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Inverse subsemigroups of free inverse semigroups

Peter R. Jones

This thesis is concerned with the properties of inverse subsemigroups of inverse semigroups, from various classes: the unifying feature is that all these classes contain the free inverse semigroups I_X for every X. Here, for simplicity, we just state our main results in this special case.

The most important result (Corollary 2.2.5: the "Basis Theorem") is that free inverse semigroups have the "strong basis property", which we now describe. Let U and S be a pair of inverse subsemigroups of I_X (for some set X), with $U \subseteq S$. Suppose A and B are both *U-bases* for S, that is, subsets of I_X minimal with respect to the property that, together with U, they generate S. *Then* A *and* B *have the same cardinality.*

By proving most of our results for this general situation (of U-bases), we can obtain considerable information about the special case of most interest: when U is empty. For example (Corollary 2.3.4), any two *bases* (minimal, or "irredundant" generating sets) for an inverse subsemigroup of I_X in fact have the same (finite) number of elements *in any given* $\mathcal{J}$*-class of* I_X.

By analysis of the $\mathcal{J}$-structure of I_X, it is shown (Corollary 3.1.6) that every inverse subsemigroup of I_X does indeed have a basis. From

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these two results, a series of "change of basis" theorems is developed.

In the final chapter, we take a rather different approach, by considering inverse subsemigroups of *reduced* inverse semigroups. (An inverse semigroup is reduced if $R \cap \sigma$ is the identity relation, where σ is the minimum group congruence.) McAlister in [3] has shown that every reduced inverse semigroup can be represented as a P-semigroup $P(G, X, Y)$ for some group G, semilattice Y and poset X. (In particular McAlister and McFadden in [4] showed that I_X is reduced for every X; their representation of I_X as a P-semigroup is described in Chapter 1.)

We show that an inverse subsemigroup of a P-semigroup $P(G, X, Y)$ is determined by a subgroup of G, a subsemilattice of Y and a pair (X', θ) consisting of a poset X' and a mapping θ of X' into X satisfying certain simple conditions. Those inverse subsemigroups (the "weakly unitary" ones) for which X' can be chosen as a *subposet* of X are found. This leads to a construction for the congruences on a P-semigroup.

We note, finally, that the material in Chapters 2 and 3 (approximately) is to appear in the author's paper [1]. The material in Chapter 5 has been submitted for publication, [2].

References

[1] Peter R. Jones, "A basis theorem for free inverse semigroups", *J. Algebra* (to appear).

[2] Peter R. Jones, "The lattice of inverse subsemigroups of a reduced inverse semigroup", submitted.

[3] D.B. McAlister, "Groups, semilattices and inverse semigroups. II", *Trans. Amer. Math. Soc.* 196 (1974), 351-370.

[4] D.B. McAlister and R. McFadden, "Zig-zag representations and inverse semigroups", *J. Algebra* (to appear).