GLOBULAR CLUSTER AGES

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1. INTRODUCTION

Within the framework of a model cosmology, the ages of the globular clusters should provide upper limits to the value of Hubble's constant. Should both the ages and H be known to high accuracy, we may ask if standard comsological models are adequate. This paper will summarize recent results obtained by many astronomers for the ages of the globular clusters, how large the uncertainties are thought to be, and will suggest some further work.

2. AGE DETERMINATION TECHNIQUES

The estimation of globular cluster ages relies on the main sequence turn-off, as is well known. The sensitivities of the observational data and the evolutionary models to a host of variables warrant attention, however, as well as the actual methods of comparing the models to the data.

2.1. Sensitivities

If the color-magnitude diagrams (CMDs) of two globular clusters are superposed, it is unlikely there will be perfect overlap. Aside from errors in the photometry and in the estimates of relative distances and reddening, the discrepancies can arise from physical differences in the two stellar populations. The table below depicts the effects of variable composition, age, and convection strength on the stellar emergent spectra, which is what we actually measure, and the luminosities and temperatures of isochrones' main sequence loci, turn-offs, and subgiant, red giant, and horizontal branches, as well as the color of the latter. We should also note that B-V may become sensitive to Z_{CNO} at lower temperatures because the G-band lies in the B bandpass. In addition, the sensitivity of the main sequence locus and turn-off to α (the ratio of the convective mixing length to pressure scale height) increases with increasing metallicity (VandenBerg 1982), and even the emergent spectrum is vulnerable to α when $T_{eff} < 5000$ K (Dennis 1968; Carney 1980).

Richard M. West (ed.), Highlights of Astronomy, Vol. 6, 255–266.

https://doi.org/10.1017/S1539299600005219 Published online by Cambridge University Press

	Emergent Spectrum	M.S. Locus	M.S. Turn-off	SGB	RGB	HB	HB Red/Blue
$Z_{Fe}\uparrow$	B−V↑ V−R--	L↑	L↓ T↓	L↓	T↓	--	---
$Z_{CNO}\uparrow$	B−V-- V−R--	L↑	L↓ T↓	L↓	--	--	R
Y↑	---	L↓ T↑	L↓ T↑	--	T↑	L↑	B
α↑	---	T↑	T↑	T↑	T↑	?	?
$t_9\uparrow$	---	L↑	L↓ T↓	L↓	T↓	--	B

2.2. Estimation of Parameters

Z_{Fe} and Z_{CNO} are in principle available from spectroscopic measurements, as well as by some photometric methods. Unfortunately, the results for the more metal-rich clusters are currently disputed, with spectroscopists favoring iron-peak metallicities lower by > 0.5 dex than proposed by the photometrists. Fortunately, the debate does not extend to the more metal-poor clusters.

The helium mass fraction Y is not available observationally except in stars and planetary nebulae that have undergone extensive helium nucleosynthesis and mixing. Y may be estimated, however, in a variety of model-dependent ways, such as the temperature of the blue edge of the instability strip. Other methods seem too vulnerable to convection. A safe method is to adopt a Y value consistent with model cosmologies ($Y > 0.2$) and the observed values of HII regions ($Y < 0.3$). The isochrones themselves may be used to estimate an appropriate value. The above table and Figure 1 show, for example, that the main sequence-subgiant branch gap is a good helium indicator. Another possibility is to use the field subdwarf locus to derive a helium abundance (Carney 1979b) and assume the field and cluster stars have similar Y values.

The appropriate α value may be set by running the stellar evolution codes for a solar mass and metallicity star to find which combination of α and Y are necessary to reproduce the Sun's age, luminosity, and radius (Gough and Weis 1976; Iben and Mahaffy 1976; VandenBerg 1982). An alternative is to select the value of α that permits the best match of the lower red giant branch, whose location in temperature is quite sensitive to α (VandenBerg 1982).

2.3. Cluster distances

Although the temperature of the turn-off is sensitive to age, it is not as reliable an absolute age estimator as the turn-off luminosity. The

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computed stellar radius for the cool stars in globular clusters depends on α and hence the latter's uncertainties introduce error in age estimates from turn-off temperatures. The model luminosities should not be sensitive to α, and VandenBerg's (1982) comparison of his $\alpha = 1.6$ models to the $\alpha = 1.0$ isochrones of Ciardullo and Demarque (1977) bear this prediction out. To use turn-off luminosities, however, cluster distances are required.

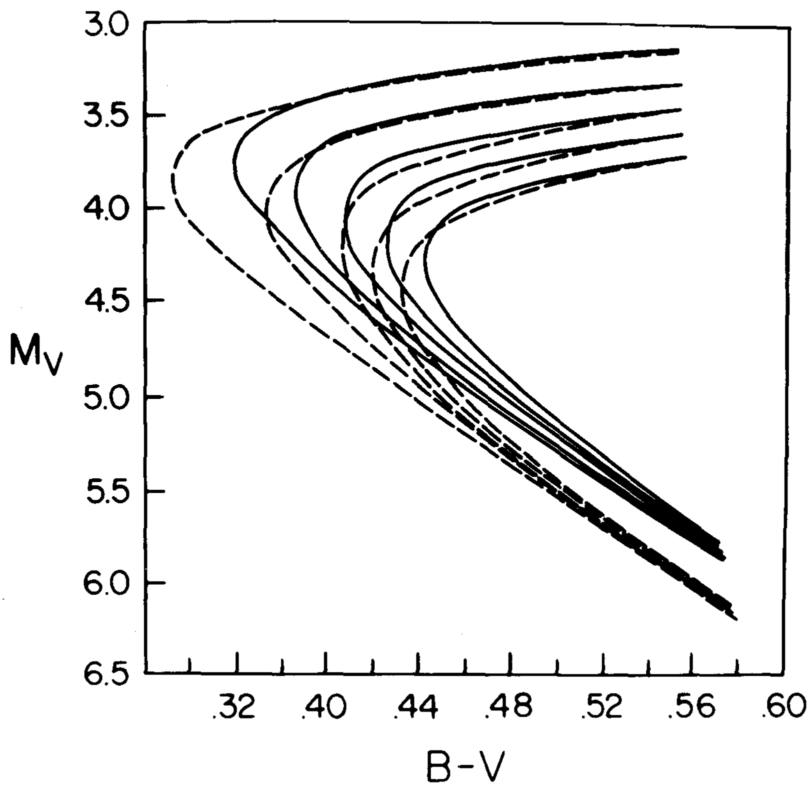

Figure 1. Y = 0.2 (____) and 0.3 (---) isochrones for ages of 12, 14, 16, 18, and 20 x 10^9 years.

Currently, two basic methods are available for estimating cluster distances, both of which rely on an assumed equivalence between field and cluster halo stars. Most frequently, an absolute visual magnitude, M_V, is adopted for the RR Lyrae variables and the horizontal branch. This value is based on statistical parallaxes and Baade-Wesselink studies of field variables, plus distances to the Magellanic Clouds derived by other means. M_V is usually taken to be +0.6 for metal-poor RR Lyraes. The second method involves main-sequence fitting (Sandage 1970; Carney 1980), which has become quite valuable due to improved trigonometric parallaxes of field subdwarfs.

3. RECENT RESULTS

3.1. Observational data

Main sequence photometric data in the BV system have been published for 15 clusters (see VandenBerg 1982 for references), and data for several others will soon be available, including some using the VR system. Of course, not all clusters are equally useful. Poor photon statisitics for faint stars, an insufficient number of observed stars, and systematic photometric errors have rendered about a third of the clusters unsuitable for age estimates. The metallicity scale uncertainties remove a few more.

3.2. Model isochrones

Several groups have published model isochrones over the past decade, but two are especially important. The work of Ciardullo and Demarque (1977), based on the evolutinary sequences computed by Mengel et al. (1979), are the most extensive. VandenBerg's (1982) work, on the other hand, is the first to thoroughly study the effects of α, as well as incorporating model atmospheres to provide more accurate surface boundary conditions.

3.3. Recent age determinations

We will focus here on absolute age determinations of the metal-poor clusters ([Fe/H] < -1.4), for which the various iron-peak metallicity estimates are at least consistent. The table below lists five sets of recent age determinations, which set of isochrones were used, the "known" parameters (measured or adopted), those solved for, the basic method of comparison of the data with the isochrones, and the results. "Overlay" means an eye estimate of the best fit, and the "6-point fit" is a 6-variable parametrization of a CMD and its comparison to a model isochrone.

Reference[a]	Isochrones[a]	"Knowns"	Unknowns	Method	t_9
DM	CD	m-M,Z,α	t,Y	overlay	13-16
C	CD	Z,α	m-M,Y,t	T_e(T.O.)	15-20
CC	CD	Z,α	m-M,Y,T	overlay	16-20
JD	CD	m-M,α	Z,Y,t	6-point	16
FJ	CD	Y,Z,α	m-M,t	stat. fit	9-17:
V	V	m-M,Z	α,Y,t	overlay	15-18

[a] DM=Demarque & McClure (1977); C=Carney (1980); CC= Caputo & Cayrel de Strobel (1981); JD=Janes & Demarque (1982); FJ= Flannery & Johnson (1982); V=VandenBerg (1982)

The agreement of the diverse techniques above seems gratifying, but we must remember that except for the last study they are based on the same observational and theoretical data. The only major disagreement is the result for NGC 6752 obtained by FJ. This is not a matter of only one cluster, however, for their method involves a sophisticated statistical fit of individual data point to isochrones, and they had such data only for NGC 6752 and M5. However, their age of 9 billion years for the former cluster is very likely a serious underestimate, and the reasons illustrate two problems in age determinations. First, there is the problem of interloping field stars and the high weight given by statistical methods to the "endpoints" (the subgiant branch in this case). The CMD published by Carney (1979a) showed two apparent subgiant branches, and the statistical method fit the brighter and younger one. Even if the dual subgiant branches were really present, this is likely to be a mistake since many physical processes (mixing, rotation, magnetic fields, mass transfer) retard the rate of stellar evolution, but few can accelerate it. If the two branches are not real, perhaps because the brighter one is due to foreground stars, we are faced with the second problem: the usual lack of any means of ascertaining the accuracy of observational data. Happily, in this particular case, we have available an independently-derived CMD, and Figure 2 shows the mean points obtained by Cannon and Lee (unpublished) plotted on top of the FJ fits to some of Carney's data. Clearly FJ underestimated the cluster's age.

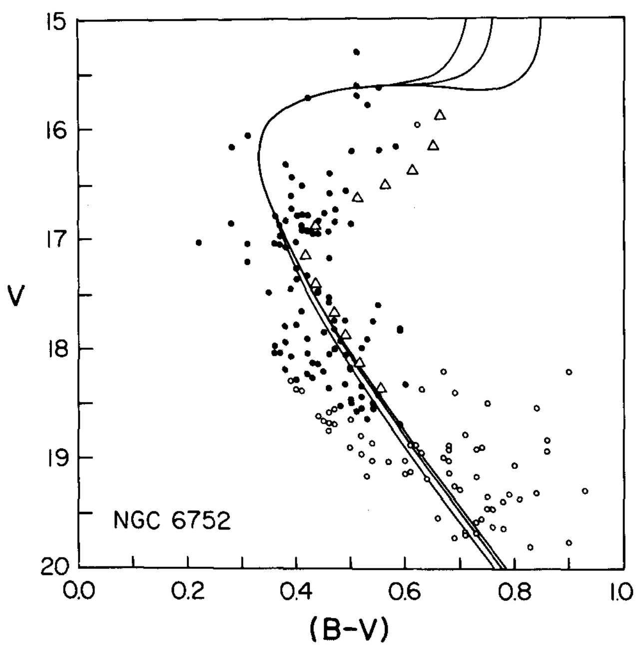

Figure 2. The preferred Y=0.2, [Z] = -1.2, -1.5, -1.8 CD isochrones of Flannery & Johnson (1982) vs. the NGC 6752 data of Carney (●,o) and Cannon & Lee (Δ)

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3.4. An optimum choice

The methods of FJ promise to be of use when more data for clusters are available. Currently, the overlay results of VandenBerg (1982) show excellent agreement between isochrones and CMDs. If we choose, however, to (a) focus only on the luminosity of the turn-off, which as we have noted is less sensitive to problems of convection; (b) work with only those clusters with a complete and accurate an abundance profile; (c) study only clusters with little or no reddening; and (d) extensive CMDs, we restrict ourselves to three clusters: M3, M13, and NGC 6752. The former two have well-detemined iron-peak and CNO abundances. The latter has an uncertain oxygen abundance, but given the near-perfect match of the CMDs of M13 and NGC 6752 and their similar iron and carbon abundances (Bell and Dickens 1980), we assume similar oxygen abundances. As already noted, NGC 6752 has two independent CMDs and thus we see from Figure 1 that the turn-off magnitude data are reliable. High resolution spectroscopic results for M3 and M13 [Fe/H] values, −1.81, −1.63 (Cohen 1978); −1.55, −1.42 (Pilachowski, Wallerstein, and Leep 1980) and photometric results (see the discussion by Suntzeff 1981) indicate comparable Z_{Fe} values. Butler's (1975) ΔS calibration yields [Fe/H] = −1.67 for M3, for example. Suntzeff (1981) and Bell and Dickens (1980) have obtained CNO abundances indicating $[Z_{CNO}/Z_{Fe}] = -0.1$ for M3 and −0.4 for M13. We adopt here $[Z] = -1.7 \pm 0.2$ for both clusters and note Suntzeff's conclusion that while the metallicities of the two clusters are comparable, their HB morphologies are not. Thus while some "second parameter" is operating, it is not Z_{CNO}. Age determinations for these three clusters may thus bracket the influence of this extra parameter on age, although age itself could be the culprit.

The helium abundance for M3 may be obtained from the temperature of the blue edge of its instability strip. Sandage, Katem, and Sandage (1981) suggested the following relation:

$$(\log T_e)_{HBE} = 3.945 + 0.274\ Y - 0.079 \log L_{HB} + 0.055 \log M\ ,$$

and Sandage (1981b) found $(\log T_e)_{HBE} = 3.875$. Uncertainties in convection do not significantly affect this temperature, and a Y value of 0.25 is obtained.

If we derive the cluster distance assuming $M_V(RR) = +0.6$, and use the above Y estimate and the empirical bolometric corrections of Carney and Aaronson (1979), we find ages of 16 ± 2 billion years for M3 and slightly greater ages for M13 and NGC 6752. The advantage of this method is that the primary influence on the RR Lyrae luminosities is the helium abundance, and that even if the field and cluster stars differ in Y, the error in the distance modulus to the cluster will just about cancel out the error in the helium abundance. The disadvantages for absolute ages are the actual Y value, which is very sensitive to $(\log T_e)_{HBE}$, the fact that a highly evolved HB star will have a higher Y value than the ZAMS stars upon which the isochrones are based ($\Delta Y > 0.02$), and that we must be certain of $M_V(RR)$. This last item is probably the most significant,

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and warrants discussion. Harris (1976) has reviewed the various determinations of M_V(RR). There are, however, some statistical parallax measures that support much larger values of M_V(RR) (+1.2, Clube and Jones, 1971, 1974; Heck 1974), which would lead to much greater cluster ages. On the other hand, slightly brighter values (+0.5) have been obtained by Manduca et al. (1981) using a revitalized version of the Baade-Wesselink method. This latter method holds great promise, for in principle it can even be extended to cluster variables, thereby circumventing the assumed field-cluster star equivalence. D. Latham and I hope to undertake such work next year, but we are currently focusing on testing the method on field stars. Our simultaneous photometry and spectroscopy of VY Ser ($\Delta S = 9$) shows a serious phase difference between the photometric and radial velocity radii, and we are for the moment uncertain of the method's reliability. We will restudy X Ari this winter.

Main-sequence fitting can yield valuable confirmations of the cluster distances. The calibration stars' distances are better determined in this case, but we have a color or temperature fit to worry about. However, the field subdwarfs have a mean [Fe/H] of −1.7, so we can fit the main sequences in B−V without worrying about line blanketing effects. Carney (1979b) has determined the field star locus from trigonometric and statistical parallaxes, and derived distances to M3, M13, and NGC 6752 (Carney 1981). M13 and NGC 6752 ages were found to be 16 ± 2 billion years (i.e., the distances were compatible with M_V(RR) = +0.5), while M3 was found to have an age of 19 ± 2 billion years (M_V(RR) = +0.85). The helium abundance appropriate for the CD isochrones was obtained by fitting the subdwarf locus to the [Z] = −1.7 CD isochrones. The temperature uncertainties due to convection are accounted for by use of this "calibrating" Y value, which was found to be 0.23 ± 0.03. As before, the derived ages are not sensitive to differences in Y between the field and clusters stars. Signs of such differences exist, we should point out, for the C+N+O results for the clusters differ from those of the field stars where $[Z_{CNO}/Z_{Fe}] \sim +0.4$ (Peterson and Sneden 1978; Sneden, Lambert, and Whitaker 1979; Clegg, Lambert, and Tomkins 1981). Although we have allowed for such CNO abundance effects in the age estimates, we should not be surprised if some other parameter is also varying. In addition, whereas VandenBerg (1982) found $\alpha = 1.6$ for the clusters, the field subdwarf locus cannot be fit with such a value if Y > 0.2 (α = 1.3 seems preferred). Perhaps CNO abundances affect the proper choice of α.

4. RELATIVE CLUSTER AGES

The relative ages of globular and open clusters are crucial to understanding the details of the formation of our galaxy and its chemical evolution, and also important in estimating the galaxy's age. Very long-lived radioactive species have been used to estimate the age of the galaxy prior to the formation of the solar system (see, for example, Clayton 1964 and Tinsley 1975 and references therein). The derived age is, however, essentially a mean age, and will

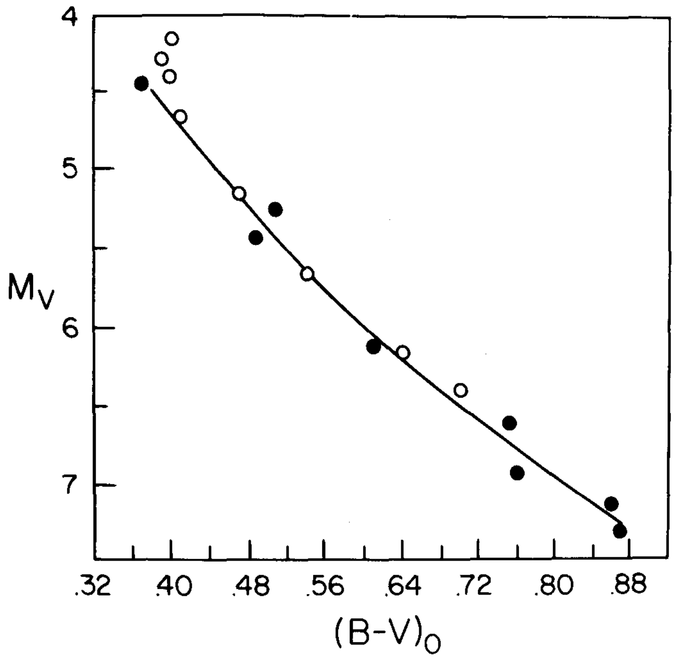

Figure 3. Sandage (1970) fiducial points of M3 (o) and subdwarfs with parallax data (●).

consequently be affected by the rapidity of galactic nucleosynthesis. Sudden nucleosynthesis will mean a smaller derived age than steady nucleosynthesis, by as much as several billion years.

The relative ages of the globular clusters is very difficult to ascertain, primarily because of the variable compositions. Demarque and McClure (1977) and Carney (1980) claimed age spreads of several billion years between the most metal-poor and metal-rich clusters, but more recent metallicity estimates have led both groups to withdraw such claims (Janes and Demarque 1982; Carney 1981). VandenBerg (1982) likewise finds no evidence for a spread in globular cluster ages, although he does see a large difference between the oldest open cluster ages and those of the globulars. The most thorough recent study of relative globular cluster ages has been that of Sandage (Sandage, Katem, and Sandage 1981; Sandage 1981a,b,c). His work was based on a period-luminosity-amplitude relation for RR Lyrae variables in clusters, and allowed him to estimate helium abundance and age variations from cluster to cluster. He has suggested an anticorrelation between Y and Z, and reports no sign of an age spread. As discussed by Carney (1981), a test of his method lies in the strong helium sensitivity of the main sequence-horizontal branch gap, and that the observed M3 and M15 gaps are inconsistent with Sandage's claim that $Y_{M15} > Y_{M3}$. However, the

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faint star photometry involved is very difficult and the discrepancy may be purely observational. If so, some doubt is therefore cast on the photometric accuracy of the turn-off, although the error most likely arises from color rather than magnitude errors. A restudy of the lower main sequences of these two clusters appears warranted.

5. SOME PROBLEMS AND SUGGESTIONS

5.1. Metallicities

If we overestimate the metallicity of a cluster, we will underestimate its age. It is obvious that more work is needed on cluster iron peak and CNO abundances.

5.2. Distances

Cluster distance determinations could be improved by using the Baade-Wesselink method studies of cluster variables, as noted earlier. The main-sequence fitting method could be improved with more parallax data, particularly at the hotter end. Parallax studies of HD 19445, HD 94028, HD 108177, and HD 188510 are recommended, although since errors must be kept below 0.005 arcsecond, such work will be difficult from the ground.

5.3. Unincluded effects

At some point, we must give thought to the variables we have chosen to overlook, and whether we can observationally test their potential effects. The increasing sophistication of model isochrones to allow for variable Y, Z_{Fe}, Z_{CNO}, and α has diminished in at least some cases the uncertainties in our age estimates, but the models could still be improved, and we should still consider such effects as diffusion, magnetic fields, and rotation.

Current stellar evolutionary models predict a larger neutrino flux for the Sun than is observed, which suggests that if the neutrino itself is not the cause of the discrepancy that the models are too hot. Ages derived from such models will therefore be underestimates. The opacities of stellar matter have been considerably improved over the years, but there remains the possibility of some unincluded opacity source. Again, if some significant opacity source has been overlooked, the models will produce underestimates of cluster ages.

Noerdlinger and Arigo (1980) have claimed that core helium diffusion may cause ages to be overestimated by up to 20%. Observers would welcome a prediction of some observational consequence of such a mechanism to test its validity, and we would also desire some confirming calculations.

Among the field subdwarfs, two searches have been made for magnetic fields. Babcock (1958a,b) measured a field of 400 gauss in HD 19445, but it was near the limit of his technique. If such a field strength could be confirmed, however, the star's parallax would prove even more important. On the other hand, with much more sensitive apparatus, Brown and Landstreet (1981) failed to detect any field in HD 103095. For now, it appears surface magnetic fields are not present.

Field subdwarfs have also been studied for signs of rotation, and Peterson et al. (1980) and Carney and Peterson (1981) found no signs of rotation above the 5 km sec^{-1} level in a large sample. However, Peterson, Tarbell, and Carney (1982) have detected rotational broadening at about the 20 km sec^{-1} level in some field horizontal branch stars, and Peterson (1982) has also found several M13 horizontal branch stars are rotating at velocities similar to the field stars. Since such stars have evolved from subdwarfs, shed 0.1 to 0.2 solar masses during the red giant stage, and expanded to 3 to 4 solar radii, these observations suggest subdwarfs harbor significant amounts of subphotospheric angular momentum. The preliminary calculations of Law (1981) indicate derived ages may thus be underestimated by as much as 30%. Rotation would also likely impede or destroy diffusion. Clearly more observational and theoretical work is required.

5.4. Color-magnitude diagrams

It should be clear that large numbers of stars should be studied in each cluster, ranging from at least the lower giant branch to the lower main sequence. Ideally, independent measures of all these stars should be obtained in order to reliably estimate the observational errors. Unbiased selection of program stars and data reduction methods that allow for the effects of crowding would also be useful, particularly in the determination of luminosity functions. These might aid identification of gaps which could be related to, for example, isochrone predictions of core hydrogen exhaustion. Finally, because metallicity is such an important variable, and because it affects both the isochrones and parts of the emergent spectrum via line blanketing, we can alleviate some of our problems by working with a photometric system less sensitive to metallicity. The Cousins VR system (see Bessell 1979 for a good description) is very suitable.

6. SUMMARY

The ages of the best-studied globular clusters are in the vicinity of 16 billion years, using current data and isochrones. Ages as small as 10-12 billion years appear ruled out unless some unstudied mechanism (such as diffusion) is accelerating stellar evolution or our abundance scale is in error by over 0.5 dex. In fact, it appears possible our derived ages are underestimates due to the neglect of rotation in the stellar models.

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