and, finally, the equation to the circle is

$$(x^2+y^2)\Pi(a_2b_3-a_3b_2)+x\Sigma c_1(a_2b_3-a_3b_2)(a_1a_2a_3-a_1b_2b_3+a_2b_3b_1+a_3b_1b_2)$$
$$+y\Sigma c_1(a_2b_3-a_3b_2)(b_1b_2b_3-b_1a_2a_3+b_2a_3a_1+b_3a_1a_2)$$
$$-\Sigma c_2c_3(a_1{}^2+b_1{}^2)(a_2b_3-a_3b_2)=0.$$

(3) Applying this to the case of the triangle, sides $x-m_1y+am_1{}^2=0$, etc. circumscribing the parabola $y^2=4ax$, we have $a_1=1$, $b_1=-m$, $c_1=am_1{}^2$.

coeff. of $x^2=\Pi(m_2-m_3)$; coeff. of x is $-a\Pi(m_2-m_3)(1+\Sigma m_2m_3)$;

coeff. of $y=-a\Pi(m_2-m_3)(\Sigma m_1-m_1m_2m_3)$;

absolute term is $a^2\Pi(m_2-m_3)\Sigma m_2m_3$;

and the equation to the circumcircle is

$$x^2+y^2-ax(1+\Sigma m_2m_3)-ay(\Sigma m_1-m_1m_2m_3)+a^2\Sigma m_2m_3=0.$$

(Cf. *Mess. Math.*, No. 352, Aug., 1900.)

(4) In the case of numerical examples it is better to proceed *ab initio.*

Find the centre of the circumcircle of the triangle formed by the lines

$x+2y+3=0$; $2x+3y+1=0$; $3x+y+2=0$. (St. Cath.'s, '96.)

The circle is $\Sigma\lambda(2x+3y+1)(3x+y+2)=0$,

where $6\lambda+3\mu+2\nu=3\lambda+2\mu+6\nu$ and $11\lambda+7\mu+7\nu=0$,

i.e. $$\frac{\lambda}{7}=\frac{\mu}{-13}=\frac{\nu}{2}=\frac{6\lambda+3\mu+2\nu}{7},$$

and the equation is $7(x^2+y^2)-80x-20y-58=0$.

Again, show that the circumcircle of the triangle formed by the lines

$$x\cos\theta+y\sin\theta=a\sec\theta+b\sin\theta \quad (\theta=\alpha,\ \beta,\ \gamma)$$

passes through the point $(0, b)$. (St. Cath.'s, '99.)

Changing the origin to $(0, b)$, the sides become

$$x\cos\theta+y\sin\theta=a\sec\theta \quad (\theta=\alpha,\ \beta,\ \gamma);$$

and the required condition is

$$\Sigma a^2\sec\beta\sec\gamma\sin\overline{\beta-\gamma}=0 \quad\text{or}\quad \Sigma\cos\alpha\sin(\beta-\gamma)=0,$$

which is clearly true, E. M. RADFORD.

CORRESPONDENCE.

Professor Hill replies to Mr. Budden as follows:

I hope you will allow me a few words of reply to Mr. Budden's further criticisms on my edition of the Fifth and Sixth Books of Euclid.

Mr. Budden *assumes* that the ratio A/B admits of an arithmetical definition as a number when A and B are incommensurable. This is an attempt to evade the whole difficulty.

Mr. Budden's point of view is that which was generally accepted as correct prior to the publication of the purely arithmetic theories of the irrational number by Weierstrass, Cantor, and Dedekind in 1872.

In order to conform to the requirements of analysis at the present time, Mr. Budden must show that when A and B are incommensurable, the symbol or operator μ, appearing in the relation $A=\mu B$, admits of arithmetical definition as a number, and so satisfies the associative, commutative, and distributive laws. This can of course be done, but the doing of it is far beyond the comprehension of beginners. Some idea of the difficulties involved may be formed by consulting the sketch of the subject given in the Second Edition of the Second Part of Chrystal's *Algebra*, pp. 97-106, together with the references he gives, especially to the works of Dedekind and Stolz. A good account by Pringsheim will also be found on pp. 49-57 of the first volume of the *Encyklopädie der Mathematischen Wissenschaften*. It is now so generally admitted that it is not permissible to assume that μ admits of arithmetical definition, and satisfies the fundamental laws of algebra, that it is not necessary to examine the rest of Mr. Budden's argument in detail.

It may not, however, be superfluous to notice two points.

The first is, that I am not entitled to the high honour of being the author of the three sets of conditions referred to on page 11 of the January number of the *Mathematical Gazette*, Art. 2, Note ii, as mine. These are the conditions stated in the Fifth Definition of the Fifth Book of Euclid. So far as I know, they were first reduced to two by Stolz in his *Vorlesungen über Allgemeine Arithmetik*, Part I., page 87, published in 1885.

The second is the reference in Mr. Budden's 5th Article to "Prof. Hill's system." On this point I desire to say that I have not invented a system. My work has been that of a commentator on Euclid, and I believe that I have accomplished two things. The first is the giving of adequate explanations of the definitions of the Fifth Book, which previously had to be learned by heart without being understood. The second is the removal of the indirectness from Euclid's line of argument *without departing from his principles*, by showing that all the properties of equal ratios can be deduced from the test for equal ratios (Euc. V., Def. 5) without the employment of the test for distinguishing between unequal ratios (Euc. V. Def. 7). The result is that the argument has been simplified to an extent which has brought it within the grasp of persons of average ability.

M. J. M. Hill.

[The reader must not assume that the Editor and his committee of co-operation agree with Mr. Budden's criticisms. The discussion has been inserted for the following reason. One aim of the *Gazette* is to keep teachers of Mathematics *au courant* with the advance of mathematical thought and methods. Mr. Budden and Professor Hill represent respectively two schools, the old and the new. With the object of showing teachers the difference between these schools, the Editor asked Professor Hill to reply to Mr. Budden's criticism. The reply involved an answer and a rejoinder. The controversy has been useful if for no other reason than that it has sent some of our readers to Stolz and Dedekind. Of course, the case *is summed up* in Professor Hill's "it is now so generally admitted that it is not permissible to assume. . . . "—W. J. G.]

NOTICES.

Eléments de Méthodologie Mathématique, by M. Dauzat. Pp. 1100. 1900. (Nony, Paris.)

If the agreeable anticipations to which the title of this volume gave rise have not been completely realised, it is no doubt because (pardon the profanity of the metaphor) there is not a one-to-one correspondence in text-books on both sides of the Channel. It is of course within the bounds of possibility that a book on Methodology in Mathematics may yet make a belated appearance in this country. But when it does, we will venture to say that, earnest and conscientious as is the voluminous compilation of M. Dauzat, an English imitator will not run to 1100 pages in covering the same ground.