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Linear stability of stellar rotating spheres

Simon Rozier[1], Jean-Baptiste Fouvry[2], Philip G. Breen[3], Anna Lisa Varri[4], Christophe Pichon[1] and Douglas C. Heggie[3]

[1]Institut d'Astrophysique de Paris & UPMC Univ. Paris 06 (UMR7095), 98 bis Boulevard Arago, 75014 Paris, France
email: rozier@iap.fr

[2]Institute for Advanced Study, Einstein Drive, Princeton, New Jersey 08540, United States of America

[3]School of Mathematics & Maxwell Institute for Mathematical Sciences, University of Edinburgh, Kings Buildings, Edinburgh EH9 3FD, UK

[4]Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, UK

Abstract. Recent observations of globular clusters encourage to revise some aspects of the traditional paradigm, in which they were considered to be isotropic in velocity space and non-rotating. However, the theory of collisionless spheroids with some kinematic richness has seldom been studied. We present here a further step in this direction, owing to new results regarding the linear stability of rotating Plummer spheres, with varying anisotropy in velocity space and total amount of angular momentum. We extend the well-known radial orbit instability to rotating systems, and discover a new regime of instability in fast rotating, tangentially anisotropic systems.

Keywords. globular clusters: general, Galaxies: kinematics and dynamics, Instabilities, Methods: analytical

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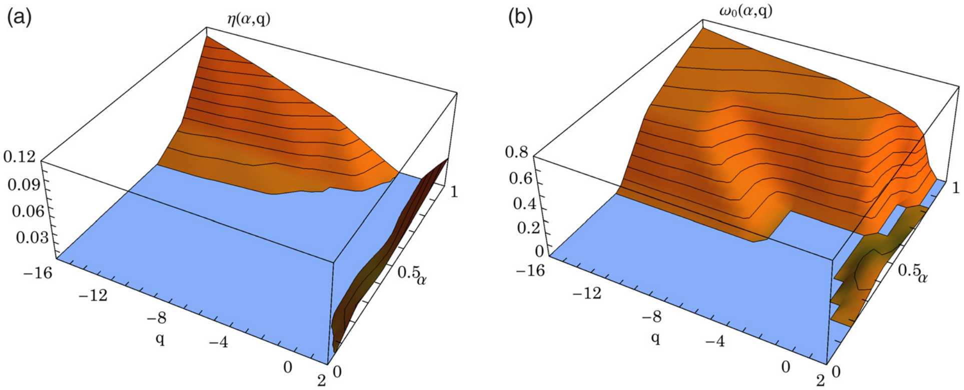
Recent astrometric measurements of globular clusters, as based on HST (Bellini *et al.* 2017) and Gaia DR2 (Bianchini *et al.* 2018), revealed that a large fraction of them are rotating significantly. While the analysis of the linear stability of spherically symmetric stellar systems has already been the subject of numerous investigations (see, e.g, the Doremus-Feix-Baumann theorem (Binney & Tremaine 2008), or radial orbit instability (ROI, see, e.g., Polyachenko & Shukhman 1981), all these investigations were limited to configurations having a spherically symmetric velocity distribution (i.e., a distribution function which is invariant under rotation in velocity space). This study sets out to lift this restriction and investigate the linear stability of rotating, anisotropic equilibria, i.e. systems with a non-zero total angular momentum. The methods, results and conclusions described here are extensively detailed in Rozier *et al.* (2019).

<u>*System and phase space distribution function:*</u> We first designed a distribution function (DF) describing a rotating anisotropic Plummer sphere at equilibrium, with tuneable rotation and anisotropy. This was achieved through the application of "Lynden-Bell's demon" (Lynden-Bell 1960) to a parametric family of non-rotating DFs described in Dejonghe (1987).

<u>*The matrix method:*</u> We applied to this series of equilibria a formalism based on the first order perturbation of the collisionless Boltzmann equation: the response matrix formalism, designed by Kalnajs (Kalnajs 1977). In this formalism, the linear response of a spherical equilibrium is described by its response matrix $\widehat{\mathbf{M}}(\omega)$, where $\omega = \omega_0 + \mathrm{i}\eta$ is the (complex) temporal frequency at which the matrix is evaluated.



Growth rate — Oscillation frequency

Figure 1. Growth rate η (left panel) and oscillation frequency ω_0 (right panel) as a function of the cluster's parameters (α, q). The parameter α controls rotation ($\alpha = 0$: no rotation; $\alpha = 1$: maximal rotation), q controls anisotropy ($q < 0$: tangential anisotropy; $q > 0$: radial anisotropy). We searched for unstable modes on a (α, q)-grid composed of the locations $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1$ and $q = -16, -12, -6, -2, 0, 1, 2$. The blue plane represents our growth rate threshold, $\eta = 0.01$, for mode selection.

Searching for instabilities: A mode of the system is identified by finding a location in ω-space where the response matrix $\widehat{\mathbf{M}}$ has an eigenvalue equal to 1. We consider a mode to be unstable when its growth rate is $\eta > 0$, then ω_0 is its oscillation frequency.

The main results of our exploration are reported in Fig. 1, where are represented the growth rates and oscillation frequencies of the m=2 instabilities which could be identified through the matrix method in a significant fraction of rotation-anisotropy space. Linear instabilities develop in surfaces coloured in orange. Two main surfaces of instability stand out in this figure: (i) The "pyramid" at the top of the $\eta(\alpha, q)$ panel ($\alpha \gtrsim 1/2, q<0$) is specific to tangentially-biased and rotating systems. (ii) The rectangle in the bottom right part of the $\eta(\alpha, q)$ panel ($q>0$) is specific to radially-biased systems, and appears as an extension of the radial orbit instability to rotating systems. These conclusions are consistent with results which we have obtained through the use of N-body simulations.

This first step in the theoretical study of rotating stellar spheroids opens some perspective in our dynamical understanding of globular and nuclear star clusters. The results might help lifting the mass-anisotropy degeneracy in rotating globular and nuclear star clusters, while the same method can be applied to Schwarzschild models of rotating spherical systems to test their linear stability.

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