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TOPOLOGICAL STATES OF MATTER AND NONCOMMUTATIVE GEOMETRY

CHRIS BOURNE

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This thesis examines topological states of matter from the perspective of noncommutative geometry and KK-theory. Examples of such topological states of matter include the quantum Hall effect and topological insulators.

For the quantum Hall effect, we consider a continuous model and show that the Hall conductance can be expressed in terms of the index pairing of the Fermi projection of a disordered Hamiltonian with a spectral triple encoding the geometry of the sample's momentum space. The presence of a magnetic field means that noncommutative algebras and methods must be employed. Higher-dimensional analogues of the quantum Hall system are also considered, where the index pairing produces the 'higher-dimensional Chern numbers' in the continuous setting.

Next we consider a discrete quantum Hall system with an edge. We show that topological properties of observables concentrated at the boundary can be linked to invariants from a boundary-free model via the Kasparov product. Hence we obtain the bulk-edge correspondence of the quantum Hall effect in the language of KK-theory.

We go on to consider topological insulators, which come from imposing (possibly anti-linear) symmetries on condensed-matter systems and studying the invariants that are protected by these symmetries. We show how symmetry data can be linked to classes in real or complex KK-theory. Finally, we prove the bulk-edge correspondence for topological insulator systems by linking bulk and edge systems using the Kasparov product in KKO-theory.

CHRIS BOURNE, Department of Mathematics,
Friedrich-Alexander-University of Erlangen-Nürnberg,
Cauerstrasse 11, 91058 Erlangen, Germany
e-mail: bourne@math.fau.de

Thesis submitted to the Australian National University in August 2015; degree approved on 30 November 2015; supervisors Alan Carey and Adam Rennie.