

Dwarfs as Cosmological Probes

Dwarf Galaxies as Cosmological Probes

Julio F. Navarro

CIFAR Senior Fellow and Professor, Department of Physics and Astronomy,
University of Victoria, Victoria, BC, Canada V8P 5C2
email: jfn@uvic.ca

Abstract. The Lambda Cold Dark Matter (LCDM) paradigm makes specific predictions for the abundance, structure, substructure and clustering of dark matter halos, the sites of galaxy formation. These predictions can be directly tested, in the low-mass halo regime, by dark matter-dominated dwarf galaxies. A number of potential challenges to LCDM have been identified when confronting the expected properties of dwarfs with observation. I review our understanding of a few of these issues, including the “missing satellites” and the “too-big-to-fail” problems, and argue that neither poses an insurmountable challenge to LCDM. Solving these problems requires that most dwarf galaxies inhabit halos of similar mass, and that there is a relatively sharp minimum halo mass threshold to form luminous galaxies. These predictions are eminently falsifiable. In particular, LCDM predicts a large number of “dark” low-mass halos, some of which should have retained enough primordial gas to be detectable in deep 21 cm or H α surveys. Detecting this predicted population of “mini-halos” would be a major discovery and a resounding success for LCDM on small scales.

Keywords. Cosmology: dark matter, Galaxies: dwarf, Galaxies: formation.

1. Introduction

There is now a well-defined paradigm for the growth of structure in the Universe. Large-scale observations of the cosmic microwave background and of galaxy clustering have helped to constrain the matter-energy content of the Universe, as well as its expansion history and overall geometry. These observations are well reproduced in the LCDM paradigm, which assumes that the recent accelerated expansion of the Universe is due to the “dark energy” contribution of a cosmological constant (“Lambda”, or “L”); that the matter content of the Universe is dominated by cold dark matter (CDM), and that the initial density fluctuation was scale-free and Gaussian, as expected from inflation ([Planck Collaboration 2016](#)).

With the cosmological parameters settled, it is then possible to predict and/or simulate the present-day clustering of dark matter on all extragalactic scales relevant to the formation of galaxies, a field where large cosmological N-body simulations have become indispensable. On very large scales, dark matter distributes itself in a foam-like network of sheets and filaments. This “cosmic web” is punctuated by high-density regions (“halos”) where the dynamical crossing time is shorter than the age of the universe. Thanks largely to N-body simulations, we now have an excellent understanding of the expected structure, abundance, and clustering of cold dark matter halos in LCDM (see; e.g., [Frenk & White 2012](#) for a recent review).

CDM halos are expected to be self-similar in structure and substructure. In terms of structure, the mass profile of all halos, regardless of mass, look alike when scaled properly ([Navarro *et al.* 1996, 1997](#)). In terms of substructure, the similarity implies that the mass function of subhalos (“satellites”) is independent of halo mass when scaled to the mass of the host (see; e.g., [Moore *et al.* 1999](#); [Wang *et al.* 2012](#)).

These are important results. If, when scaled, all halos look alike, then, unscaled, all halos should look different. For example, the density profiles of two LCDM halos of different mass do not cross at any radius. This implies that measuring the halo properties at *one* radius (e.g., density, circular velocity, etc) allows a full characterization of the halo at *all* radii. In terms of substructure, it implies that the expected numbers of subhalos of a given system scales directly with the system's total mass. In other words, a more massive halo has more subhalos of all masses.

In the LCDM paradigm, galaxies are assumed to form at the centers of dark matter halos and satellites are assumed to inhabit their subhalos. Since the mass function of CDM halos and the stellar mass function of galaxies are both known, the above assumption implies that it should be possible to infer the relation between the stellar mass of a galaxy and the mass of its surrounding halo using simple, but robust, approximations. The most widely used approximation is referred to as “abundance-matching” (AbMat), where galaxies are assigned to halos respecting their relative ranks, by mass (see, e.g., Behroozi *et al.* 2013, and references therein). Thus, in a given (large!) volume, the most massive galaxy should inhabit the most massive halo, the 2nd most massive galaxy the 2nd most massive halo, and so on. Because the *shape* of the galaxy and halo mass functions are quite different, this assignment results in a strongly non-linear dependence of galaxy mass on halo mass.

In particular, because the faint end of the luminosity function is relatively flat, and the low-mass end of the halo mass function is rather steep, this implies that *most* dwarf galaxies are assigned to halos of similar mass. Indeed, the AbMat model predicts that dwarfs spanning a factor of 10,000 in stellar mass ($10^5 < M_{\text{gal}}/M_{\odot} < 10^9$) all inhabit halos spanning a rather narrow range of halo virial[†] mass, roughly between $10^{10} < M_{200}/M_{\odot} < 10^{11}$. Further, it predicts that few *field*[‡] luminous galaxies must form in halos less massive than $10^{10} M_{\odot}$, and essentially *none* in halos under $10^9 M_{\odot}$.

The latest cosmological hydrodynamical simulations of galaxy formation in LCDM have confirmed the AbMat predictions (see; e.g., Vogelsberger *et al.* 2014; Schaye *et al.* 2015), at least in the $M_{\text{gal}} > 10^7$ - $10^8 M_{\odot}$ regime, where some of the simulations are able to reproduce the galaxy stellar mass function to within a factor of two. It is unclear whether this success extends to lower masses, since the galaxy mass function is not well constrained on that regime and the simulations have insufficient resolution to provide robust results on cosmologically significant volumes.

At the very faint end, the shape of the luminosity function is probably best constrained in the Local Group (LG); the loose association of the Milky Way (MW) and Andromeda (M31) galaxies, their satellites, and surrounding field. The inventory of LG galaxies brighter than the Draco dwarf spheroidal (dSph), which has an absolute magnitude of $M_V \sim -8$ and a stellar mass $M_{\text{gal}} \sim 10^5 M_{\odot}$, is thought to be quite complete, at least within 2-3 Mpc from the MW-M31 barycenter (McConnachie 2012). It is also possible to measure the kinematics of these nearby dwarfs, enabling meaningful constraints on their dark matter content (and hence on their halo masses), at least in the region where kinematic tracers exist.

These facts make the Local Group an ideal environment to test the relation between galaxy mass and halo mass predicted by LCDM, and have prompted a number of groups to simulate volumes tailored to reproduce either the overall configuration of the massive galaxies of the Local Group (see; e.g., the ELVIS and APOSTLE projects; Garrison-Kimmel *et al.* 2014; Fattahi *et al.* 2016a), or to focus on the satellite population of individual halos with masses comparable to that of the Milky Way or M31 (see; e.g.,

[†] We use a mean density of $200\times$ the critical density to define the virial boundary of a halo.

[‡] Satellites may lose large fractions of their dark matter to tidal stripping, so this statement does not apply to the halo masses of satellites of more massive systems.

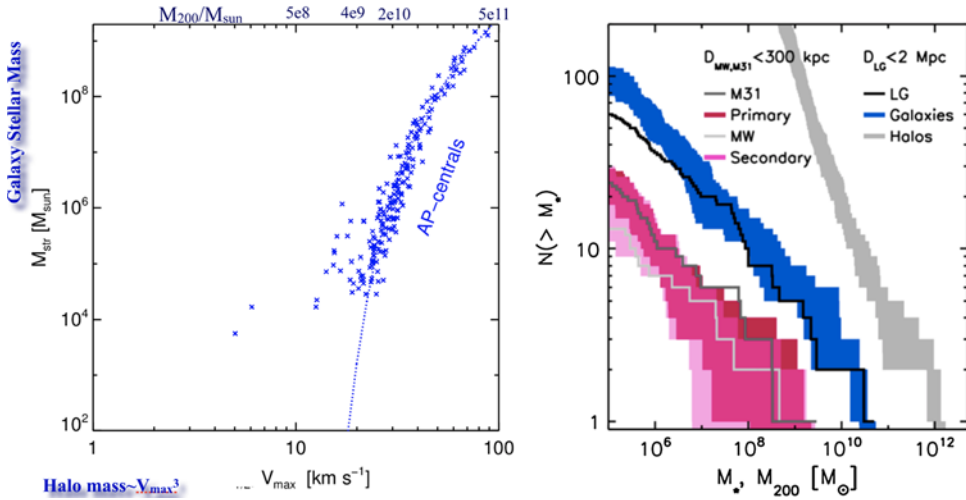


Figure 1. Left: Stellar mass of APOSTLE field dwarfs vs halo maximum circular velocity (V_{max} , scale at bottom) or virial mass (M_{200} , scale on top). Adapted from Fattahi *et al.* (2018). Right: Cumulative number of APOSTLE halos and subhalos within 2 Mpc from the Local Group barycenter (grey band) as a function of stellar mass. Blue band correspond to luminous satellites in the same volume, as a function of stellar mass. Red/pink bands are satellites of the MW and M31 analogs. Solid lines are observational data for the Local Group. Adapted from Sawala *et al.* (2016).

Brook *et al.* 2014, Brooks & Zolotov 2014, Wang *et al.* 2015, Wetzel *et al.* 2016, Grand *et al.* 2017). These simulations are able to resolve the formation of satellite and field dwarfs down to roughly $10^5 M_{\odot}$, enabling direct comparison with the Local Group dwarf galaxy population. I will discuss below how these simulations resolve the “missing satellites” and “too-big-to-fail” problems. I will focus my discussion on the results of the APOSTLE collaboration (see; e.g., Sawala *et al.* 2016, and references therein), but similar results have also been reported by other groups. An excellent recent review of these topics may be found in Bullock & Boylan-Kolchin (2017).

2. The Missing Satellites Problem

The left panel of Fig. 1 shows the galaxy stellar mass-halo mass relation for APOSTLE field dwarfs; i.e., those that are outside the virial boundaries of any other more massive system. Note the sharp cutoff in the relation, which implies that most “luminous dwarfs” (defined as those in the stellar mass range 10^5 - $10^9 M_{\odot}$) form in halos above a threshold of about $M_{200} \sim 5 \times 10^9 M_{\odot}$, corresponding to $V_{max} \sim 20$ km/s. (The few outliers scattering towards lower masses are just misidentified satellites, not field galaxies.) The threshold mass arises mainly because of the effects of cosmic reionization, which prevents gas from cooling and condensing at the center of halos whose potential wells is shallower than that characteristic mass (see; e.g., Ferrero *et al.* 2012, Fitts *et al.* 2017).

The presence of this threshold implies that, to first order, the total number of dwarfs in the Local Group within, say, 2 Mpc from the MW-M31 barycenter, should be comparable to the total number of halos and subhalos above the threshold mass. This is shown by the grey band in the right-hand panel of Fig. 1, which shows the cumulative number of halos and subhalos in APOSTLE in that volume, as a function of mass. This indicates that the LG APOSTLE realizations contain, on average, about ~ 100 halos above the threshold, and, consequently, about 100 dwarfs above $10^5 M_{\odot}$ in stars (blue band in same panel). By comparison, there are 60-70 known dwarfs within the same volume in the Local Group.

This factor-of-two agreement is encouraging, since such constraint was never used when selecting the Local Group volumes for the APOSTLE project. The same “threshold” mass yields between 15 and 30 luminous satellites around each of the M31/MW analogs in the APOSTLE volumes (red and pink bands), again in reasonable agreement with observation (shown with grey lines). Note that this agreement is sensitive to the virial mass assumed for the LG primary galaxies. In APOSTLE, the combined virial mass of the M31+MW system is between 2 and $3 \times 10^{12} M_{\odot}$ (Fattahi *et al.* 2016a). For example, leaving all else unchanged, doubling the virial mass of the primaries would double the number of satellites, degrading the agreement between observation and the APOSTLE results.

Note that if this mass threshold did not exist, and, for example, luminous dwarfs could form in halos with virial masses as low mass as $10^8 M_{\odot}$, then we would expect about 1,000 such dwarfs in the Local Group, vastly exceeding the number of known systems. This is a clear manifestation of the “missing satellites” problem. Fig. 1 thus shows that, largely because of the effects of reionization and feedback from evolving stars (another crucial heating mechanism included in the simulations), the number of predicted satellites is in good agreement with observations. The presence of a threshold halo mass for luminous dwarf formation thus provides a simple and compelling resolution to the “missing satellites” problem in LCDM. A more thorough discussion of this result may be found in Sawala *et al.* (2016).

3. The Too-Big-To-Fail Problem

The discussion above shows that there is no obvious overabundance of field luminous dwarfs or satellites in LCDM realizations of the Local Group, provided that dwarf galaxies form only in relatively massive halos. This assumption may be probed observationally by estimating the dark matter content of *individual* systems. In the case of dwarf spheroidals, for example, this is accomplished by using the line-of-sight velocity dispersion of the stars to estimate the total mass within the stellar half-mass radius, $r_{1/2}$ (Walker *et al.* 2009; Wolf *et al.* 2010). Since these are dark matter-dominated systems, this is a direct measure of the circular velocity of the dark matter halo at $r_{1/2}$. As discussed in Sec. 1, this single measurement may be used to estimate the total mass (or maximum circular velocity, V_{\max}) of individual halos.

This is shown in the left panel of Fig. 2, where circular velocity estimates at $r_{1/2}$ are shown for 9 satellites of the Milky Way (symbols with error bars). These estimates are compared with the average circular velocity profiles of subhalos selected from the Aquarius Project simulation suite of dark matter halo formation (Boylan-Kolchin *et al.* 2012). Subhalos are binned by their maximum circular velocity, as listed in the figure legends.

This comparison illustrates a number of important points: (i) several MW satellites appear to inhabit halos with $V_{\max} < 20$ km/s; i.e., below the “threshold” value we discussed in Sec. 2; (ii) there does not seem to be a monotonic dependence of galaxy mass with V_{\max} ; for example, Draco apparently inhabits a halo more massive than Fornax’s despite being $\sim 100\times$ less luminous; and (iii) there are a number of subhalos that are apparently quite massive (i.e., $V_{\max} > 25$ km/s) but yet do not have luminous counterparts. The last issue, in particular, is what prompted Boylan-Kolchin *et al.* (2011) to argue that such subhalos would be “too big to fail” (TBTF) to form luminous satellites. This issue has often been cited as a severe observational challenge to LCDM.

How are these puzzles explained in LCDM? Let us first address the issue of numbers of massive subhalos. As discussed in Sec. 1, the number of subhalos of given mass (or V_{\max}) is a strong function of the host halo mass. According to Wang *et al.* (2012), the average number of subhalos with V_{\max} exceeding a certain value $\nu = V_{\max}/V_{200}$ (where V_{200} is

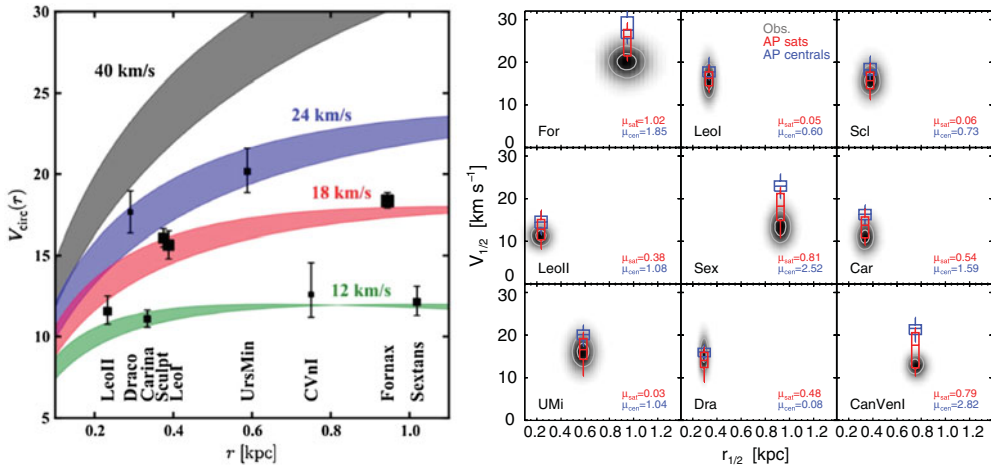


Figure 2. Left: The estimated circular velocities at the stellar half-mass radii of some MW satellites, compared with the circular velocity curves of CDM subhalos of fixed V_{\max} , as specified in the legend. Adapted from [Boylan-Kolchin et al. \(2012\)](#). Right: Circular velocities at the stellar half-mass radii of MW satellites. Grey contours indicate observational constraints; the red box-and-whisker symbols indicate the results for APOSTLE satellites of matching stellar mass at the same radius. Adapted from [Fattahi et al. \(2016b\)](#).

the virial velocity of the host) is given by $\langle N_{\text{sub}} \rangle = 10.2 (\nu/0.15)^{-3.11}$. This implies that we would expect roughly 10 subhalos with $V_{\max} > 30$ km/s if $V_{200} \sim 200$ km/s, but only ~ 4 if $V_{200} = 150$ km/s.

Since (i) the virial velocity of the Milky Way is poorly constrained, (ii) the small expected number of massive subhalos is subject to Poisson fluctuations, and (iii) the Milky Way has at least 3 satellites with $V_{\max} > 30$ km/s (the LMC, SMC, and the Sagittarius dSph), the apparent overabundance of massive subhalos that have “failed” to form luminous dwarfs is intriguing but not necessarily damning, and certainly highly sensitive to the assumed virial mass of the Milky Way. In addition, the $\langle N_{\text{sub}} \rangle (\nu)$ expression was derived for dark matter only simulations, and should be corrected when applied to dwarfs. The correction arises because dwarf galaxy halos expel most of their baryons at early times, reducing the depth of their potential wells, and reducing the values of ν by $\sim 20\%$ ([Sawala et al. 2016](#)). Because of the steepness of the subhalo mass function (see, e.g., the right-hand panel of Fig. 1), even this modest correction leads to a factor of about two reduction in the expected number of massive subhalos. Considering all of these arguments, it is hard to conclude that the number of subhalos more massive than $V_{\max} \sim 30$ km/s poses a substantial challenge to LCDM.

The above discussion addresses only one of the three items that, in my opinion, define the TBTF conundrum. The other two issues (i.e., why satellites populate halos below the “threshold” mass, and why the dependence between halo mass and stellar mass is not monotonic) are explained by the effects of tidal stripping by the host halo, as well as by the very steep dependence of stellar mass on halo mass shown in the left-hand panel of Fig. 1. This panel shows that it is indeed possible for dwarfs of widely differing stellar mass to be formed in halos of similar virial mass. If affected by tides differently, then a more luminous satellite could easily today have less dark matter than a fainter one, as is the case of Fornax and Draco.

This is because tides tend to strip less bound material first, leading to a reduction in the dark matter halo content of a satellite. The reduction affects not only the outskirts of a satellite but also the dark matter *within* the stellar half-mass radius, even when

the stellar component remains bound. This happens because, unlike stars, dark matter particles found *within* $r_{1/2}$ have apocentric radii far outside the luminous radius of the satellite and are therefore more vulnerable to stripping. In loose language, these inner particles get stripped while they are at apocenter, and are not replaced, leading to a reduction of dark matter inside $r_{1/2}$ (see; e.g., Peñarrubia *et al.* 2008).

Are tidal effects enough to reconcile the data in the left panel of Fig. 2 with LCDM? This has been examined independently by a number of authors who broadly agree that they do, although there is still some debate about the exact role of halo mass, small number statistics, baryon-induced core formation, and tidal stripping (see, e.g., the discussion and references in Fattahi *et al.* 2016b, which complements and extends that of Sawala *et al.* 2016). The analysis in many of those papers is statistical, and largely based on reproducing the left-hand panel of Fig. 2 with data from hydrodynamical simulations to argue that there are no large numbers of obvious “failures” (i.e., curves that are always above the observational data).

The recent analysis of Fattahi *et al.* (2016b) goes beyond that, and compares the predictions of APOSTLE satellite and field dwarfs with each of the nine MW satellites. This is shown in the right-hand panel of Fig. 2, where the grey contoured area indicate the observed circular velocity constraint for each individual satellite, as well as the predictions from the APOSTLE simulations *at the same radius* for systems matching the stellar mass of each satellite. Blue box-and-whisker symbols are predictions for field dwarfs; red ones indicate results for satellite systems. Note that red symbols are always below the blue ones, as expected for tidal stripping of dark matter. Overall, there is fair agreement between the APOSTLE results and observations, for all individual MW satellites. We conclude that, when including tidal effects, there is no obvious difficulty matching the observed constraints with the predictions of LCDM cosmological hydrodynamical simulations. The “too-big-to-fail” LCDM problem has thus, in my opinion, been successfully resolved.

4. RELHICs: Reionization-Limited HI Clouds

One important consequence of the threshold virial mass for luminous dwarf galaxy formation discussed in the previous subsections is that there should exist some halos just below the threshold (i.e., “mini-halos” with $M_{200} \sim 3 \times 10^8$ to $\sim 5 \times 10^9 M_{\odot}$) which, although unable to form stars, are still able to retain and bind some of the gas heated by cosmic reionization. Indeed, it is straightforward to compute the total gas mass, density profile, and temperature profile of photoionized primordial gas bound, at $z = 0$, to a low-mass LCDM halo. This gas (i) should be near hydrostatic equilibrium with the gravitational potential of the dark matter; (ii) should be essentially free of metals; (iii) have negligible velocity dispersion; and (iv) be confined by the gravity of the halo and the ambient pressure of the ionized intergalactic medium (Benítez-Llambay *et al.* 2017).

The total gas mass expected within the virial radius of “mini-halos” is shown, as a function of halo virial mass, by the magenta solid line in the top-left panel of Fig. 3. The total gas mass bound to such low-mass halos is, as expected, well below the universal cosmic average, which is indicated by the dashed line. The virial mass upper limit of these “RELHICs” (REionization-Limited HI Clouds) is determined by the central cooling time of the gas, which becomes shorter than the age of the Universe above $\sim 5 \times 10^9 M_{\odot}$. This corresponds, in Fig. 3, to the threshold halo mass above which star formation can proceed and luminous galaxies form (see, e.g., blue solid line).

The APOSTLE simulations contain a large number of low-mass halos in the “mini-halo” mass range. Some of these halos contain RELHICs; (red open circles in the left

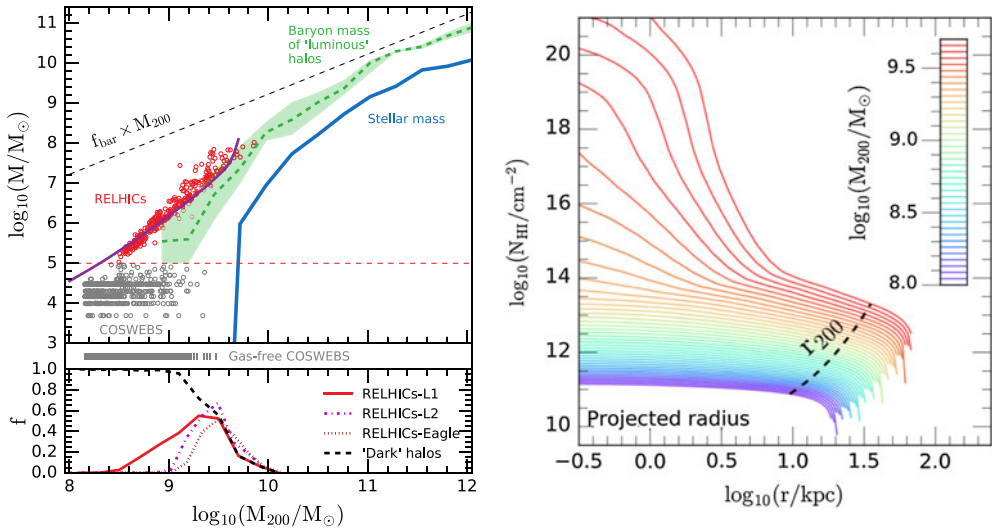


Figure 3. Left: The virial mass dependence of the baryonic mass of galaxies in the APOSTLE simulation suite. The blue solid line traces the median galaxy stellar mass. Note the threshold at $\sim 5 \times 10^9 M_{\odot}$, below which no luminous galaxies form. The red and grey open circles indicate the baryonic (gas) mass within the virial radius of “dark” halos. Right: The HI column density profile of RELHICs in the APOSTLE simulations. The color bar indicates the virial mass of the surrounding halo. Adapted from Benítez-Llambay *et al.* (2017).

panel of Fig. 3), but others are basically gas-free† (COSWEBS, or “cosmic web-stripped systems”; grey open circles). As discussed in Benítez-Llambay *et al.* (2017), the difference between RELHICs and COSWEBS is their location within the Local Group. Mini-halos that are relatively close to the two LG primary galaxies have had their gas content efficiently removed by ram-pressure from the “cosmic web” (Benitez-Llambay *et al.* 2013), which gives origin to the COSWEBS population.

On the other hand, mini-halos in low-density environments well away from the primaries are able to retain their gas, and constitute excellent examples of the RELHICs population hypothesized above. Their thermodynamic properties are well specified, and their gas density and temperature profiles may be predicted in detail. Gas in RELHICs is nearly fully ionized but with neutral cores that span a large range of HI masses and column densities and have negligible non-thermal broadening. Their predicted HI column density profiles are shown in the right-hand panel of Fig. 3.

A full analysis of the simulated RELHICs population in APOSTLE is provided in Benítez-Llambay *et al.* (2017)‡, who argue that Local Group RELHICs (i) should typically be beyond 500 kpc from the Milky Way or M31; (ii) have positive Galactocentric radial velocities; (iii) HI sizes not exceeding 1 kpc, and (iv) should be nearly round. Indeed, it is possible that some have already been detected in blind HI surveys, like ALFALFA, which have identified Ultra Compact High Velocity HI Clouds (Adams *et al.* 2013). The simulations provide guidance to identify which of those systems might be “mini-halos” and which are just random condensations of neutral gas in the Galactic halo. One way of discriminating between the two would be to measure H α emission in

† Recall that one gas particle in the highest-resolution APOSTLE runs corresponds to about $10^4 M_{\odot}$.

‡ Note that the ALFALFA data in Fig. 10 of that paper is incorrectly plotted and that its discussion will soon be revised in an erratum. My thanks to Yakov Faerman and Betsy Adams for pointing this out.

some RELHICs candidates, which should show a distinct “ring” marking the sharp transition between the inner neutral core and the outer ionized envelop of RELHICs. No such object has been reported yet, but it should be clear that the detection and characterization of RELHICs would offer a unique and exciting probe of the small-scale clustering of cold dark matter.

5. Concluding Remarks

The discussion above shows how the properties of dwarf galaxies may be used to probe some of the distinctive predictions of the LCDM cosmological paradigm on small scales. I have reviewed how cosmological hydrodynamical simulations of dwarf galaxy formation have helped to clarify the interpretation of some observations that are often cited as major “challenges” to LCDM. In particular, I showed how the “missing satellites” and “too-big-to-fail” problems can be resolved without appealing to *any* modification to the cold dark matter paradigm.

This short review is, like any and all, incomplete, and does not address the full list of small-scale LCDM worries that may be found in the literature. For example, I have not discussed the “cusp vs core” controversy but recent work has highlighted promising progress on that respect (see, e.g., the contribution of Alyson Brooks elsewhere in this volume). Less well understood are potential challenges that may arise from some of the faintest satellites of the Milky Way (i.e., the “ultra-faint” population of galaxies with stellar masses well below $10^5 M_{\odot}$), which are still beyond the capabilities of current simulations. According to the discussion above, these ultra-faints should also form in fairly massive halos, at odds with the very low velocity dispersion measured for some. This may, in principle, be resolved by appealing to the effects of extreme tidal stripping (see; e.g., the discussion in [Fattahi et al. 2018](#)), but the issue is far from settled so far.

Another issue that is far from settled concerns the origin of the morphological diversity, scalings, and scatter in the dwarf galaxy population. What sets their size and stellar mass profiles? Why do some rotate and others do not? Why do some have gas and others not? What determines their star formation history? Is the observational evidence consistent with the sharp threshold in virial mass for dwarf formation espoused above? These are issues that concern us now and will keep us busy for some time to come. Making sense of the wondrous diversity of dwarf galaxies, and contrasting it with the relatively featureless and self-similar context of their dark halos, is bound to yield interesting clues not only about the nature of dark matter, but also of the physics behind the assembly of the faintest galaxies.

References

- Adams, E. A. K., Giovanelli, R., & Haynes, M. P. 2013, *ApJ*, 768, 77
 Behroozi, P. S., Marchesini, D., Wechsler, R. H., et al. 2013, *ApJL*, 777, L10
 Benítez-Llambay, A., Navarro, J. F., & Abadi, M. G., et al. 2013, *ApJL*, 763, L41
 Benítez-Llambay, A., Navarro, J. F., & Frenk, C. S., et al. 2017, *MNRAS*, 465, 3913
 Boylan-Kolchin, M., Bullock, J. S., & Kaplinghat, M. 2011, *MNRAS*, 415, L40
 Boylan-Kolchin, M., Bullock, J. S., & Kaplinghat, M. 2012, *MNRAS*, 422, 1203
 Brook, C. B., Di Cintio, A., & Knebe, A., et al. 2014, *ApJL*, 784, L14
 Brooks, A. M., & Zolotov, A. 2014, *ApJ*, 786, 87
 Bullock, J. S., & Boylan-Kolchin, M. 2017, *ARAA*, 55, 343
 Fattahi, A., Navarro, J. F., & Sawala, T., et al. 2016a, *MNRAS*, 457, 844
 Fattahi, A., Navarro, J. F., & Sawala, T., et al. 2016b, [arXiv:1607.06479](#)
 Fattahi, A., Navarro, J. F., & Frenk, C. S., et al. 2018, *MNRAS*, 476, 3816
 Ferrero, I., Abadi, M. G., Navarro, J. F., Sales, L. V., & Gurovich, S. 2012, *MNRAS*, 425, 2817
 Fitts, A., Boylan-Kolchin, M., Elbert, O. D., et al. 2017, *MNRAS*, 471, 3547

- Frenk, C. S., & White, S. D. M. 2012, *Annalen der Physik*, 524, 507
- Garrison-Kimmel, S., Boylan-Kolchin, M., Bullock, J. S., & Lee, K. 2014, *MNRAS*, 438, 2578
- Grand, R. J. J., Gómez, F. A., & Marinacci, F., *et al.* 2017, *MNRAS*, 467, 179
- McConnachie, A. W. 2012, *AJ*, 144, 4
- Moore, B., Ghigna, S., & Governato, F., *et al.* 1999, *ApJL*, 524, L19
- Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, *ApJ*, 462, 563
- Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, *ApJ*, 490, 493
- Peñarrubia, J., Navarro, J. F., & McConnachie, A. W. 2008, *ApJ*, 673, 226
- Planck Collaboration, Ade, P. A. R., & Aghanim, N., *et al.* 2016, *A&A*, 594, A13
- Sawala, T., Frenk, C. S., & Fattahi, A., *et al.* 2016, *MNRAS*, 457, 1931
- Schaye, J., Crain, R. A., & Bower, R. G., *et al.* 2015, *MNRAS*, 446, 521
- Vogelsberger, M., Genel, S., & Springel, V., *et al.* 2014, *Nature*, 509, 177
- Walker, M. G., Mateo, M., & Olszewski, E. W., *et al.* 2009, *ApJ*, 704, 1274
- Wang, J., Frenk, C. S., Navarro, J. F., Gao, L., & Sawala, T. 2012, *MNRAS*, 424, 2715
- Wang, L., Dutton, A. A., & Stinson, G. S., *et al.* 2015, *MNRAS*, 454, 83
- Wetzell, A. R., Hopkins, P. F., Kim, J., Faucher-Giguere, C. A., Keres, D., Quataert, E., *et al.* 2016, *ApJ*, 827,23
- Wolf, J., Martinez, G. D., & Bullock, J. S., *et al.* 2010, *MNRAS*, 406, 1220