ON THE TASSOUL APPROXIMATION SCHEME FOR DETERMINING THE STRUCTURE OF ROTATING STARS

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The method used by Tassoul and Tassoul assumes a "turbulent viscosity" which is important in determining the dynamics, but is unimportant in heat transport. This approximation is inconsistent, the argument is as follows:
The rotation is expanded about uniform rotation in a series.

$$\Omega = \Omega_0 + \Omega_1 + \ldots, \text{ where } \Omega_1/\Omega_0 << 1 \tag{1}$$

The leading term Ω_0 drives a circulation, the Eddington Sweet circulation with velocity

$$V_{circ} \simeq \varepsilon V_{thermal} = \varepsilon \frac{\kappa}{R} \tag{2}$$

where κ is the thermometric conductivity, R the radius of the star, and ε the ratio of the kinetic energy of rotation to the gravitational energy (or equivalently the ratio of centrifugal force to gravity).
The toroidal component of the equation of motion requires that the advection of angular momentum be balanced by the viscosity acting on the differential rotation Ω_1

$$v_{circ}\, \Omega_0 R \simeq \nu\Omega_1 \tag{3}$$

hence

$$\frac{\Omega_1}{\Omega_0} \simeq \frac{V_{circ}R}{\nu} = \frac{\varepsilon\kappa}{\nu} = R_e \tag{4}$$

where R_e is the Reynolds number of the circulation. Thus the approximation scheme is actually an expansion in the Reynolds number which must therefore be small.

$$R_e = \frac{\varepsilon\kappa}{\nu} << 1 \tag{5}$$

A. Maeder and A. Renzini (eds.), Observational Tests of the Stellar Evolution Theory, 517–518.

If now it is assumed - as is done by Tassoul and Tassoul that $\Omega_1/\Omega_0 = 0(\varepsilon)$, then it follows that $\kappa/\nu \sim 1$.
But $\kappa\nabla T$ is the energy flux in the form of radiation, $\nu\nabla T$ is the energy flux carried by the turbulence, hence with $\kappa \sim \nu$ it follows that the energy transport by the turbulence has a major effect on the structure of the star. Neglecting this turbulent flux is incorrect.
However it does not follow that $\Omega_1/\Omega_0 = 0(\varepsilon)$. It is in fact $0(\varepsilon/\sigma)$ where $\sigma = \nu/\kappa$ is the Prandtl number. Condition (5) must be satisfied for an expansion procedure of this form to be self consistent, hence

$$\frac{\nu}{\kappa} >> \varepsilon \tag{6}$$

Consequently the turbulent energy flux $\nu\nabla T$ is always greater than $\varepsilon\kappa\nabla T$, that is the variation of the radiative flux that drives the circulation. Now the turbulent viscosity ν is assumed to be produced by the instabilities of the differential rotation. The differential rotation Ω_1 varies from one part of the star to another, and in particular with latitutde. We must therefore expect that ν varies by a factor of order 2 over a spherical surface. In this case the variation of the turbulent energy flux is larger than the variation of the radiative flux that drives the circulation. To neglect the turbulent energy flux is therefore inconsistent.