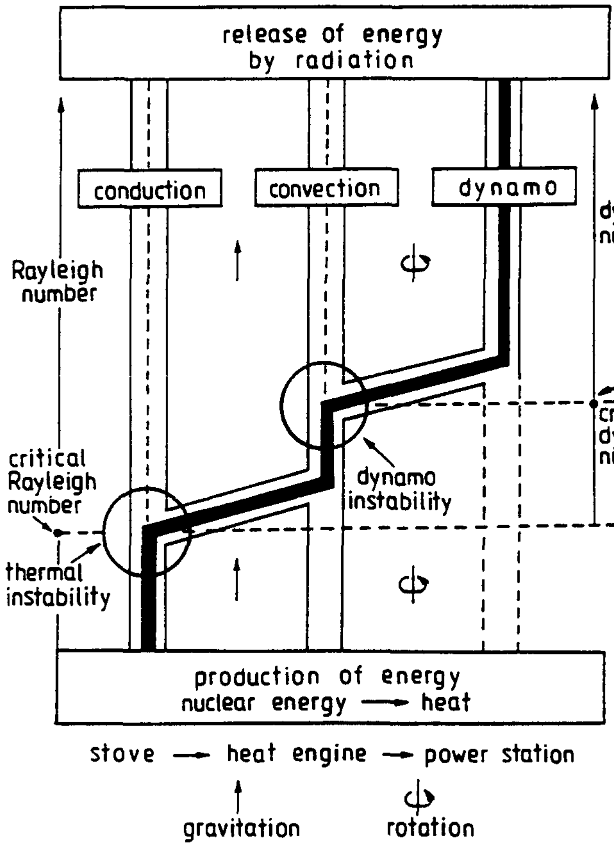

Fig. 1. The transport of energy in a star like the Sun is characterized by two basic bifurcations: the thermal instability marks the onset of convection and the dynamo instability that of the excitation of a magnetic field.

mean quantities. The nonlinear interactions of the fields involved in this energy transport provide new cooperative processes which enter the mean-field equations. The best known examples are characterized by the eddy viscosity and the α-effect.

The goal here is to determine the space-time behaviour by determining stable solutions of the mean field equations. The other way - possible in the future because of the growing power of computers - is the direct determination of the (irregular, unstable) solutions of the original equations. Basically both methods must lead to the same results and so there must be the possibility of checking one against the other.

3. The mean-field concept

The physical quantities will be represented by the sum of a mean part, denoted by a bar, and a fluctuating part, denoted by a dash:

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}', \quad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}', \quad p = \overline{p} + p', \dots \tag{8}$$

The averaging procedure need not to be specified, however some general rules have to be fulfilled, the so-called Reynolds rules:

$$\overline{F'} = 0, \quad \overline{\overline{F}} = \overline{F}, \tag{9}$$

$$\overline{FG} = \overline{F} \cdot \overline{G} + \overline{F'G'}, \ \overline{\overline{F} \cdot \overline{G}} = \overline{F} \cdot \overline{G}, \ \overline{\overline{F} \cdot G'} = 0, \tag{10}$$

where F and G denote arbitrary random fields.

These rules are fulfilled for statistical averages. For real measured quantities time or space averages have to be used. In the latter cases the validity of (9) and (10) will at best be guaranteed approximately; in particular the condition $\overline{F'} = 0$ is questionable. This is also the case if unstable solutions of the system (1)–(4) have been determined by numerical integration.

Because of the first rule in (10) the nonlinear terms in the equations (1)–(4) give rise to additional terms in the mean-field equations. We will illuminate this procedure with the Navier-Stokes equation for incompressible turbulence and the induction equation, assuming a weak magnetic field, i.e. that

$$\frac{\overline{B^2}}{2\mu} < \frac{\varrho}{2}\overline{u^2}; \tag{11}$$

the Lorentz force is neglected in (2).

According to (10) we have

$$\overline{(\mathbf{u} \cdot \nabla)\mathbf{u}} = (\overline{\mathbf{u}} \cdot \nabla)\overline{\mathbf{u}} + \overline{(\mathbf{u}' \cdot \nabla)\mathbf{u}'}, \tag{12}$$

$$\overline{\mathbf{u} \times \mathbf{B}} = \overline{\mathbf{u}} \times \overline{\mathbf{B}} + \overline{\mathbf{u}' \times \mathbf{B}'}, \tag{13}$$

and so find from the original equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\frac{p}{\varrho} + \mathbf{F} + \nu \Delta \mathbf{u}, \tag{14}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}(\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu\sigma}\Delta \mathbf{B}, \tag{15}$$

the following mean-field equations

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + (\overline{\mathbf{u}} \cdot \nabla)\overline{\mathbf{u}} = -\nabla\frac{\overline{p}}{\varrho} + \overline{\mathbf{F}} - \overline{(\mathbf{u}' \cdot \nabla)\mathbf{u}'} + \nu \Delta \overline{\mathbf{u}}, \tag{16}$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \operatorname{curl}(\overline{\mathbf{u}} \times \overline{\mathbf{B}}) + \operatorname{curl}(\overline{\mathbf{u}' \times \mathbf{B}'}) + \frac{1}{\mu\sigma}\Delta\overline{\mathbf{B}}. \tag{17}$$

Two additional terms are present in the equations (16) and (17):

$$-\overline{(\mathbf{u}' \cdot \nabla)\mathbf{u}'} = \frac{\partial Q_{ij}}{\partial x_j}, \tag{18}$$

with the correlation tensor

$$Q_{ij}(\mathbf{x}, t) = \overline{\mathbf{u}'_i(\mathbf{x}, t)\mathbf{u}'_j(\mathbf{x}, t)} \tag{19}$$

and the turbulent electromotive force

$$\mathcal{E} = \overline{\mathbf{u}' \times \mathbf{B}'}. \tag{20}$$

The appearance of these two new quantities mathematically means that the system is not closed, that is additional relations have to be derived which relate these second order statistical moments back to the mean fields. This problem leads deep into the theory of turbulence with all its solved and unsolved problems.

However, by making a few very natural assumptions, it is possible to derive in a rather simple but rigorous way to quite general explicit expressions for these quantities. For the case under consideration the turbulent motion is given. Subtracting (17) from (15) we find

$$\frac{\partial \mathbf{B}'}{\partial t} - \operatorname{curl}(\overline{\mathbf{u}} \times \mathbf{B}') - \frac{1}{\mu\sigma}\Delta\mathbf{B}' = \operatorname{curl}(\overline{\mathbf{u}} \times \mathbf{B}') + \operatorname{curl}(\mathbf{u}' \times \mathbf{B}' - \overline{\mathbf{u}' \times \mathbf{B}'}) \tag{21}$$

Without explicit integration we can see that $\mathbf{B}'$ is a linear functional of $\overline{\mathbf{B}}$, and the same is true of $\mathcal{E}$ (c.f. Krause and Rädler, 1980). In addition, we assume local dependence, i.e. the characteristic space and time scales of the fluctuations, say correlation length λ_{cor} and correlation time τ_{cor}, are small compared with the corresponding scales $\overline{\lambda}$, $\overline{\tau}$ of the mean fields:

$$\lambda_{cor} \ll \overline{\lambda}, \;\; \tau_{cor} \ll \overline{\tau}. \tag{22}$$

Hence $\mathcal{E}$ can be represented by the expression

$$\mathcal{E}_i = a_{ij} \cdot \overline{B}_j + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k}, \tag{23}$$

where the pseudo-tensors a_{ij}, b_{ijk} depend on the properties of the turbulent motion, i.e. on the physical quantities influencing the turbulence.

The simplest, therefore basic, model is that of an absolutely structureless turbulence, i.e. a turbulence that is homogeneous, isotropic and steady. In this case only the two tensors characterizing the Euclidean space, namely the Kronecker tensor δ_{ij} and the Levi-Civita tensor ϵ_{ijk}, are available; hence

$$a_{ij} = \alpha\delta_{ij}; \;\; b_{ijk} = \beta\epsilon_{ijk}, \tag{24}$$

and so from (23)

$$\mathcal{E} = \alpha\overline{\mathbf{B}} - \beta\,\mathrm{curl}\,\overline{\mathbf{B}}. \tag{25}$$

The properties of the turbulence are now contained in the pseudo-scalar α and the scalar β: the best known effects, the α-effect and the turbulent magnetic diffusion, characterized by the corresponding diffusivity β, are thus derived in a simple way. It should be noted that this derivation is a rigorous one, but based on the assumption (11) of weak magnetic field, and (22) of locality. A pseudo-scalar α is not simply available in real systems. It needs at least rotation and stratification. The latter may be due to gravity and then we can construct a pseudo-scalar

$$\alpha = \alpha_o(\mathbf{g}\cdot\Omega). \tag{26}$$

Eventually this does mean that the α-effect is quite a natural effect in a rotating systems. However, it has to be taken into account that the system is now anisotropic and the complete expression for a_{ij} reads

$$a_{ij} = \alpha_o(\mathbf{g}\cdot\Omega)\delta_{ij} + \alpha_1 g_i\Omega_j + \alpha_2 g_j\Omega_i \tag{27}$$

with scalars $\alpha_o, \alpha_1, \alpha_2$ determined by the turbulence. The original form (25) has to be replaced by a much more complex expression with a certain number of parameters (Rädler, 1980). This is also true if the restriction (11) is removed when nonlinear effects allow for further tensorial constructions.

The method illuminated here for the turbulent emf $\mathcal{E}$ has also been developed for the Reynolds stresses $-\varrho Q_{ij}$ which appear in the Navier-Stokes equation for the mean velocity field. In particular, a relation

$$Q_{ij} = \Lambda_{ijk}\Omega_k, \tag{28}$$

has similarly been considered in the context of differential rotation (Rüdiger, 1989). With the required symmetry of Λ_{ijk} one finds as the simplest form in a stratified medium (Λ-effect)

$$\Lambda_{ijk} = \Lambda(\epsilon_{ipk}g_j + \epsilon_{jpk}g_i)g_p \tag{29}$$

4. Success of mean-field models

Especially in connection with the question of the origin of the magnetic fields of cosmic object the mean-field concept has proved itself to be very effective. This need not be discussed here in detail. As a striking example Fig. 2 shows the observed butterfly diagram of the sunspot phenomena and one derived from an $\alpha\omega$-dynamo model based on mean-field magnetohydrodynamics (Steenbeck and Krause, 1969).

The, at least qualitative, agreement showing the latitudinal migration of the magnetic activity towards the equator is obvious. During the last 20 years many

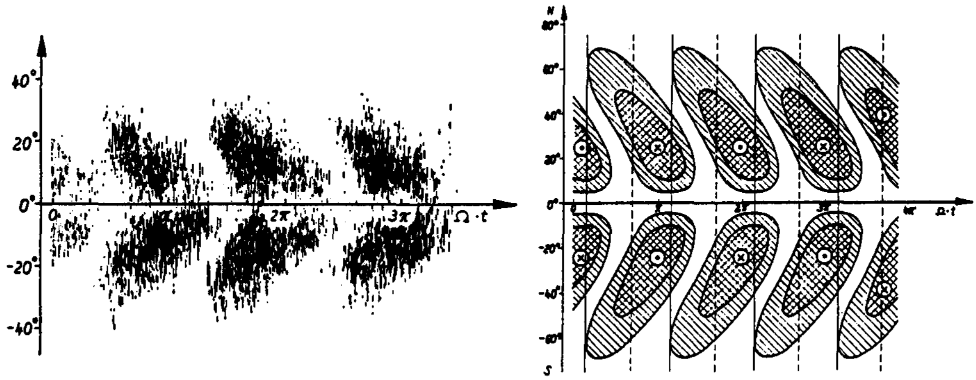

Fig. 2. Representation of the butterfly diagram of the Sun and of a kinematic $\alpha\omega$-dynamo model. A modelling of basic features of the solar cycle is possible, however, α-effect and differential rotation are chosen independently here.

models of this kind have been elaborated with the intention of optimizing the fit to observation. Here a selection of papers may be quoted: Roberts (1972), Roberts and Stix (1972), Yoshimura (1975a, b), Rädler (1980, 1986); and in addition there are monographs discussing the issue: Moffatt (1978), Krause and Rädler (1980), Zeldovich *et al.* (1983). Recently, observations have provided more data and in this way more conditions on the free parameters can be imposed (Brandenburg *et al.*, 1990). For example, one important new challenge to dynamo theory is given by the statements concerning the dependence of the differential rotation with depth, which are derived by means of helioseismology.

The accuracy of the relation (25) becomes questionable if nonlinear effects are taken into account, i.e. if the condition (11) is dropped and the magnetic field significantly influences the motion. A simple modelling is possible by reducing α with the growing magnetic field strength (α-quenching). These investigations show that the stability of a solution in the nonlinear regime is decisive in deciding which magnetic field will be excited and maintained by the dynamo. The growth rates determined from the linear problem are of less significance (Krause and Meinel, 1988; Brandenburg *et al.*, 1989).

A scenario is thus possible that predicts stable regular magnetic fields with certain symmetries (axisymmetry, equatorial symmetry) just beyond the marginal dynamo number, but for values of C that substantially exceed the marginal number, non-symmetric or even irregular solutions have to be expected.

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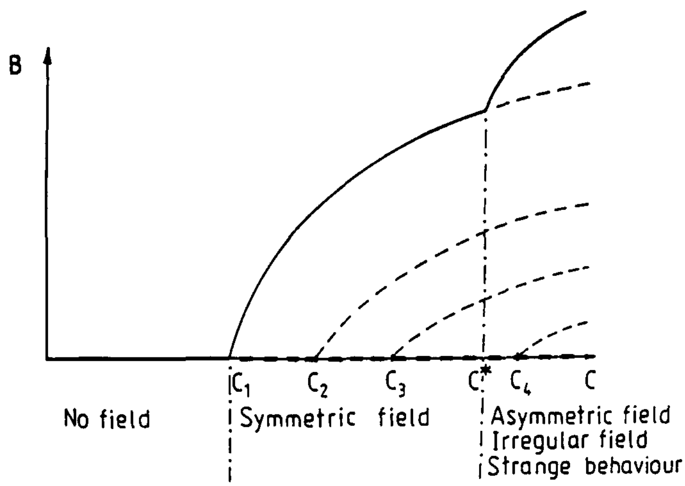

Fig. 3. Schematic dependence of dynamo excited cosmic magnetic fields on the dynamo number C. The solution bifurcating at the marginal dynamo number proves to be stable and so will be excited. Solutions bifurcating at higher values C are generally unstable and therefore never realized. The first solution reflects to some extend the symmetries of the cosmical object operating as a dynamo. For higher dynamo numbers symmetry breaking and irregularity have to be expected.

5. Numerical simulations of the solar cycle

The first attempts to derive solutions of the hydrodynamic equations (1)–(3) by direct numerical integration were carried out by P. Gilman in the early seventies (Gilman, 1972). The model consisted of a rotating spherical shell heated from below and the Boussinesq approximation was adopted. As the first step it was possible to find differential rotation, caused by the convection, that basically corresponded to that observed at the solar surface. In the next step the calculations of the hydromagnetic equations (1)–(4) were brought to the point that dynamo excitation of a magnetic field occurs (Gilman and Miller, 1981; Gilman, 1983).

The magnetic fields excited in these models deviate significantly from those observed on the Sun. The models of Gilman (1983) do show cyclic behaviour with time, but, in contrast to the solar field, the toroidal belts migrate toward the poles and not to the equator. A number of reasons can be listed, that might be responsible for these discrepancies:

(i) First of all, Boussinesq approximation must be mentioned, which does not allow the steep density gradient in the solar convection zone to be modelled.

(ii) The number of grid points is restricted by the capacity of the available computer. In the calculations quoted here the cell size lies above that of the super granules. The physics of the subgrid scales is taken into account by a parametriza-

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tion according to the mean-field concept. In this case there is simply an eddy viscosity and turbulent magnetic viscosity.

(iii) If the boundary conditions are taken into account in a rigorous way they have to guarantee a fit to a solution of the problem outside the sphere. Because magnetic fields are long range physical quantities, the description of their influence generally requires non-local boundary conditions. For example, mean-field models such as those mentioned in section 3 consider the sphere to be embedded in an insulating space. If a grid point method is used for integration non-local boundary conditions require an additional mathematical effort. Therefore the local boundary condition $\mathbf{B}_{tan} = 0$ is used in all cases. There is no physical justification.

(iv) The most important reason may arise from the fact that differential rotation represented by, say, the Λ-effect and the α-effect are caused by the same convective motion. In the simulations both effects are simultaneously generated and the excited magnetic field is produced by their common action. In the mean-field model quoted in section 3 that shows a nice agreement with the real sun, the differential rotation and α-effect are independently chosen: there is a rotational shear at the bottom of the convection zone and an α-effect in the upper regions. The differential rotation that is generated in the models of Gilman does not show this structure.

Further efforts in the field of numerical simulations have been directed towards removing of the Boussinesq conditions. G. Glatzmaier attacked the problem using the anelastic approximation (Glatzmaier, 1984, 1985a, b), which allows more compressibility effects to be taken into account. However, it is still restricted to the low Mach number regime. The final results of these investigations are very similar to those of Gilman. In particular, a poleward migrating field was still found.

6. Numerical experiments

A new development has started in recent years with the elaboration of computer codes for fully compressible convection. Firstly two-dimensional models were considered (e.g. Chan *et al.*, 1982; Hurlburt *et al.*, 1984), and these have recently been extended to three dimensions (Chan and Sofia, 1986; Stein and Nordlund, 1989).

These investigations are carried out in rectangular boxes. They do not really pretend to simulate the solar cycle, but rather are numerical experiments carried out in order to study convection, including the effects of magnetic fields (Hurlburt and Toomre, 1988; Nordlund and Stein, 1989). With the aim of comparing results with those of mean-field magnetohydrodynamics Brandenburg *et al.* (1990) simulated conditions where the α-effect and turbulent magnetic diffusivity may be determined.

The most interesting question clearly is whether the well-known relation

$$\alpha = -\frac{\tau_{cor}}{3}\overline{\mathbf{u}' \cdot \operatorname{curl} \mathbf{u}'} \tag{30}$$

can be confirmed. This relation has been derived on the basis of the second order correlation approximation in the high-conductivity limit (c.f. e.g. Krause and

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Rädler, 1980). It relates the parameter α to the helicity $\overline{\mathbf{u}' \cdot \mathrm{curl}\, \mathbf{u}'}$ of the turbulent motions. It should be noted that the proportionality of α to $(-\overline{\mathbf{u}' \cdot \mathrm{curl}\, \mathbf{u}'})$ has been confirmed in an experiment with liquid sodium (Steenbeck *et al.*, 1968).

At first glance the results derived by Brandenburg *et al.* (1990) contradict relation (30) in so far as the calculated α is directly proportional to the helicity. What can be the reason? The discrepancy is surely due to the fact that the calculation of α in (30) is based on isotropic conditions where the boundaries are distant. Both conditions are clearly violated in the numerical experiment. The direction of gravity is preferred in the model and downdrafts extend from the top to the bottom of the layer (cf. Stein and Nordlund, 1989).

However, homogeneity is guaranteed in the horizontal planes. If the anisotropy is taken into account, relation (25) must be replaced by

$$\mathcal{E} = \alpha_V \mathbf{B}_V + \alpha_H \mathbf{B}_H. \tag{31}$$

A similar derivation to that leading to (30) then gives

$$\alpha_H = -\frac{1}{3} \int_0^\infty \overline{u'_z(\mathbf{x}, t) \cdot (\mathrm{curl}\, \mathbf{u}'(\mathbf{x}, t+\tau))_z}\, d\tau \tag{32}$$

which may be evaluated as

$$\alpha_H = -\frac{\tau_{cor}}{3} \overline{\mathbf{u}'_z \cdot (\mathrm{curl}\, \mathbf{u}')_z}. \tag{33}$$

The horizontal α is related to the vertical helicity. The latter relation is, indeed, confirmed by the above mentioned numerical experiment.

The authors continued their investigations by introducing a non-vanishing mean-current and a gradient of the mean velocity. As a result a comparison with the standard expressions of the turbulent diffusivities becomes possible, which gives a fairly good agreement.

7. Conclusion

The comparison of mean-field results with those of numerical experiments is not in any case possible in a simple way.

Mean-field theories are generally based on assumptions that provide a practicable, or even comfortable, process of analytical deduction. The latter include the use of (i) statistical averages, (ii) two-scale property, i.e. weak variations of the mean quantities in space and time over scales of the fluctuations, and (iii) distant boundaries. In addition it has to be noted that a certain closure is always used, i.e. a decision concerning the neglect of higher order statistical moments.

Numerical simulations are generally restricted by the efficiency of the computer. Even the best of today's computers are not capable of solving a sufficiently well posed problem of magnetoconvection in a spherical shell. Restriction of the model, e.g. to a rectangular box, to artificial boundary conditions, etc., have to be taken into account. In addition, devices have to be introduced to guarantee numerical stability.

For these reasons the discrepancies revealed by a comparison are not surprising. In future modelling of situations corresponding more closely to reality by numerical simulations will become increasingly possible. Then it will become possible to compensate for the weak aspects of mean-field theories, especially the badly known parameters and the difficulties with nonlinearities.

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