TRANSACTIONS OF
THE FACULTY OF ACTUARIES

VOL. XI.—SUPPLEMENT NO. 4.

EXAMINATION PAPERS

SET AT THE

FACULTY'S EXAMINATIONS

HELD IN

OCTOBER 1926 AND APRIL 1927

HALL: 14 QUEEN STREET, EDINBURGH

THE FACULTY OF ACTUARIES

EXAMINATION PAPERS, 4 October 1926

(The time allowed was two hours.)

Board of Examiners

A. E. KING, *Chairman.*

C. M. DOUGLAS, *Hon. Secretary.*

J. C. BOYD. C. KEITH GRANGER.
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D. GIBB, M.A., B.Sc. G. C. HUTTON, M.A.
C. S. PENN.

PART I—Section A

FIRST PAPER

1. Find the value of :—

$$8 \cdot 136\left[(0 \cdot 2847)^{(3 \cdot 1)^{\frac{1}{3}}} - (1 \cdot 034)^{-\frac{1}{3}}\right].$$

2. If 12 men could do a certain piece of work in 9 days of 10 hours each, how many men should be set to do the same work in 15 days of 8 hours each if 3 men join them at the beginning of the fourth day and 4 more at the beginning of the tenth day?

3. If A, G, H be the arithmetic, geometric and harmonic means between a and b, show that $AH = G^2$ and arrange them in descending order of magnitude assuming a and b to be positive.

If between any two quantities there be inserted n arithmetic means $A_1, A_2 \ldots A_n$, m geometric means $G_1, G_2 \ldots G_m$, and p harmonic means $H_1, H_2 \ldots H_p$, show that

$$G_1 G_m = (A_1 + A_n) \times \frac{H_1 H_p}{H_1 + H_p}.$$

4. Find the condition that the roots of the equation $ax^2 + bx + c = 0$ should be (1) equal in magnitude and opposite in sign and (2) reciprocals.

Solve the equation

$$x^4 - 6x^2 + 1 = 0.$$

5. If there be 7 objects of which 3 are all alike of one kind, 2 are alike of another kind, the remaining 2 being different from each other and the rest, find

    (*a*) the total number of arrangements which may be made of the 7 objects ;

    (*b*) the total number of selections which can be made from the 7 objects ;

and (*c*) the total number of ways in which three objects may be selected and permuted.

<div align="center">SECOND PAPER</div>

6. Find the coefficient of x^3 in the expansion of the product of

$$\frac{x^3+1}{(x+2)(x-1)^2} \text{ and } (x^3+x+1).$$

7. What is a logarithm? Find $\log_{0\cdot001} 0\cdot0001$ and $\log_{9\sqrt{3}} 0\cdot\dot{1}$ and calculate $\log_2 0\cdot0001$ given $\log_{10}2 = 0\cdot30103$.

8. Determine the general form of the graph of $x(y-x-1)=1$: in particular obtain the co-ordinates of its maxima and minima, if any, and show that the function $x(y-x-1)-1$ does not vanish for any real values of x and y.

9. Prove that the square root of the cosine of an angle is a mean proportional between the sum and difference of the cosine and sine of half the angle.

Show that

    (i) $\tan 4A = \dfrac{4\tan A - 4\tan^3 A}{1-6\tan^2 A + \tan^4 A}$;

    (ii) $\text{arc-cot } 2 + \text{arc-cosec } \sqrt{10} = \dfrac{\pi}{4}$.

10. Find all the angles between 0° and 500° which satisfy the equation

$$\sin^2\theta = \tfrac{3}{4}.$$

Trace the changes in sign and magnitude of $\sin\theta - \cos\theta$ as θ varies from zero to 2π: point out the values of θ for which the function is a maximum, a minimum, and zero.

EXAMINATION PAPERS, 26 TO 29 APRIL 1927

(*The time allowed for each paper was three hours,
except Part I. Section A—two hours each.*)

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PART I—Section A

FIRST PAPER

1. Express $\dfrac{(2x-5)}{(x-1)(x^2+1)^2}$ as the sum of partial fractions.

2. Two trains going in opposite directions take 4 seconds to pass entirely clear of each other, and 32 seconds when going in the same direction. If the speed of the faster of the two trains is 45 m p.h., what is the speed of the other?

3. (*a*) If the quantities x, y, and z be in H.P., and x, y, and w in A.P , show that

$$\frac{w}{z}=1-2\frac{(x-y)^2}{xy};$$

(*b*) If w be an imaginary nth root of $+1$, show that
$$w+w^2+ \ldots +w^n=0.$$

4. If l and m are the roots of the equation
$$ax^2+bx+c=0$$
find the equation whose roots are $\dfrac{l}{m}$ and $\dfrac{m}{l}$.

5.* Evaluate the following, using where necessary the table of logarithms supplied:—

$$\left[\cdot 7216^5+(\cdot 3612\div\cdot 4160)^{\frac{1}{5}}-\cdot 06291^{7^{\frac{1}{2}}}\right]^{-\frac{1}{3}\cdot}$$

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

6. Discuss the graph of

$$y = \frac{x^2 - 2x + 1}{x}$$

and determine its general form.

7. In how many different ways could 5 diamonds, 6 opals and 7 emeralds be arranged for a bracelet, the stones of each kind being regarded as the same?

8. (*a*) Write down the 10th term of the expansion of $\left(x^2 + \frac{2}{x}\right)^{12}$;

(*b*) Show that the equation

$$x^4 - 6x^3 + 11x^2 - x - 4 = 0$$

has at least two real roots.

9. Prove that :—
   (*a*) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$;

   (*b*) $\arctan\theta_1 + \arctan\theta_2 = \arctan\dfrac{\theta_1 + \theta_2}{1 - \theta_1\theta_2}$.

10. Show that $2\cos 11°\ 15' = \sqrt{2 + \sqrt{(2 + \sqrt{2})}}$.

PART I—Section B

1. Forty candidates enter for an examination. Each candidate must answer one, and only one, out of three questions in a certain section of the examination paper. Find the probability that at least five candidates select the same question.

2. Two persons toss a sovereign alternately on condition that the first who gets 'head' wins the sovereign. Find their expectations.

3. Two urns are exactly similar. One contains 2 white and 3 black balls, the other 1 white and 4 black balls. A ball is taken from one of the urns and proves to be white. What is the probability that it was taken from the second urn?

4. Obtain, from first principles, the first derivative of $\tan x$. Differentiate

   (i) $\dfrac{2 + x}{\sqrt{(1 + x)}}$:   (ii) $\log \dfrac{x\sin x}{\sqrt{(1 + x^2)}}$:

   (iii) e^{e^x} :   (iv) $\arctan(\sin x)$.

5. If $\dfrac{x}{e^x-1}=1-\tfrac{1}{2}x+\dfrac{B_1}{2\,!}\,x^2-\dfrac{B_2}{4\,!}\,x^4+\dfrac{B_3}{6\,!}\,x^6-\ \ldots$

show that $B_1=\dfrac{1}{6},\ \ B_2=\dfrac{1}{30},\ \ B_3=\dfrac{1}{42}.$

6. A Norman window consists of a rectangle surmounted by a semi-circle. Given the perimeter, find the height and breadth of the window when the quantity of light admitted is a maximum.

7. (i) Taking the constant increment of x as unity, define the operators Δ and E.
If $u(x)=a.3^x+b.2^x$, show that $u(x+2)-5u(x+1)+6u(x)=0$.

(ii) Form a difference table for the following observations, the order given being retained :—

x	2	6	4	8	7
$f(x)$	5	205	57	497	330.

Hence find the analytical expression for $f(x)$.

8. From the following table of values of $f(x)$ determine the value of $f(4\cdot5)$, and find x when $f(x)$ has the value $0\cdot223106$:—

x	$f(x)$
1	$0\cdot208460$
2	$0\cdot237702$
3	$0\cdot266731$
4	$0\cdot295520$
5	$0\cdot324043.$

SECOND PAPER

9. Show that the series $1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\ \ldots$ is divergent.
Discuss the convergency of the following series :—

(i) $1^2+2^2x+3^2x^2+4^2x^3+\ \ldots$

(ii) $e^{-1^2x}+e^{-2^2x}+e^{-3^2x}+\ \ldots$

10. (i) Sum, to n terms, the series :—

(a) $\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+\ \ldots$

(b) $1.3.5+2.4.6+3.5.7+\ \ldots$

(ii) Expand n^3 in the form $an+bn(n-1)+cn(n-1)(n-2)$, and hence show that $\displaystyle\sum_{n=1}^{\infty}\dfrac{n^3}{n!}=5e$.

11. Prove that $(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)(\cos\theta_3+i\sin\theta_3)=\cos(\theta_1+\theta_2+\theta_3)+i\sin(\theta_1+\theta_2+\theta_3)$.

If $x = \cos\theta + i\sin\theta$ obtain the values of $x + \dfrac{1}{x}$ and $x^n + \dfrac{1}{x^n}$.
Hence show that $\cos^4\theta = \tfrac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$.

12. Evaluate

(i) $\displaystyle\int \frac{x^2+1}{(x-1)^2(x^3+1)}\,dx$: (ii) $\displaystyle\int \cos^4 x\, dx$: (iii) $\displaystyle\int_0^a \sqrt{(a^2-x^2)}\,dx$.

13. If D denote differentiation with respect to x show that $D^n e^{ax} u = e^{ax}(a+D)^n u$, where u is a function of x.

Assuming this result to hold when $n = -1$, and that $D^{-1}f(x)$ denotes a function whose derivative is $f(x)$, find the value of $\displaystyle\int e^{-x} x^3 dx$.

14. Find the area included between the parabola $y^2 = ax$ and the circle $y^2 = 2ax - x^2$.

15. If $\ldots\ u_{-2},\ u_{-1},\ u_0,\ u_1,\ u_2,\ \ldots$ be the values of a quantity for individual years, and if $\ldots\ W_{-1},\ W_0,\ W_1,\ \ldots$ be the corresponding quinquennial sums so that

$$
\begin{aligned}
W_{-1} &= u_{-7} + u_{-6} + u_{-5} + u_{-4} + u_{-3}\\
W_0 &= u_{-2} + u_{-1} + u_0 + u_1 + u_2\\
W_1 &= u_3 + u_4 + u_5 + u_6 + u_7
\end{aligned}
$$

use the Newton-Stirling formula

$$u_n = u_0 + n\frac{\Delta u_{-1}+\Delta u_0}{2} + \frac{n^2}{2}\Delta^2 u_{-1} + \frac{n(n^2-1)}{6}\frac{\Delta^3 u_{-2}+\Delta^3 u_{-1}}{2} + \frac{n^2(n^2-1)}{24}\Delta^4 u_{-2} + \ .$$

to show that $u_0 = 0.2 W_0 - 0.008 \Delta^2 W_{-1}$.

Find u_0 when W_{-1}, W_0 and W_1 are 37057, 39888 and 42422 respectively.

16. Obtain Simpson's formula of numerical integration by evaluating $\displaystyle\int_{-1}^1 u_x dx$, where u_x is assumed to be a function of x of the third degree.

Hence find the area of a quadrant of a circle of radius $1''\cdot 2$, ordinates being taken at every fifth of an inch.

PART II

FIRST PAPER

1. In published tables the values of $a_{\overline{n}|}$ are given at all rates of interest from 1 to 8%—the rates increasing by 2s. 6d.%. State clearly the chief uses of the fractional rates of interest.

Find the value of an annuity due of 1 per annum for 20 years payable quarterly at nominal rate of interest $\cdot 04$: given $\log 1\cdot 01 = \cdot 00432$ and $\log \cdot 45563 = \bar{1}\cdot 65872$.

2. Derive a formula for the value of a temporary annuity of 1 per annum payable yearly for t years, the first payment to be made $(n+1)$ years hence—the investment to yield interest at rate i during the whole period and a sinking fund to be set up which will, allowing interest at rate j, repay the capital at the end of the term.

3. n years ago a loan of X was made to be repaid by equal annual instalments over t years with annual interest on the outstanding loan at rate i. Calculate from first principles the sum now payable to the lender who is to receive the commuted value of all future payments: interest to be taken at rate j. The nth payment under the original agreement has just been paid.

4. If X be the price per unit of a debenture redeemable at par in n years and bearing interest at rate g per annum payable half-yearly, show that the nominal annual rate j convertible half-yearly yielded by the security is approximately

$$\frac{1-\mathrm{X}+ng}{n+\dfrac{n(2n+1)}{4}g}.$$

Show how by means of this formula you would construct tables showing the approximate yield per cent. of Debentures redeemable at par in any given number of years carrying various rates of interest for prices either above or below par.

5. Prove that the net annual premium for a capital redemption policy increases with a decrease in the rate of interest used in its calculation, and that if the reserve be the accumulation of all such premiums at the rate of interest used in their calculation then that reserve will also increase with a decrease in the rate of interest.

6. A loan of £300,000 is to be discharged by the payment of an annuity of £20,000 payable yearly, the nominal rate of interest to be taken as 5% convertible half-yearly. When will the debt be extinguished?

$$\text{Given } \log 1\cdot0506 = \cdot02144$$
$$\log \cdot24100 = \bar{1}\cdot38202.$$

7.* A bond for £1000 repayable in twenty equal annual instalments (the first being due one year hence) and bearing interest at 3% is to be sold. What price can a purchaser offer if he wishes to receive 5% on his investment?

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

A 2

8.* A debenture issue of £100,000 bearing interest at 6% payable half-yearly is to be redeemed at 120% by equal half-yearly payments including principal and interest, within ten years. Find the amount of the half-yearly payment and the amount of the loan outstanding after 2 years' payments have been made.

<div align="center">SECOND PAPER</div>

9.* A whole life assurance of £1000 deferred 10 years is to be issued on the life of a child aged 11 subject to a level annual premium payable throughout life. A guaranteed uniform reversionary bonus of £2% will be added in respect of each premium paid from and including that due at age 21, and should the child die before age 21 a return will be made of the net premiums paid, with compound interest thereon at the rate of $2\frac{1}{2}$% per annum. Assuming a constant rate of mortality of ·005 for all ages under 21 and H^M mortality thereafter, find the net annual premium on a 3% basis.

10. Define $P_{x\overline{n}|}^{(m)}$ and $_tP_{x\overline{n}|}^{(m)}$. Deduce expressions for the values of these functions, and give a verbal interpretation of your results.

11. Explain in detail how you would evaluate $\bar{a}^4_{\underset{3\,2\,1}{uvwx}|y}$ by Simpson's formula of approximate integration, showing clearly the form of your working sheet.

12 * From the following service table find in respect of a life aged 25
   - (a) the rate of mortality;
   - (b) the central rate of withdrawal; and
   - (c) the probability that he will be pensioned before age 28.

Age.	On Active List.	Withdrawals.	Deaths.	Pensioned.
x	l_x^s	w_x	d_x^s	r_x
25	10000	550	44	3
26	9403	435	41	4
27	8923	360	38	5
⋮	⋮	⋮	⋮	⋮

13.* Given $_{|n}Q_{x\cdot x+n}=\cdot9$ and $_np_x=\cdot5$, find the probability that of three lives all aged x
   - (a) at least two will die between ages $x+n$ and $x+2n$, and
   - (b) the third death will take place between ages $x+n$ and $x+2n$.

* *A Short Collection of Actuarial Tables* is supplied for use in answering these questions.

14.* Find approximately the value of $\bar{A}_{\underset{30.40}{1}}$ by the H^M table at 3%, and give a rationale of the method you adopt.

15. Demonstrate whether the effect of a constant increase in the force of mortality under a table following Makeham's law is to increase or decrease policy values.

16.* An insurance company grants a policy to a select life aged 30 securing the following benefits:—

    (*a*) In the event of death before the attainment of age 60, an annuity certain payable yearly of £100 per annum for 10 years or until the end of 30 years from the commencement, whichever is the shorter period, the first payment to be made at the end of the year of death.

    (*b*) On the attainment of age 60, £1000, with the option to apply this amount, without medical examination, as a single premium to secure a whole life non-profit assurance at ordinary rates.

Find on an $O^{[NM]}$ 3% basis the net annual premium for the assurance.

<div align="center">THIRD PAPER</div>

17.* Explain the benefit denoted by $_{|30}\hat{\ddot{a}}^{(4)}_{30|30}$ and find its value on an H^M 3% basis.

18. Show that approximately

$$_{|n}\bar{A}_{xy}=\mu_{xy}\bar{a}_{x\;\;y:\overline{n}|}+\tfrac{1}{2}\big(\bar{a}_{x-1:y-1\;\;\overline{n}|}-\bar{a}_{x+1\;\;y+1:\overline{n}|}\big).$$

19. A 'service' table is subject to decrement from three causes: death, withdrawal, and retirement. Given the central death rate and the central rates of withdrawal and retirement, show how to construct the table.

20.* Calculate on an $O^{[NM]}$ basis the probability that out of four lives each aged 27 two at least will be alive at the end of four years.

21.* A Simple Reversionary Bonus office makes a bonus distribution as at 31st December 1926 and is to pay an interim bonus of 37s. 6d.% for each annual premium paid during the succeeding quinquennium. Assuming it wishes to set up a reserve for this interim bonus at the end of 1926, how much should it hold in the

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case of an endowment assurance of £1000 to which £156 of bonus now attaches effected by annual premiums on 30th December 1915, the assured being 29 at entry and the policy maturing at age 59? Basis H^M 3%.

22.* Find on an H^M 3% basis the single premium for an assurance payable if (20) die before (45) or within 25 years of the latter's death with a return of the single premium as soon as it is certain that no claim can arise by the death of (20).

23.* In a stationary population supported by 1000 births annually one quarter of those attaining age 21 then emigrate, of whom there return at age 65 an annual number equal to 10% of the remaining population attaining age 65 in any year. Find the total of the population under 70 on an H^M basis.

24.* Find an approximate present value at H^M 5% of an annuity due of £30 to accumulate during the life of (59) at 1% and to be paid in one sum at the end of the year of his death.

<div align="center">FOURTH PAPER</div>

25. Prove that $Q^1_{xy} = \frac{1}{2}\left(1 + \frac{e_{x-1\ y} - \frac{e_{x\ y-1}}{p_{x-1}}}{p_{y-1}}\right)$ approximately, and state the assumption on which it is based. How would you apply the formula to select tables?

26. Give an expression, as far as possible in terms of contingent probabilities dependent on first death, for the probability that (30) and (40) will both die before attaining age 50 and in the lifetime of (25), (20) having predeceased the survivor of (30) and (40).

27. If $l_x = a - bx$ show that $\bar{a}_x = \bar{a}_\infty - \mu_x(I\bar{a})_x$.

28. It is found that as x increases, y and z remaining constant, $\bar{a}_{xyz}$ increases to a maximum where $x=w$ and thereafter decreases. Show that $^\infty\bar{P}^1_{wyz} = \mu_w$ and prove that this relation would hold for all ages if μ were constant for all ages.

29.* Find on an H^M 3% basis the gross annual premium, payable until the termination of the contract, for an assurance of £1000 payable immediately on the death of A provided B survive him and C predecease him. It is to be assumed that A, B and C are all aged 50 and that the gross premium is equal to the net premium with the addition of 5/- per £100 assured and 10% of the gross premium.

* *A Short Collection of Actuarial Tables* is supplied for use in answering these questions.

30.* Find on an H^M 3% basis the value of $\overset{\scriptscriptstyle 2}{a}\,\overset{(4)}{\underset{30.35.60}{}}$ [2]

31. Deduce an expression for the value of an assurance of 1 payable on the death of (x) if he should die within 10 years after the death of the survivor of (y) and (z), (w) having died second. If all four lives are of the same age, show that your expression may be written in terms of an assurance payable on the death of the last survivor of the four lives and contingent assurances payable on the death of a specified life should that life be the first to fail.

32.* Given P_{20} at 3% = ·01494 and at 7% = ·01000, find approximately the value at 5%.

PART III

FIRST PAPER

1. Describe the "Census" method of constructing a mortality table from the experience of Life Offices, and discuss any statistical errors which may be introduced by the use of this method. Explain a simple method of similar character which avoids these errors.

2. Explain shortly the uses of intercensal estimates of population. Define two main classes into which methods of arriving at such estimates may be divided, stating in what circumstances each type may be most useful.

3.* Given the following data, calculate the rates of mortality experienced in year 1925 at ages 0, 1, 2, 3, and 4. Migration is negligible.

Age last Birthday.	Population by Census at 30th June 1925.	DEATHS IN YEAR.				
		1921.	1922.	1923.	1924.	1925.
0	7229	894	933	905	932	1021
1	6487	237	248	267	261	255
2	6648	74	61	79	87	103
3	5959	45	42	50	49	62
4	5772	33	29	31	37	38

* *A Short Collection of Actuarial Tables* is supplied for use in answering these questions.

Year.	Births in Year.
1920	7276
1921	7012
1922	7910
1923	7306
1924	7740
1925	8254

4. State fully the reasons for attempting to forecast the mortality of (*a*) Annuitants and (*b*) Assured Lives, and compare the methods adopted in two recent Annuitant Experiences.

5. A Life Office with a large experience of Total Permanent Disability business desires to investigate the rates of disability, and the rates of mortality among disabled lives. Explain in detail how you would carry out the investigation, and draft cards for use in connection therewith.

6. Explain very briefly in what, if any, respects the function known as the "expectation of life" as generally determined and used may be misleading.

SECOND PAPER

7. Discuss briefly the justification of the process of graduation.

8. Describe (1) the graphic method of graduation, and (2) the summation method. Under what circumstances do you consider the latter method preferable to the former ?

9. What is meant by a series being "under-adjusted," and what steps would you take after making a graduation to determine whether your figures were under-adjusted ?

10. Indicate characteristics you would expect to find in curves representing the mortality of the following classes and compare them :—

    (1) Female assured lives resident in Britain
    (2) Male   ,,   ,,   ,,   ,,   ,,
    (3) Male Annuitant ,,   ,,   ,,   ,,
    (4)   ,,   (European) assured lives resident in India.

11. Why was a hypothetical mortality table constructed to represent the mortality experienced by the British Offices in the case of female annuitants in the 1893 investigation? Describe briefly the method employed in constructing this hypothetical table.

12. Define "standard deviation" and "skewness," and indicate a measure of the latter.

13. Give a short account of the general variations in mortality as shown by the life tables of England and Wales.

<p style="text-align:center">THIRD PAPER</p>

14. An association of nurses invites several Life Offices to quote for a scheme of Deferred Annuities, commencing at age 55, with the alternative of a lump sum at that age. Premiums are to be paid monthly, and in event of death before age 55 the amount paid in premiums is to be returned with compound interest. No agency commission will be payable.

As Actuary to one of the Life Offices, state concisely the bases and formulae you would use in calculating the premiums and cash options per £10 of annuity, the surrender values, and returns at death before age 55.

15. A composite office which has not hitherto transacted Life Assurance business decides to commence a Life Fund—95 per cent. of profits to go to Policyholders. Describe in detail how you would arrive at the annual and single premium rates for Whole Life and Endowment Assurances, with and without profits; the bonus aimed at being £2 per cent. per annum simple.

What special difficulties would have to be considered, and how would you deal with them ?

16. State, giving reasons, what *method* you consider best in each case for dealing with proposals for Whole Life Assurances by annual premiums on male lives involving respectively the following types of extra risk :—

(*a*) Flying officer, Royal Air Force.
(*b*) Age 25, light weight, good personal history, but one case of phthisis in family history.
(*c*) Mercantile Marine officer on large passenger liner.
(*d*) Bar steward on same type of vessel.
(*e*) Age 40, suffering from Pyorrhoea but no other adverse point except general condition rather "run-down."
(*f*) Age 50, heavy-weight, large abdominal girth, one private friend reports unfavourably as to habits.

17. A Mutual Life office with no agencies outside the United Kingdom has under consideration a proposal to abolish all extra premiums for travel or residence in tropical climates. State concisely the arguments for and/or against this proposal.

18. A large industrial concern decides to establish a Staff Pension Fund. The records contain full statistics as to withdrawals, deaths during service, and retirals through illness or superannuation in the past. The benefits to be granted through the Fund are :—At death or withdrawal, return of full past contributions without interest. On retirement, either by superannuation at age 65 or earlier disablement, a _uniform_ percentage of the salary or wages being received at date of such retirement.

The contribution for future entrants to the Fund, who will all come in at age 25, is fixed at 8 per cent. of salaries or wages throughout membership. Describe fully how you would arrive at the percentage of salaries which could be allowed on retirement. (Present employees aged 25 and over are to be dealt with separately, and may be left out of account.)

19. The membership of a District Thrift Club is about equally divided between Colliery Workers and Farm Hands. It is decided to open a voluntary Sickness Insurance Scheme, among the members. You are asked to draw up a table of contributions for certain specified Sickness Benefits. What bases would you consider suitable for use in your calculations?

FOURTH PAPER

20. The Directors of an Assurance Company desire an analysed report on the stock exchange securities held in order that they may be satisfied as to the distribution of these. What information do you think should be given? Draft any statements which you would incorporate in your report, and state what references, if any, you would make to the other investments held by the Company.

21. Mention the various headings in the Bank of England Weekly Return, and explain briefly what is meant by the items "Government Debt," "Rest" and "Public Deposits."
When was the Weekly Return made compulsory?

22. What post-war direct obligations of the British Government are quoted in the Daily Official List?
Describe briefly, "Securities guaranteed under the Trades Facilities Acts of 1921-25." How should the yields obtainable on these compare with the yields obtainable on direct obligations of the British Government?

23. Mention the *method* in London of quoting exchange between London and the following towns :—

Paris ; Brussels ; Bombay ; Mexico City ; Oslo ; Budapest.
In what instances has there been a change since 1914 ?

24. Criticise as investments for an Assurance Company any *two* of the following securities :—

	Offered to yield, with redemption.		
Transvaal Govt. 3% Guaranteed Stock 1958 .	.	£4 12	0%
Mashonaland Railway 5% G'teed. Mort. Deb. Bonds 1955 (G'teed. British South Africa Co.)	.	. 5 12	0%
Belgian Govt. 7% Stabilisation Loan of 1926 (Redeemable by 1956)		7 4	6%
Buenos Ayres and Pacific Rly. 4½% Cons. Deb. Stock		5 10	0%
Manila Rly. (1906) 5% Debenture Stock 1939 (Interest guaranteed by Philippine Govt.) .	.	8 5	0%

25. A premium fell due on a policy and was not paid to the agent during the days of grace, the relative receipt being thereafter returned to the Company by the agent. The subsequent period during which the policy might be revived without medical examination having expired, the policy was written off the books, and the free surrender value transferred to an unclaimed surrender value account. The amount of the surrender value was intimated to the assured and subsequently paid to him.

Draft the various Cash Book, Journal and Ledger entries that would be made during the period mentioned.

26. How would you deal in your books with payments of interest and principal received in respect of—

(1) Debentures redeemable at par at a fixed date purchased under par ;

(2) Debentures redeemable at par at a fixed date purchased over par ;

(3) Indian Railway " A " Annuities.

PART IV

FIRST PAPER

1. An office has in the past valued its business on the O^M basis at 3% interest. It has declared regularly for some time a compound reversionary bonus at a uniform rate. It has just completed its quinquennial investigation on the old basis, and is

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contemplating a change in basis by using O^M 3% with $O^{[M]}$ 3% net premiums.

Show in schedule form a method of estimating the cost of this change, and state what effect it is likely to have on the future rate of bonus.

2. An office values its general Endowment Assurance classes by the O^M Table at 3% interest, employing Lidstone's Z^M method. State in detail the method you would adopt in valuing those With Profit Endowment Assurances under which the premiums are paid by single and limited payments. State whether your method is likely to underestimate or overestimate the true reserve.

3. In the past a mutual office has declared a simple reversionary bonus at a uniform rate on all classes of assurance. At the forthcoming valuation it is anticipated that a surplus will be yielded sufficient to increase substantially the former rate, but before deciding upon the allocation of this surplus it is desired to investigate the equity of giving the same rate to both Whole of Life and Endowment Assurance classes.

(1) State how you would carry out such an investigation; and
(2) If it is found that the Endowment Assurance class is not earning so high a rate as the Whole of Life class, state what considerations would affect your choice between
  (*a*) allotting a higher rate to Whole of Life Policies, and
  (*b*) allotting the same rate to both classes, and increasing the premium rates for Endowment Assurances.

4.* The following particulars have reference to a group of Whole of Life non-profit policies:—

	Sum Assured.	Gross Premiums.	Net Premiums.
Business in force at 1st January	£20,000	£400	£385·0
Written off during year—			
(1) *Claim by death, after payment of year's premium*	£750	£13	£12·2
(2) *Surrender before payment of year's premium*	1,000	19	18·7
(3) *Paid-up policy of £750 granted before payment of year's premium*	2,000	45	43·7

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

New Business during year . .	£5,000	£190	£186·6

Business in force at 31st December .	£22,000	£513	£497·0

Revenue Account—

Fund at 1st January .	£5,600	*Claims* . . .	£750
Premiums received .	526	*Surrenders* . .	250
Interest (net) received .	274	*Expenses* . . .	25
		Fund at 31st December	5,375
	£6,400		£6,400

All lives assured are aged 50 nearest birthday at 31st December. Premiums are assumed to be payable at the middle of the year. Cancellations, alterations and entry of New Business take place at the middle of the year.

Value the group at the beginning and end of year on the basis of H^M (Text Book) 3%. Analyse the profit or loss during the year as far as possible, and if any untraced balance remains, indicate the probable source of such balance.

SECOND PAPER

5. A fund providing pensions on retirement at age 65 to the employees of a large industrial company has been valued and found at present market values to provide exactly the reserves necessary.

All contributors enter at age 18 and pay into the fund yearly a small percentage of their salary, the employers paying a similar amount. The contributors in number and age distribution have been stationary for many years.

It is proposed to alter the salary scales either (*a*) by a 100% increase, or (*b*) by a £100 constant addition, both at all ages, and new valuations are made on the new salary bases assuming (1) that the same percentage of salary continues payable in future by all, and (2) that new entrants and the employers in respect thereof pay the actual percentage required to secure the pension provided by the scheme, namely one-sixtieth of final salary for each year of service.

Discuss on broad lines the relative deficiencies arising on each assumption, and state the various ways in which they might be dealt with, taking into account equity as between old and new members.

6. Set out and explain the forms in which returns of outstanding Employers' Liability claims require to be made in terms of the Assurance Companies Act 1909.

Is any basis of valuation laid down by the Act for these claims or any section of them, and do any of the forms require to be signed by an actuary?

7. An annual premium non-profit endowment assurance was effected, on a life aged x, t years ago. It matures at age $x+n$. It includes a disability clause providing for (a) waiver of all premiums falling due after proof of permanent total disability, and (b) the payment commencing a year after the policy anniversary next succeeding the proof of total permanent disability of yearly instalments of $\frac{1}{K}$th of the sum assured, the instalments ceasing when an amount equal to the sum assured has been paid. In event of the policy becoming payable through the survivance or death of the assured, the sum assured is paid subject to deduction of an amount equal to the amount paid in instalments and no further instalments are payable.

Obtain a formula for valuing the disability benefit reserve, assuming (x) is not a disabled life.

8. Explain in regard to pensions based on average salary how you would obtain a formula for the valuation factor required to value prospective pensions due to past contributions.

9. Explain briefly the Friendly Society system known as the Individual Account or Holloway system, pointing out its advantages and its disadvantages.

10.* A Friendly Society valuing and calculating its rates on the basis of the Manchester Unity Sickness and Mortality Experience at 4% printed in the tables supplied, has on investigation shown a deficiency of £10,000. The benefits granted are 40s. a week for the first six months' sickness, 15s. a week for the second six months, and 10s. a week thereafter. From age 65 sickness benefit ceases and a pension of 10s. a week becomes payable.

It is found that the deficiency has arisen through the first six months' sickness cost exceeding the Manchester Unity 4% cost by 30%, the sickness in other periods and the mortality being in accordance with the Manchester Unity experience. The deficiency

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* *A Short Collection of Actuarial Tables* is supplied for use in answering this question.

is to be met by reducing the sickness benefit during the first six months.

Assuming the effect of any reduction will be to reduce the weeks of sickness in that period by 50%, and also that the original rates of contribution remain unaltered, what amount of reduction in the first six months' sickness pay will be required to wipe out the deficiency ?

The following are particulars of the Society :—

Age attained.	Number of members (All entitled to immediate benefit.)
30	250
50	150
70	100

11. A widows' fund charges the following fines on first marriage in addition to annual contributions. How would you estimate the value of these fines in a valuation?

Age of member at marriage.	Fine	
40–50	£4	for each year over 5 that his age exceeds his wife's.
50–60	£5	,, ,, ,,
Over 60	£6	,, ,, ,,

THIRD PAPER

12.* A, a male aged 70 and B a female aged 65, have during their joint lives and the life of the survivor the Liferent of an Estate consisting of—

  £10,000 $3\frac{1}{2}\%$ Conversion Loan
  £5,000 Dominion of Canada 3% Stock 1938
  £5,000 5% National War Bonds payable September 1928 at 105%.

The market values of these stocks may be taken as 75, $83\frac{1}{2}$, and $105\frac{1}{4}$ respectively.

After the death of the survivor of A and B the Estate passes to C, a male now aged 30, if he be then alive. What is the maximum amount which you would advance to C on security of his reversion and what rate of interest would you charge? The annual premium for an assurance payable should C predecease the survivor of A and B may be taken as £1, 5s. 6d.%.

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question. Calculations should be made at 3% interest with the H^M Mortality Table (Text Book Graduation), using actual ages, but candidates are required to state the rates of mortality and interest which they would employ in practice.

13.* A sold a well-secured reversion of £5000 payable on the death of B, a male aged 70, retaining the option of redemption for 5 years. At the end of the 5 years he applies for an extension of the option period for a further 5 years. What fine would you charge? The £5000 may be assumed to be payable free of all expense.

14. A Trust Estate consists of—

£5000 New South Wales Govt. 6% Stock 1930/40
(market value 100%).

£4000 New Zealand Govt. $3\frac{1}{2}$% Stock 1940
(market value 85%).

£3000 Dunlop Rubber Co. $5\frac{1}{2}$% 1st Mort. Deb. Stock
(market value 103%).

£5000 Port of Karachi 6% Bonds 1952
(market value 106%).

£5000 London Midland & Scotttish Ry. 4% Preference
Stock 1923   (market value 72%).

and £5000 in Bonds over Heritable Property which presently
carry interest at $5\frac{1}{2}$%.

How would you arrive at the income to be valued if a purchase of the life interest was contemplated, or at the amount of capital that could be taken into account if asked to quote for purchase of the reversion?

What special enquiries would you make regarding the Trust Deed, and the Trustees?

15. A, a male aged 55, who is unmarried, has the Life Interest in an Estate valued at £40,000 which is invested in Trustee Securities. If he should marry and be survived by his wife an annuity of £500 will be payable to her until her death or remarriage. On the death of A the Estate will pass into the possession of his issue, if any, or failing such will go to B, a male aged 60, if he be then alive. If A leaves no issue, and B has predeceased him, the Estate will fall into the possession of C, a female aged 25, or her heirs.

How would you value the respective interests of A, B, and C for a mutual apportionment? What tables of mortality, etc., and what rate of interest would you employ? Give the formulae which you would use.

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* *A Short Collection of Actuarial Tables* is supplied for use in answering this question. Calculations should be made at 3% interest with the H^M Mortality Table (Text Book Graduation), using actual ages, but candidates are required to state the rates of mortality and interest which they would employ in practice.

16.* Six years ago A, then aged 50, effected a 15 years Endowment Assurance of £1000 with profits at an annual premium of £74. The policy now carries vested bonus additions of £90 and the rate of bonus declared at the last three annual valuations has been £1, 10s.% p. a., simple. Application is made for conversion of the policy into a Whole Life Assurance by premiums ceasing at age 65.

Make the necessary calculations and draft a letter to the assured making your quotation. The annual premium for a Whole Life Assurance with profits by premiums ceasing at age 65 may be taken as £6, 3s. 6d.% at age 50 and £9, 19s.% at age 56.

17. On what basis would you calculate surrender values and paid-up policies for—

    (1) Children's Deferred Assurances commencing at age 21, with a return of premiums in the event of death before that age and with the option, at age 21, of taking either a cash payment, a Whole Life Assurance with profits, or an Endowment Assurance with profits. The premium is constant throughout, the sum assured being adjusted at age 21 to meet the class of assurance.

    (2) Double Endowments.

    (3) Single Premium Assurances.

<div align="center">FOURTH PAPER</div>

18. It is intended that a Life Assurance Company should be started registered as a Limited Company under the Joint Stock Companies Acts : state the various steps that must first be taken.

19. Explain the difference between payment into Court under the Life Assurance Companies (Payment into Court) Act 1896, and proceeding by interpleader, stating the advantages and disadvantages of each.

20. Write a brief note on what constitutes an insurable interest. Quote generally the provisions of the various Acts of Parliament dealing with the subject of insurable interest.

21. Under what circumstances, and for what reason, must a Life Assurance Company satisfy itself of the payment of Death Duties before settling a claim arising on the death of one of its Lives Assured?

* *A Short Collection of Actuarial Tables* is supplied for use in answering this question. Calculations should be made at 3% interest with the H^M Mortality Table (Text Book Graduation), using actual ages, but candidates are required to state the rates of mortality and interest which they would employ in practice.

22. State briefly the law regarding the Presumption of Life Limitation as it affects the presumed deaths of Lives Assured and of Assignees.

23. How would you deal with the title to a policy of Life Assurance for £1000 which has become a claim by death, and under which only one Notice of Assignment has been received if that notice be of

> (*a*) An Absolute Assignation to a Scottish Bank stamped 10s. without any monetary consideration being shown and the policy being subject to Scottish Law;
>
> *or* (*b*) A Mortgage to an English Bank to cover all advances stamped 10s., the policy being subject to English Law;
>
> *or* (*c*) An Assignment executed in 1912 by the deceased in favour of his son in consideration of natural love and affection stamped 10s.?

24. Give briefly the conditions that govern relief of Income Tax to a Life Assurance Company (having *inter alia* a foreign Life Assurance Fund) in respect of its expenses of management, assuming the profit/loss arising from Reversions over a series of years to be as follows:—

Year of Assessment.	Office Year.	Profit.	Loss.
1922/23	1922	—	400
1923/24	1923	100	—
1924/25	1924	350	—
1925/26	1925	—	500
1926/27	1926	2000	—

State what sums (if any) would fall to be deducted from the expenses of management claim in respect of year of assessment 1923/24, 1925/26 and 1926/27.

State also what statutory limit there is to the relief to be given to a Life Assurance Company in respect of its expenses of management. How does this limit apply in the case of a *mutual* Company?