NONLINEAR EFFECTS AND THE LIMITATION OF ELECTRON STREAMING INSTABILITIES IN ASTROPHYSICS

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ABSTRACT

Nonlinear effects, such as soliton collapse, will result in evolution of hydromagnetic waves excited by a field-aligned charged particle beam. If the time scale for such evolution is comparable to, or shorter than, linear time scales such as those for wave growth or pitch angle isotropization, then nonlinear effects may limit the instability. For conditions appropriate to relativistic electron streaming in a radio galaxy, the nonlinear time scale may be comparable to the linear time scales.

A topic of considerable discussion in this symposium is the stability of a relativistic, magnetic field-aligned electron beam. Linear plasma theory predicts that such an electron distribution is unstable to the growth of Alfvén waves. Quasilinear theory then predicts the electrons will resonantly interact with these waves, resulting in pitch angle diffusion, and thereby isotropization of the distribution. The net result of the linear and quasilinear processes is that the electron streaming speed is reduced to the Alfvén speed. A controversy continues as to whether the aforementioned sequence of events occurs. To date, theoretical arguments against the Alfvén streaming speed have tended to invoke linear processes, i.e., Holman, Ionson, and Scott (1979).

In this paper we consider possible nonlinear processes which might limit this instability. Nonlinear hydromagnetic waves in a finite-beta plasma are described by the Derivative Nonlinear Schrödinger equation (Spangler and Sheerin 1982),

$$i \frac{\partial \varphi}{\partial t} + i \frac{\partial}{\partial x} \left\{ \frac{1}{4} \varphi \left[4 + \frac{|\varphi|^2}{(1-\beta)} \right] \right\} \pm \mu \frac{\partial^2 \varphi}{\partial x^2} = 0 \quad , \tag{1}$$

where φ is a circularly-polarized hydromagnetic wave, x is the direction of wave propagation, β is the ratio of plasma pressure to magnetic energy

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density, and μ a parameter determining the strength of dispersion.

The DNLS possesses a number of envelope soliton solutions (Spangler and Sheerin 1982). We now consider the consequences if the Alfvén waves excited by streaming electrons resemble these Alfvénic solitons. The most profound effects would probably occur if they undergo collapse, as do Langmuir solitons. Langmuir solitons are stable in one-dimensional systems. In two or three dimensions, however, they are unstable in the sense that they become narrower and of greater amplitude with time. At the present time we do not know if such a collapse will occur for Alfvén solitons. For the present we will assume that such a collapse will occur. In order to discuss the phenomenon in the context of the above theory, we will further assume that as the soliton contracts, it makes a transition between one-dimensional soliton states.

Three characteristics of the wave packet both insure that the soliton will collapse and furnish a means for estimating the collapse time scale. (1) A constant of the motion of the DNLS is the total energy in the soliton,

$$C_o = \int_{-\infty}^{\infty} |B|^2 \, dx \simeq 2B^2 \ell \quad , \tag{2}$$

where B is the wave amplitude and ℓ is the length scale of the soliton. (2) If the plasma beta exceeds unity, there will be an anticorrelation between the wave energy density and the plasma density, i.e.,

$$\delta n \propto \frac{B^2}{(1-\beta)} \quad , \tag{3}$$

where δn is the change in the plasma density. (3) Associated with the nonlinear wave field, there will be a pondermotive force which tends to drive plasma out of the soliton.

We envision the evolution of the nonlinear wave packet or soliton to proceed as follows. The pondermotive force causes a flow of plasma out of the soliton, thus lowering the plasma density and making δn more negative. The proportionality in (3) indicates that for $\beta > 1$, this entails an increase in the wave energy density. From equation (2) we see that if B^2 increases, conservation of the constant of the motion requires that ℓ become smaller, i.e., the soliton must contract. A larger energy density in the soliton and a smaller length scale mean that the pondermotive force will be greater. The above sequence of events will continue, but at an accelerated rate. The result is, therefore, a runaway collapse.

The above arguments can be used to obtain an equation for the temporal evolution of the soliton length scale, which goes to zero on a time scale (Spangler and Sheerin 1983)

$$\tau_c = \frac{4}{3}\frac{\ell_o}{V_A}\sqrt{\frac{B_o^2}{B^2(0)}} \quad . \qquad (4)$$

In equation (4) ℓ_o is the initial length scale of the soliton, and V_A is the Alfvén speed. The quantity $B^2(0)/B_o^2$ is the energy density of the initial wave packet divided by the energy density of the static field.

The time scale (4) has been derived by considering hydromagnetic waves to be solitons, but we feel that it is probably of much greater generality, and gives a good estimate of the time scale on which nonlinear effects cause a substantial change in a wave packet. In support of this statement, we note that the numerical results of Steinolfson (1981) for the time scale for steepening of waves into hydromagnetic shocks are in good agreement with (4). This time scale is also similar to that obtained by Goldstein (1978) for the decay of a large amplitude Alfvén wave by coupling to random density and magnetic fluctuations in a plasma.

The importance of soliton collapse, or related nonlinear processes, to the electron streaming instability may be determined by how the nonlinear time scale (4) compares with the familiar linear time scales, such as the time for wave growth (reciprocal of the growth rate) and pitch angle isotropization (reciprocal of the pitch angle diffusion coefficient).

Spangler and Sheerin (1983) compared the linear, quasilinear, and nonlinear time scales for conditions appropriate to an extragalactic radio source. For a plausible subrange of conditions, the nonlinear collapse time scale can be comparable to or less than the linear time scales when the wave amplitude is 1 - 10 percent of the static field. This suggests the possibility that the streaming instability would begin in the linear phase, and proceed until the excited waves reached a certain amplitude. Thereafter, nonlinear processes would determine the evolution of the waves, and might "decouple" the waves from the beam responsible for their excitation.

There remains the question of the process by which this decoupling would occur. One possibility is wave damping. Amplitude-modulated waves in an envelope of shrinking size will be spread out in wave number, possibly enhancing the mechanism discussed by Holman, Ionson, and Scott (1979). Another possible mechanism is resonance breaking, or resonance broadening. The wavenumber spreading alluded to above imposes a coherence time on the wave-particle interaction, and thereby breaks the resonance. An additional source of resonance breaking is the nonlinear phase modulation characteristic of the DNLS. If the Alfvén wave excited by an electron beam is described by the DNLS then (possibly random) phase shifts, due to this modulation, will occur in the wave train.

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As pointed out by Spangler and Goertz (1981), the occurrence of such phase jumps can drastically change the magnitude and even sign of the Alfvén wave growth rate, and it seems likely that the quasilinear diffusion coefficient would be similarly effected. The net result might be an inhibition of the quasilinear relaxation.

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DISCUSSION

Hasegawa: Alfvén waves have a very small dispersion and, hence, are unlikely to form an envelope soliton (because it can decay before the envelope soliton is formed) unless $\omega > \omega_{ci}/4$. Also, in the presence of B, collapse is unlikely.

Spangler: The dispersion may be small, but is not ignorable. Formally, this is true because the dispersive term is of a different order than the other terms in the equation, and is therefore reminiscent of the boundary layer problem in hydrodynamics. The magnitude of the dispersion sets the scale of the soliton. With regard to the remainder of your comment, I would contend that issue such as decay and collapse will be unclear until a multidimensional treatment with realistic ion dynamics is undertaken.

Benford: We have observed electromagnetic emission in laboratory relativistic beam-plasma experiments, perhaps caused by Langmuir solitons. The observed plasma line emission is suppressed or broadened by quite small magnetic fields, $\omega_c \sim \omega_p/10$. The galactic magnetic field may inhibit your Alfvén soliton collapse. Can you take this into account?

Spangler: No. As I mentioned in my talk, our treatment utilizes 1-D soliton properties and the assumption that collapse occurs. Questions regarding the details of the collapse, such as stabilization etc., and even a rigorous demonstration of its existence must wait a multidimensional treatment. We are presently working on this.

Bratenahl: I am trying to understand the structure of Alfvén solitons - they imply a pair of anti-parallel currents, do they not? I like the collapse argument (negative feed back), but I don't understand what happens to the anti-parallel currents.

Spangler: I have a somewhat different view, picturing these solitons as due to a balance between nonlinearity and dispersion due to a finite ion gyrofrequency.

Papadopoulos: Two Comments: (1) The 2-D theory of solitons, as shown by Rowland et al. Phys. Rev. Lett. 1981, shows that soliton collapse can be prevented by transverse magnetic field effects. (2) The presence of growth and damping transforms the solutions of the nonlinear Schrödinger equation to stochastic solutions, invalidating the soliton picture.

Spangler: The role of a transverse magnetic field in preventing Langmuir soliton collapse is not so clear. While an infinitely strong field would certainly prevent collapse, the work of Weatherall et al (1981, Ap. J., 246, 306) and Weatherall et al (1982, JGR, 87, 823) shows that soliton collapse does occur in the presence of a magnetic field.

With regard to your second point, I disagree that the occurrence of a transition to chaos invalidates the soliton picture. Poolen et al.(1983, Phys. Rev. Lett., 51, 335) show that chaotic solutions of the Zakharov equations possess "caviton" properties.

Finally, your comments refer to the properties of Langmuir waves and turbulence. It is not clear that they are applicable to nonlinear Alfvén waves.

Henriksen: We have looked at single relativistic particles interacting with non-linear wave packets at various scattering angles. We find that resonance persists even for very short "coherence lengths" or "phase lengths" of the packet.

Spangler: At least some of the quantities of interest, such as wave growth rates, are determined by the particle distribution function. A single particle approach may not be sufficient to reveal modifications due to large wave amplitudes, shocks, etc.

Kennel: What can you tell us about multidimensional soliton solutions that propagate at an angle to $\vec{B}_o$?

Spangler: Solitons propagating at an angle with respect to the magnetic field correspond to the kinetic Alfvén mode, and differ, for example in the nature of the dispersion, from those I have discussed.

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