THE VELOCITY-DISTANCE RELATION AND THE HUBBLE CONSTANT FOR NEARBY GROUPS OF GALAXIES

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Abstract. The distances of about 100 nearby groups, clouds and clusters of galaxies ($\Delta < 52$ Mpc) have been derived from the apparent magnitudes and diameters of the 3 to 5 major members. The linearity of the distance scale was verified for $\Delta < 40$ Mpc by comparison with luminosity classifications and diameters of ring structures. The velocity-distance relation of these groups is apparently non-linear for $\Delta < 30$ Mpc. The velocity/distance ratio increases from $H \simeq 50$ to 150 km s^{-1} Mpc^{-1} when Δ increases from $\Delta \simeq 5$ to $\Delta \simeq 25$ Mpc. Such local departures from linearity are predicted in condensed regions of an inhomogeneous 'big bang' hierarchical model obeying the universal density-radius relation. The galactic apex varies with the apparent magnitude of galaxies used for reference in a manner generally consistent with a model of the Local Supercluster in differential rotation and expansion. The best-fit rotation-expansion constants are $\omega_1 R_1 = 400 \pm 50$ km s^{-1}, $\varepsilon_1 R_1 = 1250 \pm 50$ km s^{-1}, $\dot{z} = -250 \pm 50$ km s^{-1}. The velocity-magnitude relation corrected for solar motion is linear with slope 0.2 in $9 < m < 14$, confirming that apart from local departures reflecting the supercluster kinematics the underlying Hubble flow is linear and isotropic. If the distance modulus of the Virgo Cluster is $(m - M)_0 = 30.5 \pm 0.2$ the Hubble constant derived from the survey of groups is $H_0 = (100 \pm 10)$ km s^{-1} Mpc^{-1}.

Except for the absolute calibration of the primary distance indicators, the two major difficulties in deriving the correct value of the Hubble constant are: (1) establishing a truly linear distance scale consistent with the adopted distances of normal spirals in the Local Group and the nearest groups; and (2) allowing for the non-linearity and anisotropy of the velocity field for bright galaxies (de Vaucouleurs, 1958, 1964, 1966). This report describes the main results of a continuing study of these problems in progress since 1964 at the University of Texas.

1. Distance Scale for Nearby Groups

The Reference Catalogue of Bright Galaxies (BGC) (G. and A. de Vaucouleurs, 1964) has been used for a systematic search of all nearby groups and clusters represented by at least one member in the BGC (de Vaucouleurs, 1965, 1967a; Corwin, 1967). Probable and possible group members were selected from a combination of criteria including apparent distribution, apparent diameters and luminosities, morphological types and radial velocities. Distances were estimated from the apparent diameters and luminosities of the 5 brightest members reduced to homogeneous scales and corrected for inclination, absorption and size-of-sample effects (de Vaucouleurs, 1965). The zero point of the distance scale rests on the generally accepted distance moduli of the major members of the Local Group (because of small differences in absorption corrections, etc. adopted values average 0.1 mag. *less* than those in Sandage, 1962; van den Bergh, 1969, 1970). The scale was extended first to 4 groups nearer than 6.5 Mpc (de Vaucouleurs and Brown, 1964) by means of all available primary (novae,

D. S. Evans (ed.), External Galaxies and Quasi Stellar Objects, 353–366.

TABLE I

Distance moduli and mean radial velocities of 13 nearest groups of galaxies

Nr.	Group Cluster Cloud	l b	SGL SGB	μ A_B	μ_0 Δ	$D \times d$	V_0 n AD	X Y Z
G1	Scl	5 −80	265 −3	27.1 0.2	26.9 2.4	25 × 20 1.0 × 0.8	194 6 99	+0.1 −2.4 −0.2
G2	M81 (N3031)	142 +41	42 +1	27.3 0.3	27.0 2.5	40 × 20 1.8 × 0.9	160 9 66	+1.8 +1.7 +0.0
G3	CVn I	162 +80	82 +5	28.1 0.2	27.9 3.8	28 × 14 1.9 × 0.9	342 9 69	+0.5 +3.8 +0.2
G4	N5128	310 +20	155 −5	28.5 0.5	28.0 4.0	30 × – 2.1 × –	319 5 55	−3.8 +1.3 −0.4
G5	M101 (N5457)	102 +60	64 +23	28.5 0.2	28.3 4.6	23 × 16 1.8 × 1.3	508 8 85	+1.8 +3.8 +1.7
G6	N2841	170 +34	45 −20	29.2 0.3	28.9 6.0	15 × 7 1.6 × 0.8	589 4 122	+3.7 +4.4 −1.6
G7	N1023	145 −20	341 −8	29.6 0.6	29.0 6.3	20 × 10 2.2 × 1.1	566 8 127	+5.9 −1.9 −1.0
G8	N2997	250 +19	133 −53	30.0 0.6	29.4 7.6	14 × 8 1.9 × 1.1	534 2 –	−3.0 +3.1 −6.3
G9	M66 (N3627)	241 +64	97 −19	29.6 0.2	29.4 7.6	7 × 4 1.0 × 0.6	592 5 74	−0.9 +7.1 −2.4
G10	CVn II	138 +75	73 +3	29.7 0.2	29.5 8.0	22 × 12 3.0 × 1.6	747 15 71	+2.4 +7.7 +0.3
G11	M96 (N3368)	231 +58	93 −26	29.8 0.2	29.6 8.3	11 × 7 1.6 × 1.0	741 9 80	−0.7 +7.4 −3.6
G12	N3184	176 +60	67 −13	30.1 0.2	29.9 9.6	10 × 5 1.7 × 0.8	629 4 122	+4.0 +8.4 −2.7
G13	Coma I	198 +86	89 +3	30.1 0.2	29.9 9.6	11 × 5 1.8 × 0.8	944 15 170	+0.1 +9.5 +0.5

l, b = galactic coordinates (degrees). SGL, SGB = supergalactic coordinates (degrees).
μ, A_B = apparent distance modulus and galactic absorption (magnitudes).
μ_0, Δ = corrected distance modulus and distance (magnitudes and Mpc).
$D \times d$ = apparent and linear dimensions of group (degrees and Mpc).
V_0 = mean radial velocity corrected for conventional solar motion (300 cos A) (km s^{-1})
n = number of velocities and average deviation (AD)
X, Y, Z = rectangular supergalactic coordinates (Mpc from Sun).

Cepheids) and secondary criteria (luminosity classes, brightest stars, H II regions) and then to a dozen groups nearer than 10 Mpc (Table I) yielding a total of 68 galaxies ($M < -16$) which served to calibrate the BGC magnitudes and diameters used as tertiary distance indicators for the more distant groups and clusters. The first survey (de Vaucouleurs, 1965) comprises 54 definite or probable groups (outside the Local Group) nearer than about 16 Mpc. The second survey (Corwin, 1967) comprises 64 other groups, clouds or clusters (most definite, some probable and a few only possible) generally between 10 Mpc and 50 Mpc; except for increasing incompleteness at larger distances the two surveys are strictly homogeneous in principles of selection and methods of distance derivation.

2. Velocity-Distance Relation of Nearby Groups

When the mean redshifts of these groups were first plotted vs the adopted distances Δ

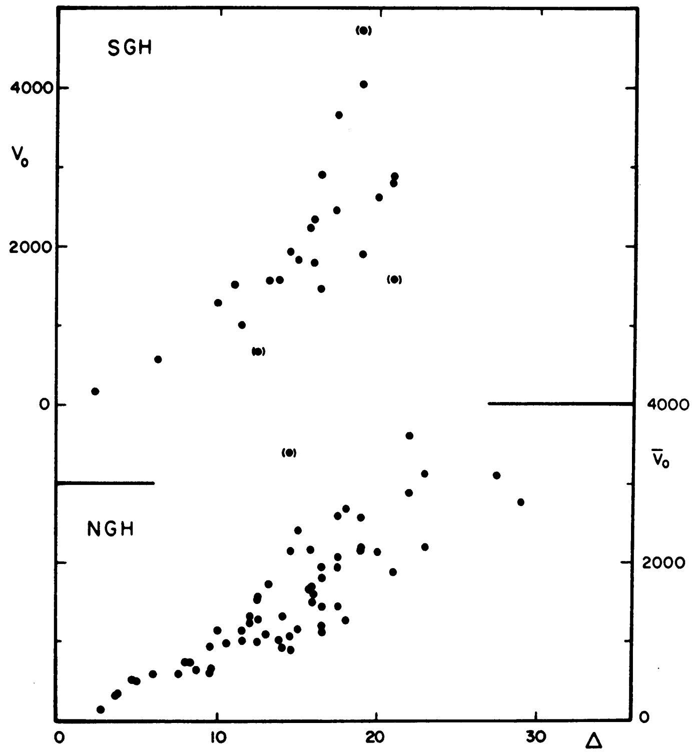

Fig. 1. Velocity-distance relation for nearby groups.

(Figure 1) a rather shocking result emerged: the apparent Hubble ratio $H = \bar{V}_0/\Delta$ was not a constant, but increased linearly with distance (Figure 2), i.e. the velocity-distance relation was parabolic within the range of the survey.

The author's initial reaction to this result was to suspect that some systematic error

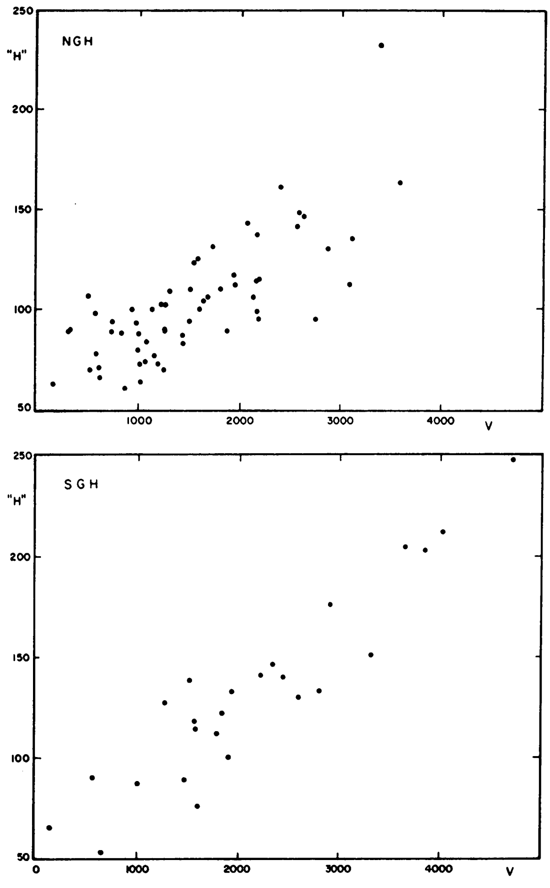

Fig. 2. Velocity/distance ratio vs velocity for nearby groups.

had crept into the derivation of distances, causing the estimated distances of the more distant groups to be *too small* relative to the distances of the nearer groups.

3. Verification of the Distance Scale

In order to check the distance scale used in the survey of nearby groups, two tests were applied: (a) comparison with distances derived independently from the van den Bergh (1960) luminosity classifications and from the Holmberg (1964) color-surface brightness data; (b) comparison of the average linear diameters of ring structures in galaxies (de Vaucouleurs, 1956) in nearby and distant groups which should be independent of Δ if the relative distance scale of the survey is proportional to true geometric distance.

Both tests confirm that the distance scale of the two surveys of groups is substantially correct.

(a) The distance moduli μ_1 derived from the van den Bergh $\mathscr{L}$ classes (with revised

TABLE II

Systematic and accidental errors of galaxy distance moduli ($\mu < 31.5$)

	$\mu_1(\mathscr{L})$	$\mu_2(C'^*S^*)$	μ_3(Groups)
Zero (mag.)[a]	$+0.033$	$+0.002$	-0.035
Mean error (mag.)	0.36	(0.72)[b]	0.15
Correlation	$\varrho_{12} = 0.794 \pm 0.061$	$\varrho_{23} = 0.802 \pm 0.075$	$\varrho_{13} = 0.907 \pm 0.030$
Mean difference	$\langle \mu_1 - \mu_2 \rangle = -0.026$	$\langle \mu_2 - \mu_3 \rangle = -0.031$	$\langle \mu_1 - \mu_3 \rangle = -0.074$
Standard deviation	$\sigma_{12} = 0.704$	$\sigma_{23} = 0.472$	$\sigma_{13} = 0.196$

[a] zero point correction to mean of 3 systems
[b] includes effect of systematic errors (Figure 3).

zero-point calibration to conform with the BGC magnitude system), μ_2 derived by Holmberg from corrected colors and average surface brightness, and μ_3 derived by the author and Corwin in the survey of groups, were intercompared 2 by 2 and analyzed for systematic and accidental errors (de Vaucouleurs and Brown, 1966).

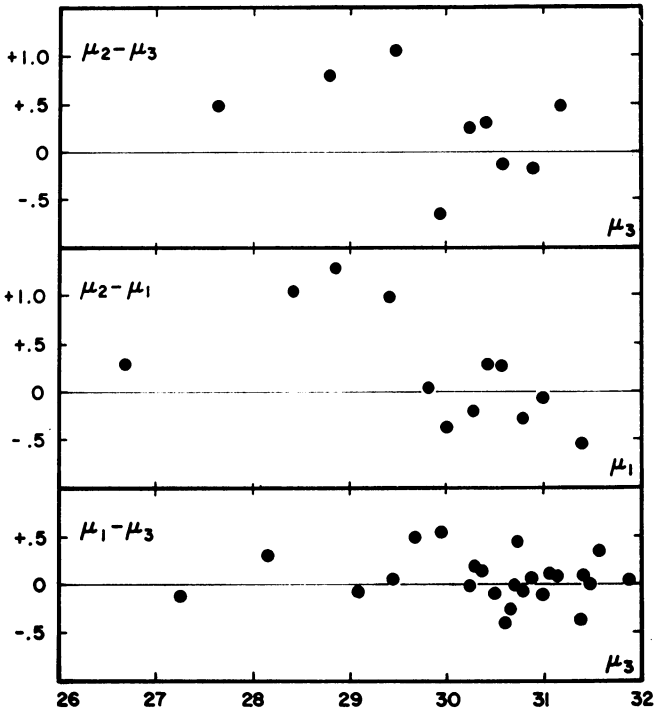

Fig. 3. Comparison of distance moduli from luminosity classification μ_1, color-surface brightness μ_2, and groups μ_3. Each point represents the mean of 10 galaxies.

The results are summarized in Table II and Figure 3 showing the average differences (mean points of 10 objects) $\Delta\mu_{ij}$ vs μ_j. The agreement is excellent between the luminosity and group distances over the whole range $27 < \mu < 32$, where $\mathscr{L}$ data exist.

A least-squares solution for all μ_1, $\mu_3 < 31.5$ (to avoid regression effects) gives

$$\mu_1 - \mu_3 = +\,0.043 - 0.056\,(\mu_3 - 30) \qquad (n = 254,\ \sigma = 0.80 \text{ mag.})$$

without rejection, and

$$\mu_1 - \mu_3 = +\,0.004 - 0.011\,(\mu_3 - 30) \qquad (n = 224,\ \sigma = 0.48 \text{ mag.})$$

after 2.5σ rejections. Both sets differ systematically from the μ_2's which have also larger accidental errors (Table II). All three sets, however, agree well in zero point in the range $29.5 < \mu < 31.5$.

(b) A detailed study was made of ring structures as distance indicators (de Vaucouleurs and Schultz, 1969) for all galaxies which had been assigned group membership in the survey of groups and for which ring diameters are given in the BGC. The average linear diameter of inner rings (r) is 4.1 kpc for lenticulars and 3.4 kpc for spirals, varying with type as shown in Table III. Within each type the average diameter does not vary much with stage (S0 → Sd) along the Hubble sequence, but individual values have a total range of 10 to 1 (corresponding to the 5-mag. range of the non-dwarf luminosity function).

Figure 4 shows the distribution as a function of distance Δ of the individual residuals

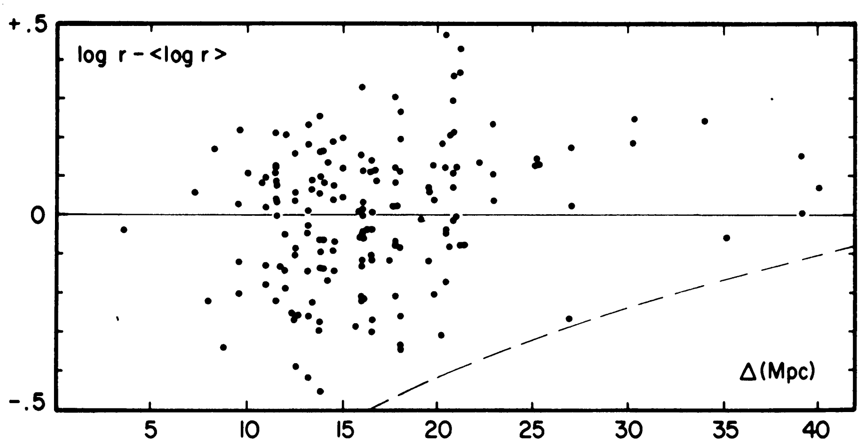

Fig. 4. Logarithmic residuals of ring diameters in galaxies vs distance from group membership. Dashed line shows observational cutoff ($\simeq 0.'25$).

from the mean values in Table III; except for the cutoff (dashed line) corresponding to the minimum angular diameter of measurable rings ($\simeq 0.'25$), there is no systematic dependence with distance over the range $4 < \Delta < 40$ Mpc. Similar graphs for each galaxy type separately confirm this result. A least-squares solution gives

$$\log(r) - \langle \log(r) \rangle = +\,0.0097\,\Delta - 0.160$$

without rejection, and

$$\log(r) - \langle \log(r) \rangle = +\,0.0080\,\Delta - 0.132$$

after 2σ rejection.

TABLE III

Diameters of ring structures in spiral and lenticular galaxies[a]

Type	SA(*r*)	SA(*rs*)	SAB(*r*)	SAB(*rs*)	SB(*rs*)	SB(*r*)	All
(*r*)	2.1	2.2	3.4	3.5	4.3	4.4	(3.4)
n	17	26	41	40	88	50	212
Type	LA(*r*)	–	LAB(*r*)	LAB(*rs*)	LB(*rs*)	LB(*r*)	All
(*r*)	4.0	–	3.9	(4.4)	3.9	4.5	4.1
n	16	–	9	2	8	13	48

[a] geometric means corresponding to $\langle \log(r) \rangle$, in kpc.

The maximum variation in the 5 to 25 Mpc range is at the most 0.2 (factor 1.6) in the sense that the average diameters of rings in the more distant groups appear to be *larger* than in the nearest groups. Taken at face value this effect suggests that the distances assigned to the far groups are *too large* relative to the near groups, which is *opposite* to the error which would have to be invoked to explain by a distance scale error the variation of H in Figure 2. However, as noted above, all or most of this effect can be accounted for by observational selection excluding the smaller rings in the more distant groups. The true scale error is certainly very much smaller.

Both tests (a) and (b) give no evidence for a systematic scale error in the group distances of the sign and magnitude required to account for the non-linear velocity-distance relation indicated by Figures 1 and 2. In both zero point and scale the adopted distance moduli appear to be systematically correct within a few tenths of a magnitude. We must, therefore, accept the parabolic red shift law of Figure 1 at face value and attempt to account for it in some other way than by the assumption of systematic errors in the distances.

4. Theoretical Velocity-Distance Relation in a Hierarchical Cosmological Model

A locally parabolic redshift law is predicted by at least two types of cosmological models:

(a) a homogeneous spherically symmetric static model with a negative pressure term such that $p + c^2 \varrho = 0$ (Hawkins, 1960, 1962a, b);

(b) an inhomogeneous hierarchical model in which local gravitation in condensed regions reduces the general expansion (Wertz, 1970).

Because of the growing evidence for hierarchical clustering and superclustering in galaxy counts (Kiang and Saslaw, 1969; de Vaucouleurs, 1970a), this is the favored interpretation. Wertz (1970) has derived the expansion law for a 'big bang' hierarchical model in which each unit of the hierarchy obeys the Newtonian equivalent of the Friedman equation

$$\frac{1}{2} V_\xi^2 - \frac{GM_\xi}{r_\xi} = E_\xi \tag{4.1}$$

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where ξ is the order of the cluster of mass M and radius r corresponding to the volume density $\varrho_v(r_\xi, t)=3M/4\pi r_\xi^3$. Inserting in Equation (4.1) the observed value $\theta=1.7$ of the exponent θ in the universal density-radius relation $\varrho_v(t=t_0)\propto r^{-\theta}$ (de Vaucouleurs, 1961; 1970a) and with the empirical values of the parameters $t_0=1.3\cdot 10^{10}$ yr and $\varrho_v=4.7\cdot 10^{-30}$ g cm^{-3} for $r_\xi=20$ Mpc, this model predicts the velocity-distance relation shown by the heavy line in Figure 5. The dashed line shows

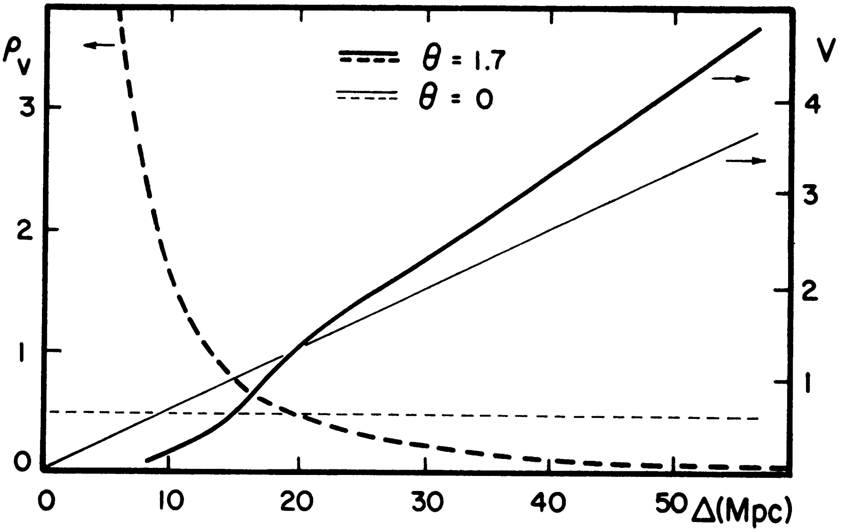

Fig. 5. Volume density ϱ_v (in 10^{-29} g cm^{-3}) and expansion velocity V (in 10^3 km s^{-1}) in 'big bang' hierarchical model for the observed value $\theta=1.7$ of the exponent of the universal density-radius relation ($\varrho\propto r^{-\theta}$) with $\varrho_v(r_\xi=20\text{ Mpc})=4.7\cdot 10^{-30}$ g cm^{-3} and $t_0=1.3\cdot 10^{10}$ yr, and in the corresponding homogeneous-isotropic model ($\theta=0$, $E\equiv 0$). (After Wertz, 1970.)

for comparison the corresponding linear Hubble law for the homogeneous-isotropic model ($\theta=0$, $E\equiv 0$). Note that, at least qualitatively, the curved part of the computed velocity-distance relation agrees with the observations of Figure 1. Detailed quantitative agreement should not be expected because of the drastic simplifications in the theoretical model (spherical symmetry, continuous representation, etc.) and, in particular the neglect of rotation and other motions in the Local Supercluster.

5. Kinematical Model of Local Supercluster

The local departures of the velocity field from linearity and isotropy are most simply and plausibly interpreted by the hypothesis of differential rotation and expansion in the Local Supercluster of galaxies (de Vaucouleurs, 1958). Critical discussions and additional velocities (de Vaucouleurs, 1964, 1966) have only strengthened the evidence in favor of this interpretation. For maximum simplicity the initial (1958) model was a thin disk model neglecting z motions and in which the rotation-expansion constants ω_1, ε_1 were not chosen to give the best fit to the data, but were merely good enough

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to illustrate the type of apparent motions predicted by such a model. The additional velocity data now available, especially in the southern hemisphere, and improved magnitudes, and absorption corrections allow a more rigorous analysis. As a first step (de Vaucouleurs and Peters, 1970) the rotation-expansion laws were assumed to be of the simple forms $\omega(R)=\omega_0 e^{-(R/R_1)^2}$ and $\varepsilon(R)=\varepsilon_\infty(1-e^{-R/R_1})$ with $\omega_1=\omega(R_1)=\omega_0/e$ and $\varepsilon_1=\varepsilon(R_1)=\varepsilon_\infty(1-1/e)$ as in the 1958 model because an attempt to derive empirically the forms of $\omega(R)$, $\varepsilon(R)$ would still be premature.

An expanding ellipsoidal system of finite thickness conserving its shape was adopted in which the expansion rate is $\varepsilon_z(\varrho)=\varepsilon_\infty(1-e^{-\varrho/R_{1z}})$ where $\varrho=\sqrt{(R^2+z^2)}$ and R_{1z} is the radius vector to the $\frac{1}{e}$ spheroid including the Sun ($a=b=R_1$, $c=0.2\ R_1$). Radial velocities were calculated for this model and trial values of $\omega_1 R_1$, $\varepsilon_1 R_1$ and $\dot{z}$ (the z velocity of the Local Group) at the actual centroids of the distribution of galaxies having known velocities in 24 approximately equal sky areas. Four intervals of magnitude $m=B(0)_c$ were considered and the distance scale was defined by the condition that $\langle m\rangle=11.8$ at $R_1=11.5$ Mpc. Finally, to minimize confusion by objects outside the Supercluster some solutions were restricted to galaxies with distances $\Delta<30$ Mpc.

The best-fit constants defined by the minimum rms deviation between observed and computed mean velocities in the 24 areas and 4 magnitude intervals are as follows (me's are estimated):

$$\begin{aligned} &\text{Rotation velocity at } R_1: && \omega_1 R_1 = 400\pm 50 \text{ km s}^{-1}\\ &\text{Expansion velocity at } R_1: && \varepsilon_1 R_1 = 1250\pm 50 \text{ km s}^{-1}\\ &Z \text{ motion of Local Group}: && \dot{z} = -250\pm 50 \text{ km s}^{-1}. \end{aligned}$$

Corresponding values for the 1958 model were 500, 1100 and 0. Stewart and Sciama (1967) derived $\omega_1 R_1=340\pm 120$ from a first-order analysis of 8 groups nearer than 8 Mpc.

The observed and computed supergalactic coordinates of the apex of the Local Group wrt galaxies in several magnitude intervals are compared in Figure 6. Considering the large uncertainty in the observed apices the agreement with the model positions for $\dot{z}=-200$ and $\dot{z}=-300$ is fairly good.

The algebraic mean O–C residuals $\langle\delta_V\rangle$ and rms deviation σ_v for the expanding ellipsoidal model using these constants are given in Table IV.

TABLE IV

$\langle m\rangle$	9.0	11.5	12.4	13.35
$\langle\delta_V\rangle$	−14	+52	+42	−96
σ_v	190	315	839	1136
n/N[a]	20/99	23/207	23/210	20/86

[a] n = number of regions, N = total number of galaxies.

Velocity-distances of groups and even individual galaxies calculated by means of the kinematical model of the Supercluster are distinctly better than those derived on

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the assumption of a linear-isotropic expansion law (de Vaucouleurs, 1967b; Rubin *et al.*, 1970).

6. Solar Motion with Respect to Galaxies Brighter than $m \simeq 14$

An analysis of the solar motion wrt about 800 bright galaxies having corrected magnitudes $B(0)_c \lesssim 14$ and 109 nearby groups (de Vaucouleurs and Peters, 1968, 1970) leads to the following conclusions:

(1) *Local Group:* (a) Assuming that the group is not expanding, the Sun's motion wrt the velocity centroid (not center of mass) of the Local Group is $V_S = 315 \pm 15$ (me) km s^{-1}, toward $l = 95° \pm 6°$, $b = -8° \pm 3°$ ($n = 13$ excluding IC 342, $\sigma = 57$ km s^{-1}). Correcting for the solar motion toward the classical apex (20 km s^{-1} toward 18^h, +30°) and the current IAU value of galactic rotation (250 km s^{-1} toward 90°, 0°) gives for the motion of the galactic center $V_G = 80 \pm 19$ toward 130° ± 24°,

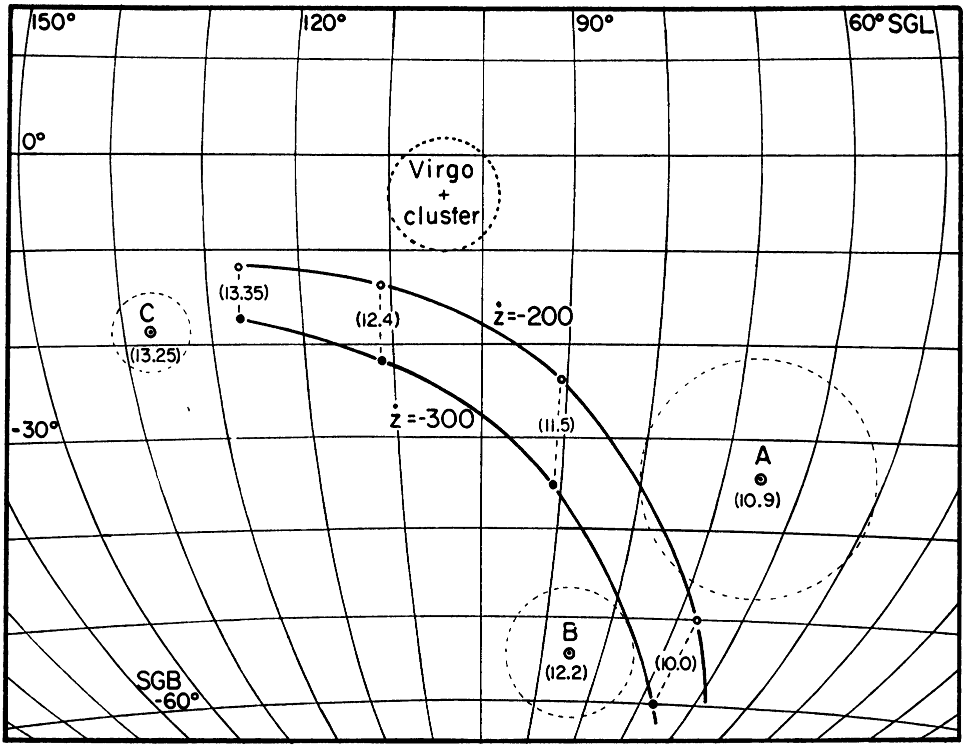

Fig. 6. Observed and computed supergalactic coordinates of apex of Local Group with respect to galaxies in several magnitude intervals. Observed points *A*, *B*, *C* with estimated mean errors (dashed circles) for magnitude intervals $10.0 < m < 11.8$ (A, $\bar{m} = 10.9$), $11.8 < m < 12.6$ (B, $\bar{m} = 12.2$), $12.6 < m < 13.9$ (C, $\bar{m} = 13.25$); each is mean apex of 4 samples of 40 galaxies each. Computed points for 2 values of $\dot{z}$ and at 4 mean magnitudes (in parenthesis) are calculated for best-fit ellipsoidal model of Local Supercluster ($\omega_1 R_1 = 400$, $\varepsilon_1 R_1 = 1250$). Location of Virgo cluster marks general direction of supergalactic center ($L_0 = 104°$).

$-42° \pm 13°$; (b) Allowing for a possible expansion of the group, i.e. including a K term proportional to distance $K = K_1 \Delta$, gives $V_S = 342 \pm 37$ toward $97° \pm 11°$, $-15° \pm 7°$ with $K_1 = 85 \pm 37$ km s^{-1} Mpc^{-1} ($n = 14$, including IC 342, $\sigma = 77$). If IC 342 is a member of the Local Group (Ables, 1968), the group is expanding, although not as fast as the surrounding field.

(2) *Nearby galaxies:* When progressively fainter galaxies are used to define the frame of reference the apices of the Sun, Galaxy and Local Group drift rapidly across the northern galactic hemisphere as shown by Figures 6 and 7; this effect reflects the

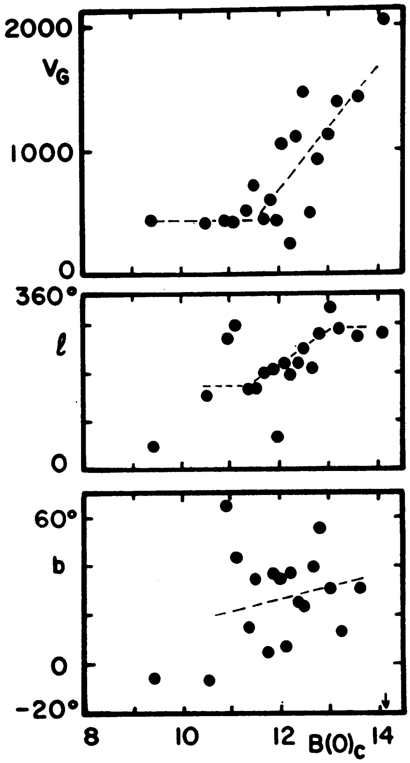

Fig. 7. Velocity V_G and galactic coordinates of apex of the peculiar motion of the Galaxy with respect to galaxies outside the Local Group vs corrected apparent magnitude. Points represent solutions for successive samples of 40 galaxies each. Note systematic drift of apex.

large-scale systematic departures from the ideal linear-isotropic Hubble expansion. An interpretation of this phenomenon in terms of the rotating-expanding model of the Local Supercluster is given in Section 5. The velocity-magnitude relation given by the constant term K_m in $V(m \pm \delta m) = K_m + V_s \cos A$ is linear (Figure 8) with slope 0.2. This result proves that apart from the local departures due to the kinematics of the Local Supercluster the underlying Hubble flow is indeed linear and isotropic within the errors of the data.

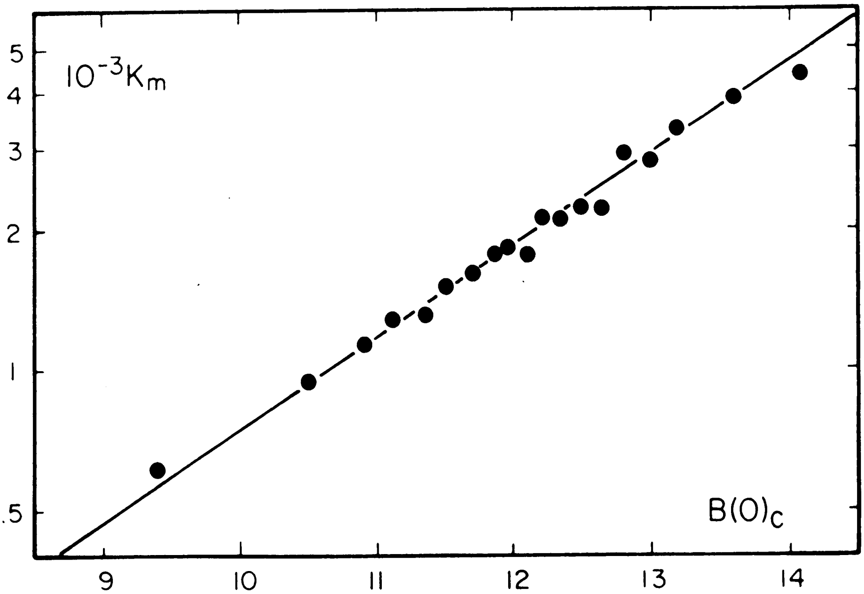

Fig. 8. Relation between corrected magnitude $B(0)_c$ and mean systematic velocity K_m corrected for solar motion. Points represent solutions from equal samples of 40 galaxies each as in Figure 6.

7. Solar Motion and Hubble Constant from Nearby Groups

Several estimates of the Hubble constant may be derived from the distances of the survey of groups:

(1) A solar motion solution with a linear K term $V = H\Delta + V_s \cos A$ with respect to 109 groups ($2 < \Delta < 75$ Mpc, $\bar{\Delta} = 19$ Mpc) gives $H = 133 \pm 3$ km s^{-1} Mpc^{-1} with $V_s = 382 \pm 104$ toward $206° \pm 29°$, $+49° \pm 20°$ ($\sigma = 649$). Restricting the solution to the nearest 52 groups ($2.5 < \Delta < 17.5$ Mpc, $\bar{\Delta} = 12.7$ Mpc) whose distances are perhaps best determined gives $H = 113 \pm 4$ with $V_s = 409 \pm 99$ toward $177° \pm 18°$, $+26° \pm 11°$ (Table V).

TABLE V

Solutions for solar motion

Reference	n	V_s	l_s	b_s	K_1, H	σ
Local Group	13	315 ± 15	95 ± 6	-8 ± 3	(0)[a]	57
	14	342 ± 37	97 ± 11	-15 ± 7	85 ± 37	77
Nearby Groups[b]	109	382 ± 104	206 ± 29	$+49 \pm 20$	133 ± 3	649
Nearest Groups[c]	52	409 ± 99	177 ± 18	$+26 \pm 11$	113 ± 4	–
Galaxies[d]	203	299 ± 45	171 ± 13	$+26 \pm 6$	109 ± 2	271

[a] assumed; K, H in km s^{-1} Mpc^{-1}, σ in km s^{-1}.
[b] $2 < \Delta < 75$ Mpc, $\bar{\Delta} = 19$ Mpc.
[c] $2.5 < \Delta < 17.5$ Mpc, $\bar{\Delta} = 12.7$ Mpc.
[d] distances from group membership (excluding Local Group).

(2) A solution for 203 galaxies treated individually but using distances from their group membership assignments gives similar results: $H=109\pm2$, with $V_s=299\pm45$ toward $171°\pm13°$, $+26°\pm6°$ ($\sigma=271$).

The value of the Hubble constant consistent with the distance scale used in the survey of groups

$$H_0=110\pm3 \text{ (m.e. exclusive of zero point)}$$

is only slightly higher than the value $H_1=95$ $(+15, -12)$ recently derived by van den Bergh (1970) without allowance for the effects of local departures, but it is much higher than the values $50\leqslant H_2\leqslant75$ derived by Sandage (1968), Abell and Eastmond (1968), Heidmann (1969) and de Vaucouleurs (1970b) from various considerations of Virgo cluster objects. The discrepancy cannot be attributed to significant differences in the adopted primary Local Group standards which agree within ±0.1 mag., nor in the magnitude systems which generally agree within ±0.2 mag. or in the treatment of the relatively minor corrections for internal and galactic absorption.

The simplest way out would be to demonstrate that (outside the Local Group) the distance moduli used in the survey of groups have a significant zero point error (but not a scale error, Section 3). Table VI compares recent estimates of the distance

TABLE VI

Distance modulus of Virgo cluster

Source	Method	μ_B	μ_0
Sandage, 1958	Brightest stars in M100	30.8	(30.55)[a]
Sandage, 1962	Largest H II region in M100	30.7	(30.45)[a]
Holmberg, 1964	Corrected color-surface brightness of spirals	30.76	30.5
de Vaucouleurs, 1965	Brightest and largest members of groups	30.5	30.3
Sandage, 1968	Brightest globular clusters in M87	31.1	30.85
de Vaucouleurs, 1970b	Brightest globular clusters in M87	31.5	31.25
Heidmann *et al.*, 1971	Diameter-luminosity relation for spirals	(30.95)[a]	30.7
	Mean of all methods	30.9	30.65

[a] for $A_B=0.25$ mag.

modulus of the Virgo cluster which has the largest number of determinations and is near the average distance of nearby groups. If the mean modulus $\langle m-M\rangle_0=30.65$ is taken as the best current estimate, the distance scale of the survey of groups requires a correction factor $\times1.17$ and the corresponding value of H is H_0 (corr.)$=94$ km s^{-1} Mpc^{-1}.

On the other hand it may be that the brightest globular cluster criterion is subject to systematic error, either in the measurement of faint apparent magnitudes or in the basic assumption that cluster luminosities have the same upper limit in distant ellipticals and in the nearby spirals used for calibration. If the 2 determinations by this method in Table VI are rejected, $\langle m-M\rangle_0=30.5$ and $H_0=100$ km s^{-1} Mpc^{-1}.

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