There is never time to mention everything in this series of compilations, so I will finish by mentioning a number of puzzles. John Conway and others pose ‘a headache-causing problem’ in game theory. Stan Wagon asks what direction will a stationary bicycle move if you pull back the pedal, and in what direction does the lower pedal move relative to the floor. Jacob Siehler discusses conditions under which it is possible to tricolour a pyramid of hexagonal cells, under certain rules.

The inclusion of Mike Askew's address made me wonder how often Mircea Pitici has managed to include something from the journal in his compendia. The answer is that only one article, that by John Conway and Alex Ryba in 2015, has made it into the selection. There is also a list of material which was considered by the Editor not chosen, and again the *Gazette* is, I think, under-represented, with only two mentions. Obviously the focus in this publication is going to be on the USA, but I cannot help thinking that there is plenty of excellent material in our journal which is never considered.

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What's the use? by Ian Stewart, pp 336, £20 (hard), ISBN 978-1-54169-949-6, Basic Books (2021)

Thinking better: the art of the shortcut in math and life by Marcus du Sautoy, pp 336, £20, ISBN 978-1-54160-036-2, Basic Books (2021)

Both of these books appear to belong in the category of maths for the general reader, by established and well-known authors in this field, which is why I decided to read both together. I did this for the first few chapters of each, at which point it became clear that they had very different flavours.

The theme of Stewart's book is Eugene Wigner's famous comments about “the unreasonable effectiveness of mathematics in the natural sciences”, though the book widens the scope far beyond the natural sciences. Stewart's distinction between reasonable and unreasonable is that it is no surprise that the equations of aerodynamics are useful in aircraft design, precisely because the equations were developed with air flow over obstacles in mind. So this spin-off is entirely reasonable. By contrast think of graph theory, invented in the eighteenth century as a clever way of solving a little recreational puzzle, but in later centuries (including the present one) finding totally unexpected applications to a wide range of topics, some of which did not exist when graph theory was invented. It is in this sense that Stewart uses ‘unreasonable’. He concentrates on how mathematics can lead us far outside anything we may have associated with it.

Du Sautoy on the other hand concentrates more directly on clever shortcuts *within* mathematics—how, for example, a task which looks computationally or logically daunting can be reduced to manageable proportions by a clever trick. Of course it is only a trick when you see it for the first time; after seeing it used in many different ways you consider it more as a standard technique.

Stewart's second chapter is ‘How politicians pick their voters’ and is largely about how to decide, fairly or otherwise, on the boundaries of electoral regions. It involves the mathematics of shape to explain the anomaly of a party achieving a large proportion of the vote but only a small proportion of elected representatives, the concept of the wasted vote, and the legal minefield of proving a boundary unfair, with reference to Markov chains and Arrow's impossibility theorem. All fascinating, but all purely descriptive; there is no encouragement for readers to do their own experiments, and even if there were, that reader would have to be already a fairly sophisticated mathematician.

Du Sautoy's first chapter is about how spotting patterns, mainly in numerical sequences, can give real insight and lead us to seek, explain and use further patterns. The story of the young Gauss's method for evaluating $1 + 2 + \ldots + 100$ in a flash is followed by the connection between the Fibonacci numbers, powers of 2 and sizes of subsets, all hidden in the entries of Pascal's triangle. The reader with at most A-level familiarity will be able to engage actively with this, suggesting to me that du Sautoy's book would be a more satisfying read. This impression was confirmed as I read on; about half of his chapters are A-level accessible, but virtually none of Stewart's.

Stewart's chapter 4 is about the bridges of Königsberg and how graph theory led to a vast array of applications. The one on which he concentrates is cycles of kidney transplants. This is followed by a chapter on security of encryption systems, concentrating on post-1970s developments in pubic key systems from RSA to elliptic curves, finite fields and the threats that quantum computing may open up. But the treatment is so much of a whirlwind that no reader who is not already an expert will be able to extract much real understanding from it. The rest will be able only to gaze in wonder at the things mathematics can now do, but as for how it does it, that remains hidden.

Another chapter starts with the travelling salesman problem: its origin and history, its significance, its spin-offs, and what counts as an approximate solution. This leads to discussion of algorithms with polynomial running times and the set P of problems for which there is such an algorithm. Suppose there is a problem, not known to be in P, for which someone claims a solution. You then need some algorithm to check whether the solution lives up to the claim. If this algorithm runs in polynomial time the problem is put in the class NP, standing for nondeterministic polynomial, though Stewart does not explain why 'nondeterministic' features in the name. Since both intuitively and as a matter of experience checking that a proposed solution is in fact a solution is much easier than finding a solution. It is entirely reasonable to believe that $P = NP$. An example familiar for cryptography is that checking that x is a factor of y is, if the numbers are big, vastly easier than finding x in the first place. But, as Stewart says, no one has a clue as to how to prove or disprove that $P = NP$. The section ends with a paragraph on NP-complete and NP-hard problems, though the explanation of these terms is not very clear. From that we move on to the fact that modern computing power has meant that methods have been devised for tackling big examples of problems possibly in the non-P category. For example, solutions were found to the travelling salesman problem with 318 towns in 1980, and this record had risen to 33810 by 2005. The practical spin-off is that these methods, when applied to smaller problems, find a solution very quickly.

Another way forward is to find solutions, not necessarily optimal but guaranteed to be close to optimal. Here we join the 'unreasonable effectiveness' theme with an account of the tumultuous changes in mathematics, and mathematicians, between about 1880 and 1920, with the logical analysis of infinity, space filling curves, dimension, the nature of curves and surfaces, and, later, changing the way in which mathematical proof is expressed so that automatic checking by computer is achievable.

Other topics covered by Stewart, in the same broad-brush way, include: computer animation via the complex plane and quaternions; chaos theory and fuzzy logic in spring manufacture; transform theory in X-rays and CT scans; and spaces with dimensions in the millions—for example, spaces of bit strings used for computer recognition of images relevant to self-driving cars. The ease with which computers can be fooled provides a powerful argument that the rush to embrace self-driving cars is dangerous.

How does this compare with du Sautoy's next sections? One is about short-cuts in calculation—the sheer power of things we take for granted: the place-value system, the invention of a symbol for zero, algebraic notation and manipulation, logarithms, binary numerals, imaginary numbers. The work of early pioneers in complex numbers can be summed up in the advice "break the rules (all squares are $\geqslant 0$) and see where it leads".

One is on the theme of mathematics as a language or languages, and the power of translating from one language to another. For example, a game described in the language of numbers may appear difficult to analyse, but when translated into geometric language it can become easier, or be recognised as an already familiar game. Even the understanding of multi-dimensional hypercubes yields to this device.

Another chapter is the geometric short cut. Here we have ancient and more recent surveying of the earth, Mercator's projection, trigonometry, astronomical measurement, triangulation, standardisation of the metre, geodesics, and worm holes in space-time. Diagrams have a chapter of their own: display of statistical information, the London underground map, structures of molecules, the Feynman diagram, and even the humble Venn diagram.

There is some overlap with Stewart, notably in the Königsberg bridges and their graph-theoretic spin-offs, and a final chapter, 'The impossible short-cut', which refers to the *P* vs *NP* problem.

Between each chapter and the next, du Sautoy has what he calls pit stops—about half a dozen pages devoted to non-mathematical topics which illustrate the short-cuts in a much wider context. They are often incidents and experiences from his own life.

Both authors write very entertainingly and both books provide an A-level student with inspirational reasons for going on to study maths at a higher level. They can also provide, for those already at undergraduate level, ideas for going on to the next level and for embarking on research.

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A prelude to quantum field theory by John Donoghue and Lorenzo Sorbo, pp. 160, £25 (paper), ISBN 978-0-69122-348-3, Princeton University Press (2022)

What is a quantum field theory? – a first introduction for mathematicians by Michel Talagrand, pp. 741, £69.99 (hard), ISBN 978-1-31651-027-8, Cambridge University Press (2022)

What is quantum field theory? Wikipedia puts it succinctly 'QFT is a theoretical framework that combines classical field theory, special relativity, and quantum mechanics.' More specifically it is what you get when you try to explain the interactions of elementary particles.

Let's agree to call the book by Donoghue and Sorbo 'The Prelude' and the book by Talagrand 'The Introduction'. The two books could hardly be more different.

The Prelude is short, written by physicists, and is for readers who already have a solid foundation in quantum mechanics (QM). It is intended to fill the 'gaps in the pedagogic literature' and so help students transition from QM to QFT.

The Introduction is long, written by a mathematician and, as the subtitle promises, is intended for mathematicians. On reaching his 60th birthday Michel Talagrand decided he had