USING HELIOSEISMIC DATA TO PROBE THE HYDROGEN ABUNDANCE IN THE SOLAR CORE

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ABSTRACT. A procedure for inverting helioseismic data to determine the hydrogen abundance in the radiative interior of the sun is briefly described. Using Backus-Gilbert optimal averaging, the variation of sound speed, density and hydrogen abundance in the energy-generating core is estimated from low-degree p-mode frequencies. The result provides some evidence for there having been some redistribution of material during the sun's main-sequence evolution. The inversion also suggests that the evolutionary age of the sun is perhaps some 10 per cent greater than the generally accepted value, and that the solar neutrino flux, based on standard nuclear and particle physics, is about 75 per cent of the standard-model value.

1. INTRODUCTION

Helioseismology provides a unique tool for investigating the chemical composition of the interior of the sun. Information can be obtained either indirectly, by comparing the observed frequencies with the eigenfrequencies of theoretical solar models (e.g. Christensen-Dalsgaard and Gough, 1980, 1981), or directly, by measuring the influence on the sound speed in the regions of ionization of abundant elements (Gough, 1984). Frequency comparisons have been carried out, for example, for models with varying initial chemical abundances, and with turbulent diffusive mixing or enhanced energy transport by weakly interacting massive particles in the core [see reviews by Gough (1983, 1985) and Christensen-Dalsgaard (1988)]. In principle, the direct method can be used to determine the helium abundance in the convection zone, though attempts to do so have not yet yielded a reliable value (Däppen, Gough and Thompson, 1988).

It should be noted that the eigenfrequencies of a spherically symmetrical hydrostatic solar model depend only on the pressure p, density ρ and adiabatic exponent γ of the equilibrium state. Therefore, any attempt to relate them to chemical composition $\underset{\sim}{X}$ must require a knowledge of the equation of state: $\gamma = \gamma(p,\rho,\underset{\sim}{X})$. In the

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radiative interior of the sun the most abundant elements are almost totally ionized, and γ ($\simeq 5/3$) is extremely insensitive to $\underset{\sim}{X}$. Therefore, only indirect methods are available to study $\underset{\sim}{X}$ in the core, requiring the seismic analysis to be supplemented with additional assumptions.

In this paper we report on a helioseismic inversion in which it is assumed that the sun is in thermal balance. By equating the generation of thermonuclear energy in the core with radiative transfer, and assuming the total heavy-element abundance Z to be given, an estimate of the deviation δX of the hydrogen abundance X in the core of the sun from that in a standard solar model is obtained. With the use of the equation of state it is then possible to estimate the temperature deviation, and thence revise the theoretical computation of the neutrino flux.

2. HELIOSEISMIC INVERSION UNDER THE CONSTRAINT OF THERMAL BALANCE

Direct inversions, based on iterative linearization of the relation between frequency and structure differences between the sun and a theoretical model, depend on satisfying linear integral constraints such as

$$\frac{\delta\omega^2_{n,\ell}}{\omega^2_{n,\ell}} = \int_0^R \left(K^{(n,\ell)}_{\rho,\gamma} \frac{\delta\rho}{\rho} + K^{(n,\ell)}_{\gamma,\rho} \frac{\delta\gamma}{\gamma}\right)dr \quad , \tag{2.1}$$

or

$$\frac{\delta\omega^2_{n,\ell}}{\omega^2_{n,\ell}} = \int_0^R \left(K^{(n,\ell)}_{c^2,\gamma} \frac{\delta c^2}{c^2} + K^{(n,\ell)}_{\gamma,c} \frac{\delta\gamma}{\gamma}\right)dr \quad , \tag{2.2}$$

(e.g. Gough, 1985; Kosovichev, 1986; Gough and Kosovichev, 1988), where $\delta\omega_{n,\ell}$ is the perturbation to the frequency $\omega_{n,\ell}$ of order n and degree ℓ produced by small deviations $\delta\rho$, $\delta\gamma$, or equivalently δc, $\delta\gamma$, in ρ, γ and sound speed c. Spherical symmetry has been assumed, so that degeneracy splitting in azimuthal order m is ignored; that also reduces integrals over the volume of the sun to integrals with respect to the single radial coordinate r. Only two of the three independent thermodynamic state variables ρ, c and γ appear in each constraint, because in deriving those constraints the equations of hydrostatic support

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2} \quad , \tag{2.3}$$

$$\frac{dm}{dr} = 4\pi\rho r^2 \quad , \tag{2.4}$$

(together with a knowledge of the mass M and radius R of the sun) have been imposed, as indeed they have been also in determining the displacement oscillation eigenfunctions $\xi_{n,\ell}$ of the theoretical model, upon which the kernels $K^{(n,\ell)}_{\alpha,\beta}$ depend.

The kernels in equations (2.1) and (2.2) characterize the sensitivity of the oscillation frequencies to deviations in the structure at different radii in the star. Using the constraints (2.1) and (2.2), those deviations can be inferred from the data $\delta\omega^2_{n,\ell}$ by a standard inversion procedure.

Our interest here is in the core of the sun, where $\gamma \simeq 5/3$. Therefore, notwithstanding possible contributions to $\delta\omega^2_{n,\ell}/\omega^2_{n,\ell}$ from γ deviations in the ionization zones, we have set $\delta\gamma = 0$ in equations (2.1) and (2.2); our hope is that from the data set with which we are working it is possible to construct averaging kernels for linear combinations $\Omega_k = \sum\alpha_{k,n\ell}(\delta\omega^2_{n,\ell}/\omega^2_{n,\ell})$ of data that are sufficiently small in the outer regions of the sun such that any contribution from $\delta\gamma$ to Ω_k is small, even though the contribution to $\delta\omega^2_{n,\ell}/\omega^2_{n,\ell}$ may not be. Thus we approximate equations (2.1) and (2.2) by

$$\frac{\delta\omega^2}{\omega^2} = \langle K_\rho, \frac{\delta\rho}{\rho}\rangle \quad , \tag{2.5}$$

and

$$\frac{\delta\omega^2}{\omega^2} = \langle K_{c^2}, \frac{\delta c^2}{c^2}\rangle \quad , \tag{2.6}$$

where the angular brackets denote integration with respect to r. For clarity, we have now suppressed the labels n,ℓ identifying the modes, and also the second subscript on the kernels.

The frequency conditions (2.5) or (2.6) do not adequately constrain the possible deviations $\delta\ln\rho$ or $\delta\ln c^2$ from solar models, since they do not demand the condition $m=M$ at $r=R$. This can be achieved simply by imposing the additional constraint:

$$0 = \langle 4\pi\rho r^2, \frac{\delta\rho}{\rho}\rangle \quad , \tag{2.7}$$

which is of the same form as the other conditions (2.5). The density kernel $4\pi\rho r^2$ can be transformed into a corresponding c^2 kernel by the same procedure as is used for transforming the other density kernels (cf. Kosovichev, 1988; Gough, 1989). In the remainder of the discussion it is assumed that this constraint is included amongst the frequency constraints (2.5) and (2.6).

Equations (2.5) or (2.6) can be transformed into a relation between $\delta\ln\omega^2$ and composition deviations by additionally imposing the constraint of thermal balance through the equations:

$$\frac{dL}{dr} = 4\pi\rho r^2\varepsilon \quad , \tag{2.8}$$

$$\frac{dT}{dr} = \begin{cases} -\dfrac{3\kappa\rho L}{64\sigma r^2 T^3} , & \text{in radiative zones} \\[2ex] \left(\dfrac{dT}{dr}\right)_c , & \text{in the convection zone} \end{cases} \tag{2.9}$$

where L is luminosity, T is temperature, ε is the energy-generation rate per unit mass, κ is opacity and σ is the Stefan-Boltzmann constant. The function denoted by $(dT/dr)_c$ is provided by the (mixinglength) formalism relating temperature gradient to energy transport in a convection zone. In addition one requires a means of determining the equilibrium values of $\varepsilon(\rho,T,X,Z)$ and $\kappa(\rho,T,X,Z)$ and their partial derivatives. The transformation then yields a new constraint, which may be written

$$\frac{\delta\omega^2}{\omega^2} = \langle K_X, \frac{\delta X}{X}\rangle + \langle K_Z, \frac{\delta Z}{Z}\rangle \quad . \qquad (2.10)$$

It is a straightforward matter to compute the kernels in equation (2.10) from K_ρ using the adjoint of the linearized structure equations (e.g. Marchuk, 1977; cf. Masters, 1979). For simplicity, and for want of an obviously more suitable procedure, we have set

$$\delta Z = 0 \quad , \qquad (2.11)$$

(we could equally well have chosen $\delta(Z/X) = 0$ without substantially complicating the analysis), under which circumstances the linearized structure equations (2.3), (2.4), (2.8) and (2.9) formally become, for example,

$$\mathcal{A}\frac{\delta\rho}{\rho} = \frac{\delta X}{X} \quad , \qquad (2.12)$$

where $\mathcal{A}$ is a linear differential operator. This equation must be supplemented with appropriate boundary conditions, which are derived from demanding regularity of the deviation at the centre of the sun and from the requirement that conditions in the photosphere are unchanged, which we adopt in the form $\delta\ln\rho = 0$, $\delta\ln L = 0$ at $r=R$. Substituting equations (2.11) and (2.12) into equation (2.10) yields

$$\frac{\delta\omega^2}{\omega^2} = \langle K_X, \mathcal{A}\frac{\delta\rho}{\rho}\rangle = \langle \mathcal{A}^* K_X, \frac{\delta\rho}{\rho}\rangle \quad , \qquad (2.13)$$

where $\mathcal{A}^*$ is the adjoint of $\mathcal{A}$. By demanding that the expressions on the right-hand sides of equations (2.5) and (2.13) are identical, one thus obtains

$$\mathcal{A}^* K_X = K_\rho \quad . \qquad (2.14)$$

Thus, the hydrogen-abundance kernels K_X are obtained as solutions of the adjoint linearized structure equations, subject to adjoint boundary conditions which are determined by requiring that the integrated parts arising from the partial integrations necessary to derive the second of equations (2.13) from the first vanish for all functions $\delta\rho/\rho$ that satisfy the boundary conditions supplementing equation (2.12).

Kernels K_Z are computed similarly. Examples of kernels K_X, K_Z, K_ρ and K_{c^2} are presented by Gough and Kosovichev (1988).

3. INVERSION OF FREQUENCIES

Inversions using the constraints (2.5), (2.6), and the constraints (2.10) with the condition (2.11), have been separately carried out to determine $\delta\ln\rho$, $\delta\ln c^2$ and $\delta\ln X$ using the optimal averaging procedure of Backus and Gilbert (1968; see also Gough (1985) for a discussion of its application to the solar interior). Only the 119 frequencies of modes with $\ell \leq 5$ reported by Jiménez et al. (1988) and Henning and Scherrer (1986) were used; these are the modes that are most important in diagnosing the structure of the core. Christensen-Dalsgaard's (1982) solar model 1 was adopted as the reference.

To test the procedure we first inverted the frequencies of the chemically homogeneous model used by Christensen-Dalsgaard and Gough (1980) and Christensen-Dalsgaard et al. (1989). The results are compared with the exact deviations $\delta \ln \rho$, $\delta \ln c^2$ and $\delta \ln X$ in Figures 1, 2 and 3. It can be judged from those figures how accurate the procedure might be when optimal averages are identified with point values. It should be borne in mind, however, that this test is not realistic: the deviations are much larger than those of the sun, which must have a deleterious effect on the accuracy of the linearized frequency constraints; on the other hand, the theoretical frequencies of the homogeneous model were computed (without deliberately introducing errors) in precisely the same manner as those of the reference model, and the values of Z are known to be the same for the two models, both of which tend to enhance the apparent reliability of the inversions.

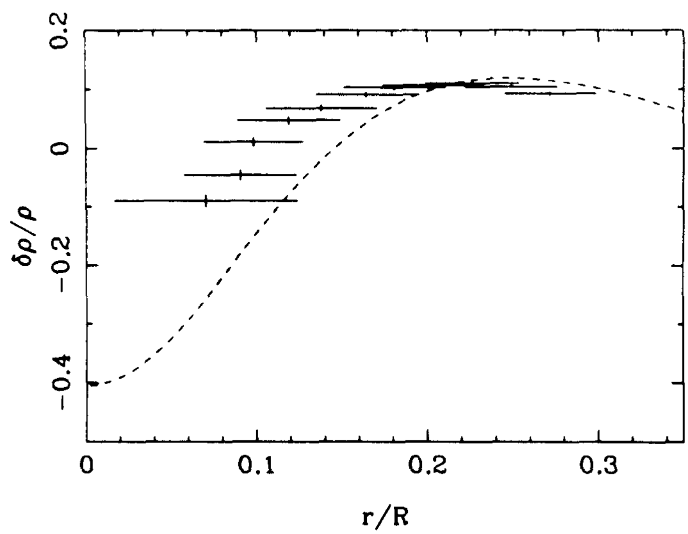

Figure 1. The dashed line is the relative difference between the density of a homogeneous model of the sun and that of a standard model. The crosses represent optimal averages deduced from the differences between eigenfrequencies of the two models corresponding to those modes that were used for the inversion of the solar data. The horizontal components of the crosses indicate the widths of the optimal averaging kernels and represent the resolution of the inversion; the vertical components indicate the standard errors. The latter were computed assuming the standard errors for the corresponding observed frequencies.

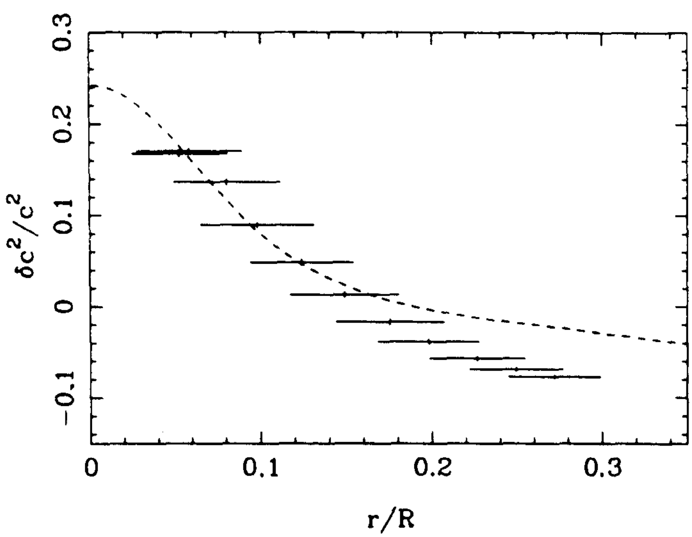

Figure 2. Difference between squared sound speeds of the homogeneous and the standard models, and the inversion results. The notation is otherwise the same as in Figure 1.

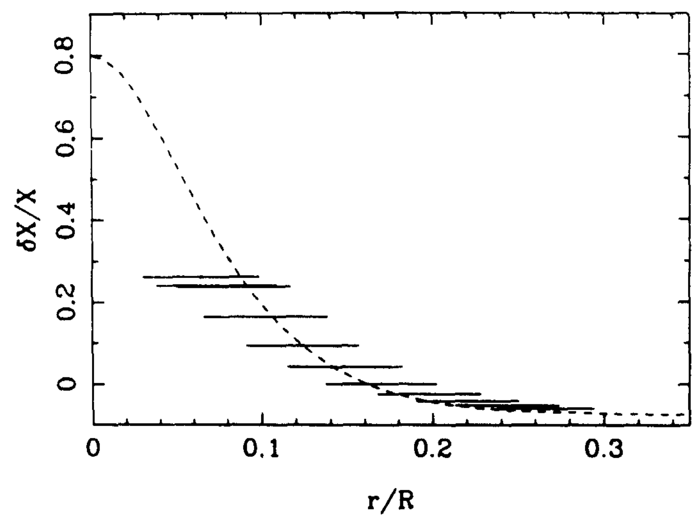

Figure 3. Difference between the hydrogen abundances of the homogeneous and the standard models, and the inversion results. The notation is otherwise the same as in Figure 1.

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Notice, however, that because the two models have different hydrogen abundances there is a substantial difference in the value of γ in the ionization zones, possibly of greater magnitude than for the sun. Therefore the inversion does test our ignoring the γ deviations in equations (2.5) and (2.6) for the mode set we have used. It is evident from the inversions that the structure of the small inner region $r \lesssim 0.05$ R cannot be resolved by the p-mode data.

Inversions of the solar data have been reported previously (Gough and Kosovichev, 1988), and are reproduced in Figures 4, 5 and 6. The sun appears to be some 10 per cent denser at the centre than the standard solar model. Moreover, the inversion for $\delta \ln X$ suggests a flatter hydrogen-abundance profile than that of the model. This is suggestive of there having been some degree of material redistribution in the core.

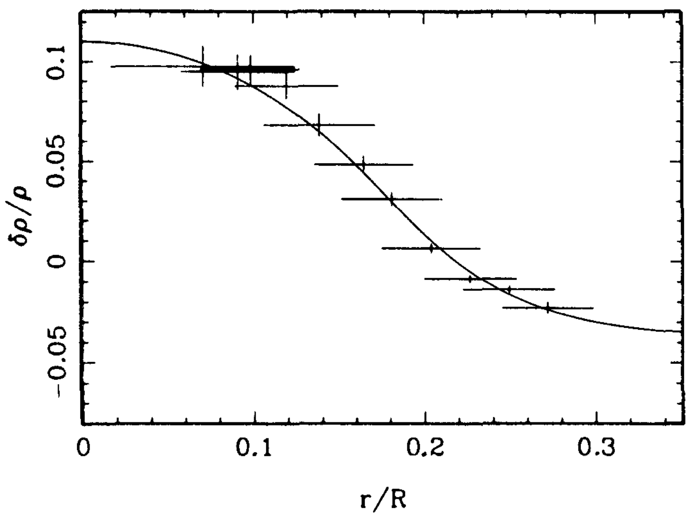

Figure 4. Optimal averages of the relative density deviation of the sun from the reference solar model, obtained by inversion of the low-degree p-mode frequencies, represented as crosses in the manner of Figures 1-3. The continuous curve was drawn freehand through the averages, and was used for estimating the neutrino flux.

4. MAIN-SEQUENCE AGE AND NEUTRINO FLUX

If one were to accept the continuous line drawn through the points in Figure 6 to be representative of the deviation of the solar hydrogen abundance from that of the standard model, one would conclude that more hydrogen has been consumed in the sun than is predicted theoretically. Since the main-sequence variation of L with time is only weakly dependent on the fine details of conditions in the cores of calibrated solar models, this would appear to imply that the sun is

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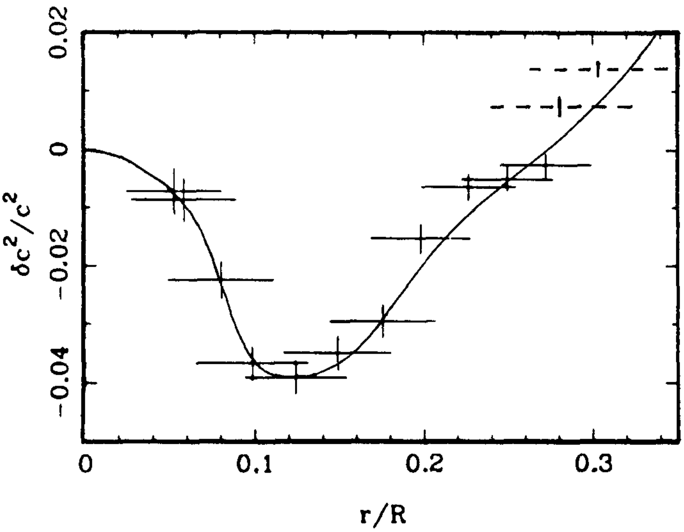

Figure 5. Optimal averages of the relative deviation of the square of the sound speed in the sun from that of the reference theoretical model. The continuous crosses denote averages obtained by inverting the constraints (2.6) imposed by the low-degree modes alone; the dashed crosses denote optimal averages, presented by Gough and Kosovichev (1988), of the modes with $\ell \leq 3$ provided by Jiménez et al. (1988) together with all the modes listed by Duvall et al. (1988) whose lower turning points are below r = 0.7 R. The continuous curve was drawn, essentially freehand, through the averages, taking account of the inversions for r > 0.35 R which lie outside the bound of this figure.

older than is generally believed, by an amount δt given approximately by

$$\delta t \simeq E\bar{L}^{-1} \int_0^M (\delta X_0 - \delta X)dm \quad , \tag{4.1}$$

where δX_0 is the deviation of the primordial hydrogen abundance from that of the model, $E = 6.3 \times 10^{18}$ erg g^{-1} is the energy released per unit mass in the conversion of hydrogen to helium, and $\bar{L} \simeq 0.85\ L_\odot$ is the mean main-sequence surface luminosity of the sun (averaged over time since the start of main-sequence evolution). If one sets δX_0 equal to the constant value attained for $r \gtrsim 0.25$ R in Figure 6, then $\delta t \simeq 6 \times 10^8$y. However, if $\delta X_0 = -0.005$, which is hardly inconsistent with the inversion, the increment δt is reduced to zero.

One can also estimate the solar neutrino flux. For this purpose it is adequate to use the perfect gas law for fully ionized material in the form

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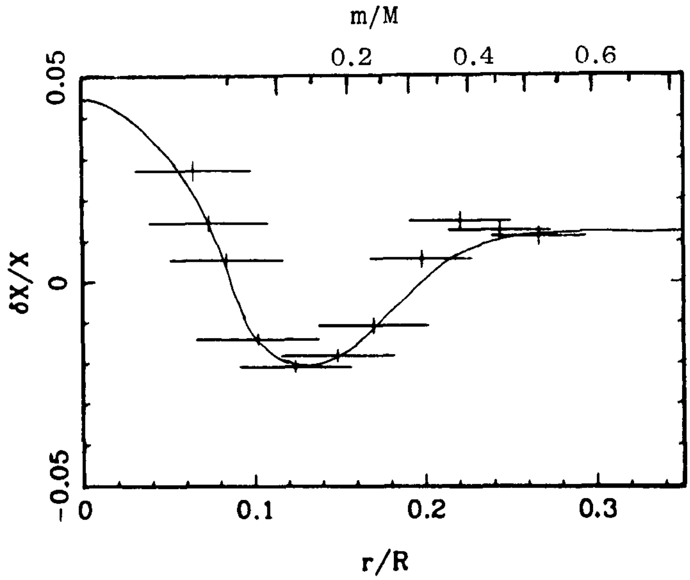

Figure 6. Optimal averages of the relative deviation of the hydrogen-abundance in the sun from that in the reference solar model, obtained by inversion of the low-degree p-mode frequencies. The continuous curve was drawn freehand, and was used together with the curve plotted in Figure 5 to estimate the temperature which is plotted in Figure 7. The upper abscissa scale is the relative mass variable for the sun, inferred from the continuous curve plotted in Figure 4. The notation is otherwise the same as in Figure 4.

$$T = \frac{\mu c^2}{\gamma R} \quad , \tag{4.2}$$

to estimate the temperature T, where $\mu \simeq 4/(5X+3)$ is the mean molecular weight and R is the gas constant. From the continuous lines drawn in Figures 5 and 6, the temperature deviation δT can be obtained by linearizing equation (4.2) in the deviations. The resulting temperature distribution we thus infer for the central regions of the sun is shown in Figure 7.

The major contribution, $F_{\nu 8}$, to the neutrino flux F_ν comes from the decay of ^8B. It can be estimated from the formula

$$F_{\nu 8} = \frac{\lambda_8}{\lambda_{8m}} F_{\lambda 8m} \quad , \tag{4.3}$$

where the subscript m denotes the value obtained from the theoretical model and

$$\lambda_i = \int_0^M \phi_i dm \quad , \tag{4.4}$$

with

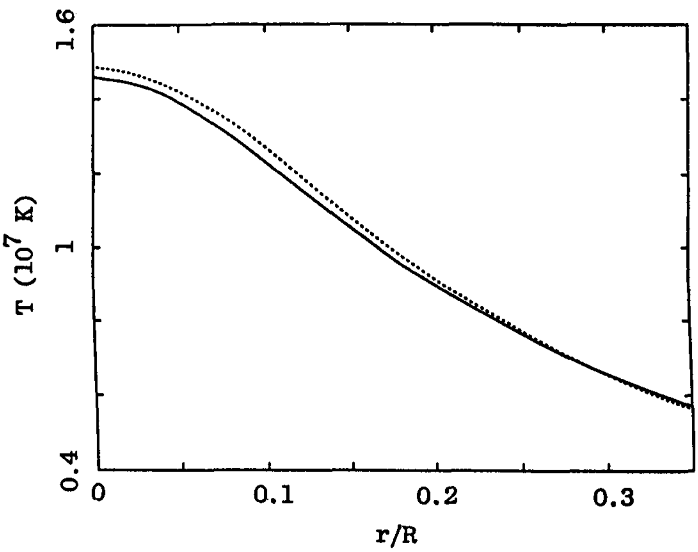

Figure 7. The continuous curve is the temperature in the core of the sun, inferred from the sound-speed and hydrogen-abundance deviations plotted as continuous curves in Figures 5 and 6. For comparison, the temperature of the reference theoretical model of Christensen-Dalsgaard (1982) is plotted as a dotted curve.

$$\phi_8 = \frac{1-X}{1+X} X^2 \rho T^{24.5} \quad , \qquad (4.5)$$

(cf. Gough, 1988). The fluxes $F_{\nu 7}$ and $F_{\nu CNO}$ from the electron capture by ^7Be in the proton-proton chain and from the decay of ^{13}N and ^{15}O in the CNO cycle were scaled similarly, with $\phi_7 = X(1-X)\rho T^{11}$ and $\phi_{CNO} = XZ\rho T^{20}$. We thus found $\lambda_8/\lambda_{8m} = 0.67$, $\lambda_7/\lambda_{7m} = 0.84$ and $\lambda_{CNO}/\lambda_{CNOm} = 0.73$. The remaining contribution, principally from the p-p reaction of the proton-proton chain, is small and is hardly changed and was taken to be invariant.

Theoretical neutrino fluxes are not available for Christensen-Dalsgaard's model 1. We therefore adopted the model values quoted by Bahcall and Ulrich (1988), namely $F_{\nu pepm} = 0.2$ snu, $F_{\nu 7m} = 1.1$ snu, $F_{\nu 8m} = 6.1$ snu and $F_{\nu CNOm} = 0.4$ snu. The resulting total solar neutrino flux is thus found to be $F_\nu \simeq 5.5$ snu.

5. DISCUSSION

The aim of the analysis summarized in this paper has been to make a first estimate of the implications of helioseismic data regarding the structure of the energy-generating core of the sun. We have carried out separate inversions for the deviations $\delta\rho$ and δc^2 of the density and the square of the sound speed in the sun from the corresponding quantities in a standard solar model, the results of which are illustrated in Figures 4 and 5. The continuous lines drawn through the results are our preliminary estimates of those deviations in the inner 35 per cent of the solar radius. They were drawn essentially freehand, and, notwithstanding any systematic errors that may have corrupted our inversions, are particularly uncertain for $r \lesssim 0.05$ R, where the structure of the star has little influence on the p-mode frequencies, and for $r \gtrsim 0.25$ R. To make inferences outside the energy-generating core would require supplementing the seismic data we have used with observed frequencies of modes of higher degree.

Our inversions have not been carried out in a wholly consistent manner. One of the most obvious approximations we have made is to neglect the deviation of γ in the ionization zones. The degree to which this spoils our inferences can be judged from Figures 1 and 2, which display the results of using theoretical eigenfrequencies to infer the structure of a chemically homogeneous solar model. In carrying out those inversions, we used the same mode set as that for the solar inversions. The homogeneous model has the same value of Z, but a different hydrogen abundance. Therefore the ionization zones are different. Since γ varies substantially only in and above the ionization zones, in the outer layers of the sun, we conclude that this experiment actually gives an indication of our ability to eliminate the influence of not solely our neglect of $\delta\gamma$, but also the effect of all the other uncertainties in the complicated physics of the upper layers of the convection zone. Notice that the relative deviations $\delta\rho/\rho$ and $\delta c^2/c^2$ plotted in Figures 1 and 2 are not extremely small compared with unity. Therefore the linearized constraints (2.5)-(2.7) used for the inversions may not be good approximations, which no doubt accounts in part for the differences between the actual deviations and those inferred. We have made no attempt to iterate the procedure.

Of course the inversions for $\delta\rho$ and δc^2 are not independent: the kernels K_ρ and K_{c^2} are related via the hydrostatic constraint, which has already been assumed to hold in the derivation of the linearized oscillation equations. Therefore, in principle, just one of the inversions should suffice; from a knowledge of $\delta\rho$ and the equilibrium state of the reference theoretical solar model one can calculate δc^2, and vice versa. However, we have carried out both inversions separately in order to provide a consistency check. We find that the continuous curves drawn through the optimal averages in Figures 4 and 5 do not satisfy the perturbed hydrostatic equations, and therefore we conclude that our inversions are not consistent. This is perhaps not surprising, in view of the systematic errors, evident in Figures 1 and 2, in our inversions of the eigenfrequencies of the homogeneous model. We have not yet carried out a systematic investigation either to assess whether to put more trust in the inferred values of δc^2 or in the values of $\delta\rho$ (though one might naively suspect that it would be more prudent to trust δc^2, since the physics of acoustic wave propagation throughout all but the surface layers of the sun is more directly related to the sound speed than to the stratification of density), or to judge how sensitive our subsequent conclusions are to the errors whose existence the inconsistency reveals.

The inversion for the deviation δX of the hydrogen abundance is even less reliable. Not only is it susceptible to all the uncertainties in the inversions for $\delta\rho$ and δc^2, but also it requires a knowledge of the nuclear reaction rates and the opacity. Moreover, and perhaps most important of all, it depends on the assumption of thermal balance, expressed by equations (2.8) and (2.9). Judging from the inversion of the eigenfrequencies of the homogeneous model, illustrated in Figure 3, our procedure might appear to be no worse for the secondary variable δX than it is for the fundamental state variables. However, it is important to realize that this figure provides a test of only the procedure for inversion, and not of the results themselves, because, like the reference model, the homogeneous solar model is known to be in thermal balance, and, unlike the sun, was constructed with identical physics to the reference model. Of course δX could have been computed from $\delta\rho$ and δc^2, simply by integrating the perturbed thermal-balance equations. However, the outcome would depend not only on the errors in the inversions required to obtain $\delta\rho$ or δc^2, but also on the errors in drawing the smooth curve through the averages and extrapolating it into the regions where no optimal averages have been determined. It also depends on the validity of equating optimal averages with point values. Therefore we have preferred to obtain averages of δX by inverting the constraints (2.10) directly.

From the determinations of $\delta\rho$, δc^2 and δX we have estimated the variation of temperature in the solar core. This is compared with the temperature in the standard theoretical model in Figure 7. Although in the core of the sun electrons are partially degenerate and the perfect gas law should not be (and, indeed, was not) used in constructing the reference model, equation (4.2) is adequate, at the present level of reliability of the inversions, for estimating the partial derivatives of T with respect to c^2 and X required for determining the quite small relative temperature deviations $\delta T/T$. In the central region of the core the magnitude of the inferred solar temperature gradient is lower than in the theoretical model.

As we have already pointed out, the shape of the profile of $\delta X/X$ plotted in Figure 6 might be regarded as evidence for there having been some degree of material redistribution in the core during the main-sequence evolution. Diffusive mixing of the kind discussed by Schatzmann et al. (1981), however, is not consistent with the seismic data (e.g. Gough, 1983), and leads to a lowering of the central density (see also Kosovichev and Severny, 1985; Christensen-Dalsgaard, 1988) which is inconsistent with the inversion plotted in Figure 4. If energy transport in the core of the sun were enhanced by wimps the density would be increased, but the resultant sound speeds in the theoretical models that have been published are not in accord with our inferences in Figure 5 (Gough and Kosovichev, 1988).

The procedure we have outlined has yielded estimates of the thermodynamic state and the hydrogen abundance in the energy-generating core of the sun. From that we have been able to estimate the factor by which the solar neutrino flux should differ from the standard-model value: about 0.7 if the continuous curves in Figures 4-6 are adopted. Because we know that our independent estimates of $\delta\rho$

and δc^2 plotted in Figures 4 and 5 are not consistent with hydrostatics, we have recomputed the flux from an estimate of $\delta\rho$ derived from the sound-speed deviation plotted in Figure 5 and the hydrostatic constraint (2.3) and (2.4). To accomplish that it was necessary to know the sound-speed deviation throughout the model. In the radiative envelope we used the results of an inversion of intermediate-degree p modes [specifically, we used the dashed curve in Figure 15 of Gough and Kosovichev (1988); this is consistent with other recent inversions by Vorontsov (1988), Christensen-Dalsgaard et al. (1989) and Shibahashi and Sekii (1988)], matched smoothly onto the optimal averages plotted in Figure 5. The resulting factor by which the standard-model flux should be multiplied is 0.83, yielding $F_\nu \simeq 6.6$ snu if Bahcall and Ulrich's (1988) standard value of 7.9 snu is adopted. The difference between this value and the value (5.5 snu) obtained by taking the independent inversions plotted in Figures 4 and 5 provides some indication of the degree of uncertainty in our result. [Had we adopted the value of the neutrino flux (5.8 snu) obtained from the standard solar model of Turck-Chièze et al. (1988) to be representative of our reference model, we would have obtained 4.1 snu and 4.9 snu as our estimates of the neutrino flux from the sun. Of course, neither the model of Bahcall and Ulrich (1988) nor that of Turck-Chièze et al. (1988) is identical to Christensen-Dalsgaard's (1982) standard model 1 which we have used as a reference, so our procedure for estimating F_ν from the factors λ_i/λ_{im} is not strictly valid; what we should do is either use the perturbation theory in the manner we have described but in addition take due account of the structural differences between the theoretical solar models, or alternatively compute F_ν directly from the distribution of ρ, X and T obtained from our inversions. We intend to carry out a more consistent analysis and report on the results in the near future.] We must emphasize, however, that these estimates depend on our having assumed, as is the case in standard stellar-evolution theory, that the sun is in thermal balance and that the fast nuclear reactions in the p-p chain and the CNO cycle are in equilibrium with the slowest reactions which control the overall reaction rate. They also depend on the assumption that the heavy-element abundance Z is not in serious error. In making these estimates we have not linearized equation (4.3) and its relatives determining the other contributions to the neutrino flux, because the exponents of T in the formulae for ϕ_i are too large for linearization to be valid.

One can also try to estimate how much hydrogen has been consumed during the main-sequence evolution. From the variation of X in the core alone, adopting the continuous curve drawn in Figure 6, it appears that that exceeds the standard value by about 0.005M, suggesting that the sun has a somewhat greater evolutionary age than the standard value. This result is consistent with previous discussions (Gough, 1983; Christensen-Dalsgaard, 1988) based on the mean frequency separation $\omega_{n,\ell} - \omega_{n-1,\ell+2}$ of low-degree modes. However, this result does depend critically on the value of the asymptote of $\delta X/X$ as r/R increases away from the core, which is ill determined by the data. It is therefore quite uncertain. It also depends, as does our estimate of the neutrino-flux factor, on the assumptions of thermal and nuclear balance used in the determination of $\delta X/X$, and which, as we have already pointed out, are brought into question by the inferred profile of X.

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