SELF-CONSISTENT MODELS OF PERFECT TRIAXIAL GALAXIES

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We have used Schwarzschild's (1979) method to study the variety of self-consistent solutions available to the family of triaxial "perfect ellipsoids" (de Zeeuw 1985). The time-averaged density at 240 points within the mass model is computed for each of 1065 orbits distributed regularly in phase space and covering all four major orbit families. The underdetermined linear system thereby defined is solved in two ways. First, Lucy's (1974) iterative scheme is used to find a "smooth" solution, lying in the interior of the (mathematically-allowed) solution space. Second, linear programming is used, with linear combinations of the x and z components of the total angular momentum as cost functions, to delineate the boundary of the projection of the solution space in the $L_x - L_z$ plane. The above procedure is applied to 21 figures, with axis ratios, b/a and c/a, chosen at equal intervals of 1/8.

Numerical solutions are found for all axis ratios investigated. In general, the solution spaces are approximately rectangular in the $L_x - L_z$ plane, with $L_{x,min}$ and $L_{z,min}$ nearly zero. The square corners of the solution spaces arise because the short-axis and long-axis tube orbits can each be exchanged for box orbits and the bounding marginal orbits without affecting the other. The simple cases of the sphere, the axisymmetric disk, and the needle can be treated analytically. The solution spaces of the triaxial figures tend to the appropriate limits and the boundaries move monotonically with the axis ratios between the limiting cases.

In the oblate and prolate limits, the projected solution spaces must collapse to zero extent in the L_x and L_z directions, respectively. Our most nearly oblate models show appropriately thin solution spaces, but those closest to the prolate limit do not. The implication is that galaxies that are, in shape, nearly prolate can still support a significant amount of internal streaming around the short axis.

The distribution functions in action space for representative solutions show the following characteristic features:

Minimum-L_x, minimum-L_z solutions have a fairly smooth distribution in the box orbits, which, at low binding energies, cuts off abruptly at the margins dividing the boxes from the tube orbits. At higher binding energies, some tube orbits, particularly the short axis tubes close to the margin are populated at high phase-space densities. Other tube orbits are completely unpopulated.

Minimum-L_x, maximum-L_z solutions have a similar distribution through the box orbits, going to zero more smoothly at the margin between the boxes and the short-axis tubes, and also a disconnected thin region of much higher phase

T. de Zeeuw (ed.), Structure and Dynamics of Elliptical Galaxies, 487–488.

density in the thin-walled short-axis tubes.

Maximum-L_x, minimum-L_z solutions have, again, the same sort of behavior in the box orbits, with the suggestion of a smoother transition to zero at the margin between the boxes and the inner long-axis tubes, and high density regions in the thin-walled outer and inner long-axis tubes.

Solutions by Lucy's method are reasonably smooth over all of phase space, with, at any surface of constant energy, the highest density toward the middle of the box orbits.

Regarding physical plausibility, the Lucy's method solutions tend to have no one orbit or orbit family preferred, and so would not require any special mechanisms. More cannot be said about the plausibility of these solutions without a detailed theory of formation. The minimum-L_x, L_z solutions would require some formation mechanism that would preferentially populate box orbits. This task would seem difficult to accomplish if the bulk of the mass of the galaxy is subject to violent relaxation. The maximum-L solutions require a mechanism to populate the thin-walled tube orbits. Gaseous dissipation will not suffice, since it would populate the elliptic closed orbits, not the orbits with finite extent out of their "equatorial" planes required in the solutions. In principle, these orbits could be populated by the ingestion of a smaller galaxy which is slowed by dynamical friction and tidally stripped as it spirals inward. However, it is not clear whether dynamical friction would damp the "radial" excursions of the victim's orbit faster than the "vertical" ones in a triaxial potential. Furthermore, there are suggestions (Casertano *et al.*, this volume) that a strongly radial velocity ellipsoid in the cannibal galaxy will cause the radial excursions to *increase* rather than decrease.

"Observed" values of v/σ are calculated, under the assumption of maximal streaming around the short axis, for lines of sight along the long and intermediate axes, as upper limits to values observed from other directions. We find that the smooth Lucy's method solutions can account for the positions of most ellipticals in the v/σ *vs.* ϵ diagram, though in some cases maximal streaming and the most favorable viewing geometry are required. A few galaxies, such as NGC 4742, require maximum-L_z models and fortuitous lines of sight, and may be candidates for true oblateness.

Finally, we note that our solutions in which tube orbits do not contribute to the global *pressure* (either because they are unpopulated or because the circulation is all in the same sense and the motion very ordered) show contours of projected velocity dispersion that tend to be elongated contrary to the elongation of the mass distribution. The effect can also be seen in Schwarzschild's Model B (Merritt 1980) and Wilkinson and James' (1982) N-body Model B, both of which are dominated by box orbits. Though it is only a 20% effect at best, it would nevertheless be interesting to look at the dispersions on both axes of some very flattened, slowly-rotating galaxies, such as NGC 4839 and NGC 6909, to determine if the counter-elongation can be useful as an indicator of the shape of the velocity distribution.

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