THEORIES FOR THE WINDS FROM WOLF RAYET STARS

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ABSTRACT

The massive and fast winds of Wolf Rayet stars present a serious momentum problem for the line-driven wind theories that are commonly used to explain the fast winds of early type stars. It is perhaps possible for the winds to be driven by lines, if multiple scattering occurs and if there are a sufficient number of lines in the spectrum so that large fraction of the continuum is blocked by line opacity in the winds. Several other mechanisms are discussed, in particular two that rely on strong magnetic fields: a) Alfven wave driven wind models like those recently developed by Hartmann and MacGregor for late type supergiants and b) the "Fast Magnetic Rotator" model that was developed by Belcher and MacGregor for the winds from pre-main sequence stars. In either model, large magnetic fields ($\approx 10^4$ gauss) are required to drive the massive and fast winds of Wolf Rayet stars. Smaller fields might be possible if the multiple scattering line radiation force can be relied on to provide a final acceleration to terminal velocity.

INTRODUCTION

The major characteristics of the mass loss from Wolf Rayet stars that I want to focus attention on are that 1) the winds are very "massive", i.e. the mass loss rates are very large $\sim 3\times10^{-5}\ M_\odot\ yr^{-1}$. 2) the terminal speeds of the winds are large, about 2000 km s^{-1} or more. This combination of massive and fast winds leads to the "momentum problem" of Wolf Rayet winds. The rate at which momentum leaves the star in the form of radiation is L/c, and if we assume that all of this radiation is intercepted in the wind, the momentum transferred to the mass outflow is $\dot{M}v_\infty$. Equating these momenta we get the well known "maximum mass loss rate for single scattering"

$$\dot{M}_{max} = L/v_\infty c. \tag{1}$$

C. W. H. de Loore and A. J. Willis (eds.), Wolf-Rayet Stars: Observations, Physics, Evolution, 173–183.

The winds of Wolf Rayet stars have a mass loss rate that exceeds this value by about a factor of ten! Let us consider the various ways that massive and fast winds might be driven.

1) LINE DRIVEN WINDS: THE EFFECTS OF MULTIPLE SCATTERING

The "upper limit" given in equation (1) is not a strict limit for radiation driven winds because it assumes that each photon is absorbed or scattered just once in the wind. The scattering of a photon by a resonance line does not destroy the radiation, but simply changes the direction of the photon and, as seen by an observer, shifts it slightly to the red. A photon emitted from the photosphere at λ, slightly shortward of a resonance line at λ_o will be intercepted in a "resonant shell" where the velocity satisfies the expression $(\lambda_o-\lambda)/\lambda_o = v/c$. The photon will be scattered many times within the resonant shell, but the net momentum transferred to the wind depends on the direction at which it leaves the shell. Assuming $v(r)$ increases monotonically outward, then once a photon leaves the shell it is never again resonant with the same line. However, it can scatter again, say by a different ion, in a line that lies at a wavelength somewhat longer than λ_o. Panagia and Macchetto (1981) have considered the efficiency of this "multiple scattering" process in some detail. The optical depth of the resonant shells must be large enough so that back scattering is more probable than forward scattering. The lines must be spaced closely enough so that several lines "overlap" within a velocity displacement, v_∞, in order that a backward scattered photon can be intercepted by another shell at the other side of the star. The wind broadened lines must intercept or "block" a large fraction of the flux from the star. If these conditions hold, then each photon emitted from the star can be back scattered many times and the momentum transferred to the wind can be multiplied accordingly. So perhaps n = 10 effective radial scatterings per photon could resolve the "momentum problem". However, Abbott (1980) has shown that there is an <u>additional</u> sink of radiative momentum that must be accounted for. In a radiatively driven wind the photons must also support the atomsphere against gravity. Integrating the momentum equation over the mass of the wind ($dm = 4\pi r^2 \rho dr$) from the photosphere outward, he finds

$$\dot{M}v_\infty = \frac{L}{c}\left[nb - \frac{(1-\Gamma)}{\Gamma}\tau\right] \tag{2}$$

where b is the fraction of frequency space blocked by lines, Γ is the ratio of continuum radiation acceleration to gravity ($\Gamma = \sigma_{es}L/4\pi cGM$) and τ is the electron scattering optical depth of the wind. Abbott (1980) and Panagia and Macchetto (1981) find that, when the support of the envelope (i.e. the second term in the bracket of equation (2)) is included, the maximum mass loss rate for single scattering is exceeded even for O4 stars, which have less massive winds than Wolf Rayet stars. Panagia and Macchetto (1981) estimate that 35 percent of the radiation from Wolf Rayet stars is blocked by lines (i.e. b = 0.35) and find that even with as many as n = 10 effective scatterings a wind with a mass loss only as large as $L/v_\infty c$ could be driven. So there is still an order of magnitude deficiency in the momentum transferred to the wind, which we will quantify by using the fraction $f = \dot{M}v_\infty/(L/c)$.

John Castor and David Friend are currently investigating what is needed to achieve the value of f = 10 for Wolf Rayet Stars. They assume that lines are distributed randomly with frequency and have found that if over 35 percent of the spectrum there are an average of 3 strong overlapping lines, they recover the Panagia and Macchetto result that f = 1. By increasing the fraction of the spectrum blocked by lines to 100 percent, they have found it possible to get the needed value of f = 10. It therefore appears that it is marginally possible for the massive and fast winds of Wolf Rayet stars to be driven by radiation forces on lines. To be fully convincing it will be important to carry out the calculations using actual line lists, such as those compiled by Abbott (1977), instead of a statistical representation of line strengths and locations.

One last comment should be made about the radiation driven models. It has been known for some time that many Wolf Rayet stars have rather "flat" continuous energy distributions (Van Blerkom 1973). This is often interpreted to mean that the stars have geometically extended continuum formation regions (Cassinelli 1971, van der Hucht et al. 1979). Even with multiple scattering in lines, it is not likely that the continuum formation regions deep in the atmosphere would be affected. Some other processes appears to be operating to extend the deeper layers of the atmospheres.

Even if line opacity cannot "establish" or "initiate" the mass loss, it might be responsible for acceleration in the outer parts of the wind. Multiple scattering would still be important because even with an initial acceleration to v_o, the extra momentum needed to drive the flow from v_o to v_∞ is $\dot{M}(v_\infty - v_o)$ and this still exceeds the single scattering mass loss rate limit of L/c as long as $v_0 < 0.9\ v_\infty$.

2) MAGNETICALLY DRIVEN WINDS

As we look at wind phenomena across the HR diagram, we see that magnetic fields are often associated with massive and/or fast winds.

a) In the case of the sun, the high speed streams ($v \approx 700$ km s^{-1}) that emerge from coronal hole regions have been attributed to momentum deposition by Alfvén waves (Belcher 1971, Jacques 1978).

b) The massive (10^{-6}-10^{-5} $M_\odot$ yr^{-1}) but slow (10-100 km s^{-1}) winds of red giants and supergiants have also been explained as being Alfvén wave driven (Hartmann and MacGregor 1980 and Leer and Holzer 1980).

c) Rapidly rotating pre-main sequence stars also exhibit large mass loss rates (de Campli 1981). These can be explained by the "Fast Magnetic Rotator" model of Belcher and MacGregor (1976), in which rotational energy is transferred to the outflowing matter by way of centrifugal and magnetic forces. Hartmann and MacGregor (1982) have shown that extremely massive winds ($10^{-4} M_{\odot}\ yr^{-1}$) with speeds of 10-100 km s^{-1} can be driven from protostars having plausible magnetic field strengths (10 gauss).

In this section some of the results of Hartmann and Cassinelli (1982) are given from their application of the magnetically driven wind theories to Wolf Rayet stars. The equations for the Alfvén driven winds are discussed in Hartmann and MacGregor (1980) and the Fast Magnetic Rotator equations are given by Hartmann and MacGregor (1982).

A) Alfvén Wave Driven Winds

There are several strong analogies with radiation driven wind theory, which is very familiar to most of us here. These similarities can be used to quickly estimate the mass loss rates that can be driven by the waves. Let us consider the case of an extremely massive wind that has the transition from a near hydrostatic structure to supersonic flow occuring very close to the photosphere of the star. Recall that in the radiatively driven case the hydrostatic equation is $dP/dr = -\rho g + \sigma\rho F/c$, and that the ratio of the outward acceleration by the radiative pressure gradient to gravity is near unity:

$$\Gamma = \sigma F/cg = \sigma L/4\pi cGM \lesssim 1 \tag{3}$$

In the case of a star with a massive wind driven by Alfvén waves, there is a similar expression for the hydrostatic equation $dP/dr = -\rho g + F_w/(\ell A)$, where A is the Alfvén wave speed $[B/(4\pi\rho)^{1/2}]$, ℓ is the damping length and F_w is the wave energy flux $[F_w = (\delta B^2/8\pi)A]$. In analogy with equation (3), we have

$$F_w/A\ell\rho g \lesssim 1 \tag{4}$$

The critical point of the flow is presumed to occur close to the star, at $r_c \approx R$, and the conditions at the critical point are given by the vanishing of the numerator and denominator of the velocity gradient equation;

$$\frac{d\ \ell n\ v}{d\ \ell n\ r} = \frac{2a^2 - \frac{GM}{r} + \frac{1}{2}\ \frac{\varepsilon}{\rho}\ (1 + \frac{r}{\ell})}{v^2 - a^2 - \varepsilon/4\rho} \tag{5}$$

where $\varepsilon = \delta B^2/8\pi$. Assuming the atmosphere to be cool so that the sound speed is negligible, we get at the critical point: $(\varepsilon/\rho)_c = gR = 4v_c^2$ and from equation (4) $\rho_c = 4\pi F_w^2/(\ell B_o g)^2$. Letting $\ell = R$, these give a scaling mass loss rate with respect to radius, mass, and magnetic field

$$\dot{M} = 4\pi\rho_c v_c R^2 = \frac{(4\pi)^2}{2G^{3/2}} \left(\frac{F_w}{B}\right)_o^2 \frac{R^{3.5}}{M^{1.5}}$$

This is quite similar to the result of our detailed calculations

$$\dot{M} = 1.6\times10^{-25} \left(\frac{F_w}{B}\right)_o^2 \frac{(R/R_\odot)^{3.5}}{(M/M_\odot)^{1.5}} \quad (6)$$

Now assuming δB is proportional to B in equation (6) gives $\dot{M} \propto B^2$.

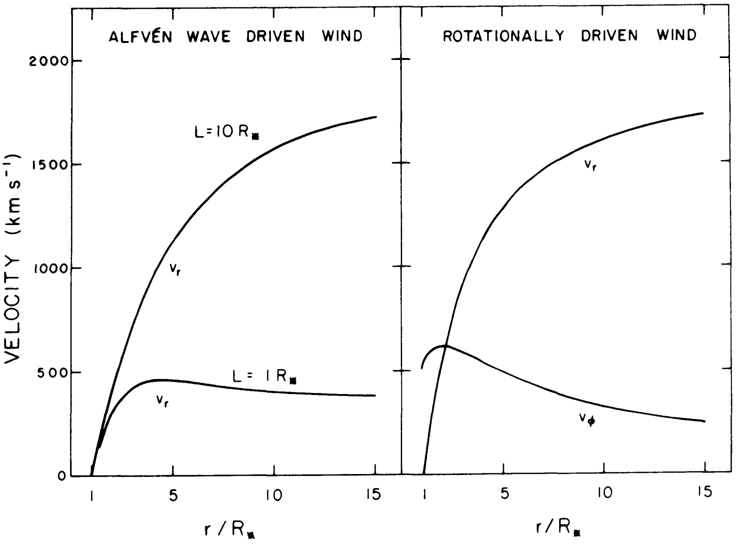

Figure 1 - shows the velocity distributions versus radius for the two magnetically driven wind models for a WN5 star. a) shows the radial velocity for Alfvén wave driven wind models with two different damping lengths, . In these models $B = 20,000$ gauss, $\dot{M} = 2\times10^{-5}$ $M_\odot$ yr^{-1}, $M(1-\Gamma) = 5$ $M_\odot$, $R = 3$ $R_\odot$ and the surface wave flux $F_w \sim 1.1\times10^{14}$ ergs cm^{-2} s^{-1}. b) shows both the radial velocity distribution v_r and the angular velocity distribution v_ϕ for a model of a rotationally driven wind from a Wolf Rayet star. The values of B and R are the same as in (a), $M = 10$ $M_\odot$ and $\alpha = 0.95$. (from Hartmann and Cassinelli 1982)

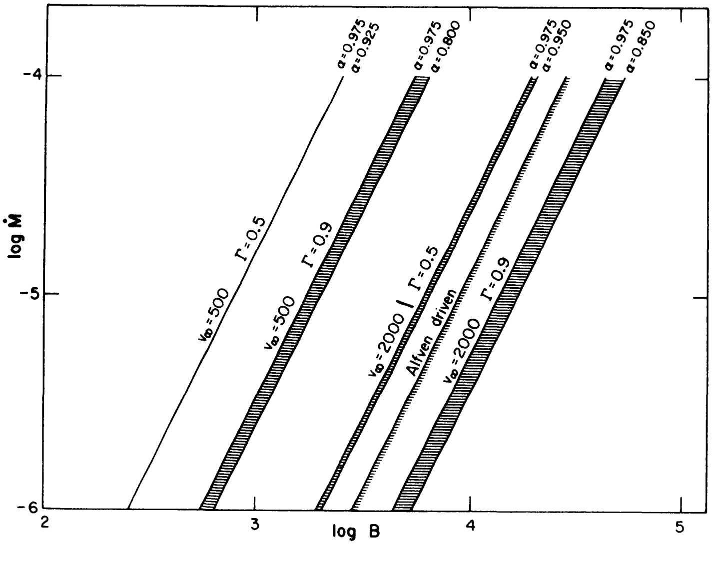

Figure 2 - shows the B_o^2 dependence of mass loss rate for both the Alfvén and Fast Magnetic Rotator models for Wolf Rayet stars. Several results for rotationally driven winds are shown which have the indicated value of the terminal velocity, v_∞, and the indicated range in values of α that satisfy the requirement of equation (9). (from Hartmann and Cassinelli 1982)

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Figure 1a shows the velocity distribution for 2 different values of the damping length ℓ. Figure 2 shows the dependence of mass loss on magnetic field ($\dot{M} \alpha B^2$). A field of about 20000 gauss is capable of driving a massive and fast Wolf Rayet wind. The surface wave flux must be substantial ($1/3\ \sigma\ T_{eff}^{4}$).

Hartmann and MacGregor (1980) explain why wave modes can be more efficient than radiation pressure in driving extremely large mass loss rates. For a given wave luminosity L_w, the wave momentum flux varies inversely as the mode speed, and this is typically a factor of 10^4 smaller than the speed of light. That is,

$$\dot{M}v_\infty = L_w/A = 0.1\ L/10^{-4}c >> L/c.$$

So the mass loss limit of equation (1) is clearly not valid here.

B) The Fast Magnetic Rotator Model

In this model the large field near the star causes corotation of the material along nearly radial magnetic flux tubes and the initial acceleration of the flow is centrifugal. Near the Alfvén point where the flux tubes develop significant curvature, there is further acceleration of the wind by $\vec{J} \times \vec{B}$ forces. There are two critical points in the solution corresponding to the speeds of the slow and fast magnetosonic waves. The flow has reached nearly terminal velocity at the outer critical point and the speed there is known as the "Michel velocity". This is given by

$$v_M^{\ 2} = \Omega^2 (r_o^{\ 2} B_o)^2 \ / \ \dot{M} \tag{7}$$

where Ω is the angular grequency of the stellar rotation which is conveniently expressed as

$$\Omega = \alpha\ \Omega_{breakup} = \alpha\ \frac{GM(1-\Gamma)}{R^3}^{1/2} \tag{8}$$

Nerney (1980) has recently used the Michel velocity expression (equation 7) to estimate upper limits to magnetic fields strengths in early type stars. He assumes v_M equals the terminal velocity of the wind, v_∞, and uses observational estimates of $\dot{M}$, v_∞ and Ω in equation (7) to solve for B_o.

He found, for example, that rather modest magnetic fields of $< 10^2$ gauss could explain the mass loss from Be stars. In addition to the Michel velocity expression used in Nerney's analysis there is another constraint that must be considered.

The Alfven speed near the star must be much larger than the sound speed, if the winds are to be driven by centrifugal and magnetic forces. That is, if gas motions are to be dominated by the field, then

$$B^2/4\pi\rho >> a^2 \tag{9}$$

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This puts a lower limit on the rotation rate parameter α (equation 8).

Figure 2 shows the results for the dependence of the mass loss rate on B for rotationally driven winds that attain the indicated terminal speed and for stars with the indicated values of Γ (i.e. $g_{eff} = (1-\Gamma)g$). A range of values of α from α = 0.80 to 0.975, were used in the calculations. The cross hatched region indicates results for the values of α that satisfied an equality of the quantities in equation (9). For example, the band at the right side of Figure 2 indicates that the star must have a rotation rate of greater than 85 percent that of equatorial breakup to drive the flow.

As was the case for the Alfvén wave driven winds, we see that the stars must have a magnetic field of $> 10^4$ gauss to drive massive and fast winds. Figure 2 also shows that much smaller fields are required if the mechanism is relied on to accelerate the winds to a slower speed of ~ 500 km s^{-1}. In this case, acceleration to the observed larger terminal velocities would require assistance by the multiple scattering line driven process discussed earlier. Figure 1b shows the run versus radius of the radial velocity v and the angular speed v_ϕ for a model with a mass loss rate of 2×10^{-5} $M_\odot$/yr.

It is plausible the Wolf Rayet stars have large magnetic fields. While they probably do not now have an outer convection zone and are not now producing a magnetic field by a dynamo action, they might have been doing so in their recent past history. This is because Wolf Rayet stars are thought to have recently been the fully convective cores of massive O stars as is evidenced by their large He abundance. Perhaps Wolf Rayet stars have strong magnetic fields which are remnants of those produced during the earlier convective phases.

Finally I would like to mention a mechanism for producing the X-rays that originate in the winds (Sanders et al., these proceedings). From the work of Lucy and White (1980) we know of at least one mechanism by which X-ray producing shocks could be formed in radiatively driven flows. If the winds are not radiatively, driven a different mechanism to produce a source of X-rays is required. A possible mechanism for shock formation in wave driven winds appears to be operating in the solar wind. Hundhausen (1973) discusses the formation of relatively steady state shock structures in a spiral pattern around the sun. This is produced by the interaction of alternating low and high speed solar wind streams. It is plausible that shocks can be formed in a similar way in magnetically accelerated flows of hot stars. The formation of steady shock structures because of Alfvén accelerated streams could possibly be the cause of the steady state high velocity absorption components often seen in UV resonance lines in O stars (Lamers and Snow 1981). Underhill (1981) has recently presented arguments that O stars have magnetic structures near the base of their winds.

3) OTHER MASS LOSS MECHANISMS

In their early papers on radiatively driven winds Lucy and Solomon (1970) and Castor, Abbott and Klein (1975) argued that fast winds could not be coronally driven because extremely high coronal temperature ($4x10^7$ K) would be required. Hearn (1975) suggested a hybrid structure might exist in which the flows from OB supergiants are initiated in a corona and accelerated further by line radiation forces. Waldron (1981) has recently constructed models of this type for Of and OB stars. I feel that the coronal model or even the hybrid coronal + radiation model will not work for the massive stellar winds of Wolf Rayet stars. It is clear that whatever is driving the winds is extremely efficient, and the coronal process is not very efficient. A large fraction of the input mechanical flux is radiated away from the coronal zone and only a small fraction is used to provide the thermal pressure gradient needed to accelerate the flow. A high density, very hot gas at the base of a Wolf Rayet wind should produce several observable effects: a) emission of a strong flux of X-rays at energies > 2keV, b) emission of coronal line radiation such as Ca XV and Fe XIV c) emission of an excess of free free radiation that will produce a bump in the infrared continuum. These effects should be looked for to provide useful constraints on such models.

Another mechanism that should be investigated for Wolf Rayet stars are pulsationally driven winds. Interesting work has recently been carried out by Willson and Hill (1979) to explain the winds of red supergiants.

SUMMARY

There are several mechanisms for driving mass loss from Wolf Rayet stars that require further investigation. More work is required on the line driven wind model to test the multiple scattering model using realistic atomic data on line strength and spectral distribution. Work has just begun on applying the magnetic and coronal wind models to Wolf Rayet stars. The pulsational mechanism should be given further consideration. There are several interesting questions that can be posed: 1) Why are the mass loss rates from Wolf Rayet stars throttled at the large value of $3x10^{-5}$ $M_\odot$/yr? 2) Can Wolf Rayet stars have large B fields? 3) Can at least some Wolf Rayet stars be fast rotators? Several observational studies should be pursued to test the various models. Rotationally distorted atmospheres are known to give rise to linear polarization which now can be searched for at both continuum and line frequencies. Satellite infrared and ground based observations at wavelengths 5-100μ should be able to provide information on $\rho(r)$ and $T(r)$ in Wolf Rayet atmospheres. Further studies of atmospheric absorption lines might be used to identify fast rotators. Coronal lines such as Ca XV, Fe XIV and further X-rays observations should be made to constrain coronal models.

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DISCUSSION

Pismis: It is gratifying to hear that you invoke magnetic fields in the problem of the mass loss from early-type stars, but what is the configuration of the magnetic fields one assumes in this case ?

Cassinelli: The theoretical models assume the B field is radial near the star. The actual field configuration might be quite complicated, maybe, even with closed magnetic loop regions as in the Sun. In the rotating model the field bends into an Archimedes spiral in the general region of the Alfven point.

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Macchetto: Is it an established fact that WR stars are stable against convection ? If they are not, convection could help in providing a mechanism for mass loss.

Cassinelli: Even if the WR stars are not convective I think it is quite plausible that they have strong magnetic fields. The stars are helium enriched, indicating that they were recently the fully convective core of an O star. Early-type stars are known to be relatively fast rotators. So the He-rich WR stars had in the recent past the two ingredients needed for a dynamo action: rotation and convection.

Massey: I guess I have a moral commitment to comment on the rotation question. There are a handful of WR stars with absorption lines which are probably intrinsic to the WR star itself. It is only from these that we could possibly measure v sin i. I've done a Fourier analysis of the very broad absorption lines in HD 193077 (Ap.J. 242, 638) and found that the broadening mechanism was clearly rotation and that v sin i was $\approx$400-500 km/s, the greatest known for an early-type star. Since then I've gotten a little suspicious of HD 193077 as my scanner data doesn't show as much H in the envelope as I originally thought from the data in the Smith and Kuhi Atlas. However, HD 193077 shows no radial velocity variations (believable ones that is), and the rotational broadening is unique, making it hard to accept that the absorption line star is a line of sight or distant companion (e.g. the Lamontagne et al. claim). As far as I know, the other ones do not have measured v sin i's. Those of us who have plates of these stars should measure them !

Conti: There is relatively unambiguous evidence for a rotation period of one WN star, HD 50896. This comes from periodic changes in the emission line profiles and from polarization variability. The period of 3.7 days gives a relatively slow rotation.

Underhill: A value of 10^4 Gauss for the magnetic field is rather high. If you considered a $\dot{M}$ that is ten times smaller, you could reduce B by a factor of 100. This would result in a more reasonable value for the magnetic field.

Cassinelli: I think Mike Barlow's arguements that the mass loss rates are high were quite convincing. The argument that the mechanism produces both the high mass loss rate and the high terminal speed is what makes it necessary to go to 20000 Gauss. Of course if one is willing to appeal to multiple scattering to provide the final acceleration to high speeds a somewhat smaller field ($\sim 10^3$ Gauss) is required.

Moffat: 1) Landstreet and collaborators have looked for polarization in ζ Pup and conclude from negative results that $B < 50$ Gauss.

2) The broad absorption lines in the WN5 star HD 193077 may be due to an unresolved OB star making this a binary system (Lamontagne, Moffat Konigsberger and Seggewiss, 1982, Ap.J., in press).

Cassinelli: The measurement that $B < 50$ Gauss for ζ Pup assumed a very simple dipole field geometry. If the field has a more complicated configuration their measurement placed a much cruder upper limit of $\lesssim 10^3$ Gauss.

Hummer: I'd like to make a comment and then ask a question. Recently I've considered the effects on a stellar flux distribution caused by ultraviolet photons scattered by a stellar wind back into the stellar photosphere. My calculation, which so far is merely schematic, shows that one consequence of multiple scattering in winds is that the colour temperature of the visible part of the spectrum will be higher. This may make possible some observational determination of the degree of multiple scattering. With Abbott and Castor, I am now doing more quantitative calculations of this effect. My question is, what is the largest ratio of mechanical to radiative luminosity in any type of star ? For the Sun it is very small.

Cassinelli: Yes, for the Sun it is negligible but for T Tauri stars the ratio is about 1/3.

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