PROBLEMS FOR SOLUTION

P 66. "Gauss' Lemma" (§ 23, vol. 1 of Modern Algebra by Van der Waerden) is essentially equivalent to the statement that a unique factorization domain R has the following property:

(*) If K is the field of quotients of R, then a polynomial over R which factors over K factors over R.

Show that the following converse holds: if R is a domain in which every element can be expressed as a product of irreducible elements - for example if R is Noetherian - and if R has property (*), then R is a unique factorization domain.

Carl Riehm, McGill University

P 67. Let

$$C = \lim_{n \to \infty} \left[\sum_{j=1}^{n} \frac{1}{j} - \ell n\, n \right]$$

denote the Euler-Mascheroni constant and let x be a real variable.

Determine the following limit:

$$\lim_{x \to 0} x^{-2} \{ C + \mathcal{R} (\Gamma' (ix)/\Gamma(ix)) \} ,$$

$\mathcal{R}$ = real part of.

H. G. Helfenstein, University of Ottawa

P 68. Find all solutions of

$$\phi(2^{2^n} - 1) = \phi(2^{2^n})$$

where ϕ is Euler's function.

David Klarner, University of Alberta

https://doi.org/10.1017/S0008439500026515 Published online by Cambridge University Press

P 69. It is a familiar fact that a cyclic permutation of length n can be written as a product of $n-1$ transpositions. Show that it cannot be done so more economically.

I. Connell, McGill University

P 70. Prove that every finite abelian group is isomorphic to a subgroup of the multiplicative group of integers relatively prime to m, mod m, for suitable m.

Carl Riehm, McGill University