ON THE ELLIPTICITY OF THE CORE-MANTLE BOUNDARY FROM EARTH NUTATIONS AND GRAVITY

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ABSTRACT. A change in the core free nutation period from its hydrostatic value of 466d to about 433d has been inferred from analysis of both earth's annual nutation amplitude from Polaris VLBI data (Herring et al., 1986) and surface gravity in the diurnal tidal band (Zürn et al., 1986; Neuberg et al., 1986). Gwinn et al. (1986) interpret this shift as due to an excess nonhydrostatic ellipticity e_d, equivalent to a change in the equatorial minus polar radius of 0.5 km. In this paper, the effect of a layer at the core-mantle boundary (CMB) on σ_f (and hence e_d) is examined. Although the potential effect of this layer on σ_f is found to be large, constraints imposed from gravimetry limit changes in e_d to 10%. In addition, the annual nutation has a significant out-of-phase component. Mantle solid friction accounts for a large fraction of the out-of-phase nutation.

1. INTRODUCTION

The forced nutation of the fluid core is sensitive to the difference in free core nutation frequency σ_f and forcing frequency σ. The spaced-fixed value of σ_f depends on earth rotation rate Ω, equatorial moments of inertia of whole earth A and fluid core A_f and core ellipticity e_f. The approximate equation is (Sasao et al., 1980)

$$\sigma_f = - \Omega \frac{A}{A_f} \left[e_f - \beta\right]. \tag{1}$$

The parameter β corrects σ_f for deformation of the core-mantle boundary. This deformation is caused by the pressure exerted by the core as it nutates relative to the mantle. The 1066B model predicts that the hydrostatic value of $e_f = 2.54 \times 10^{-3}$, $\beta = 6.17 \times 10^{-4}$ and a free nutation period of 460d.

The variation of a parameter x (e.g. surface gravity or mantle nutation) in response to differential core nutation is described by the following formula.

A. K. Babcock and G. A. Wilkins (eds.), The Earth's Rotation and Reference Frames for Geodesy and Geodynamics, 317–322.

$$x = x_o + x_1 \frac{\sigma_f}{\sigma - \sigma_f} \tag{2}$$

Herring et al (1986) infer from analysis of the annual nutation that σ_f is complex: They account for this change in the real part by increasing e_f. Presumably this excess ellipticity e_d (where $e_f = e_h + e_d$) is dynamically supported by mantle convection. Hydromagnetic and viscous friction at the CMB appear to be too weak to account for the imaginary part (Gwinn et al., 1986). An alternative is solid friction in the mantle. This can be modeled by replacing β with β $(1 + i/Q_c)$.

2. CORE-MANTLE BOUNDARY LAYER

Suppose there is a thin, possibly viscous boundary layer at the base of the mantle with thickness d and density ρ_ℓ. The layer density shall be constrained to lie between the density at the base of the mantle (5.563) and the top of the core (9.977). If the core nutates with respect to the mantle, the layer may follow the mantle or core depending on its viscosity η. Clearly, the layer will be glued to the mantle if its Ekman layer thickness $(\eta/\Omega)^{1/2}$ is large compared to d. On the other hand η can be chosen to be sufficiently small such that shear within the layer does not inhibit its deformation. Also, d cannot be too small, otherwise currents generated by periodic deformation would contribute to the stress at b_o. A d $\gtrsim$ 1 km satisfies the last condition for both tides and core nutation. In this study, it shall be assumed that that layer's lower boundary is determined from a balance of rotational and gravity forces. Also, d is small enough that changes in gravitational potential across the layer are negligible.

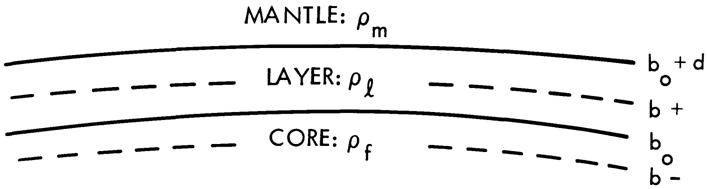

Figure 1. Boundary layer at base of mantle between radius b_o and b_o + d; b- and b+ are equilibrium surfaces. b_o is also an equilibrium surface for body deformations caused by tides, surface load and core nutation where the layer also nutates.

The elastic equations (e.g. Chinnery, 1975) have been solved for second harmonic tidal deformation and core nutation with the added layer. Obviously, if the layer follows the core's nutation, then the effect of the layer on deformation should vanish in the limit ρ_ℓ equals core density, as shown in the first line of Table 1.

ρ_ℓ	Core Nutation $\beta \times 10^4$	$\delta_1 \times 10^2$	Static Tides δ_o	h_o
(layer nutates with core)				
9.977	6.17	3.20	1.1575	0.6093
9.728	6.25	3.10	1.1580	0.6099
5.563	8.36	0.29	1.1712	0.6253
(layer is glued to mantle)				
5.563	23.1	-5.65	1.1712	0.6253

Table 1: Solutions for tidal displacement η_o, gravimetric factor δ_o $(1 + h_o - 3/2\ k_o)$, core nutation β and δ_1 $(h_1 - 3/2\ k_1)$ factors.

The layer has a strong effect on β and is increased in every example studied. If such a layer exists, then the dynamic ellipticity inferred from VLBI must be correspondingly increased. However, the observed gravimetric correction δ_1 for core nutation agrees with the expected value to within $\pm\ 2 \times 10^{-3}$ (Neuberg et al., 1986). This would limit the increase in e_d to less than 2×10^{-5} for the case where the layer nutates with the core. This is equivalent to an equatorial-polar radius change of only 60 m. Observed values of h_o could also be used to limit the boost in e_d, but this constraint is much weaker. It would seem that the case where the layer follows the mantle is completely ruled out because of the incompatible value obtained for δ_1. Allowing layer viscosity to inhibit deformation at b_o may slightly alter this conclusion. Still it is unlikely this change will significantly affect the bound on e_d. However, it opens the possibility that shear in the layer might be a significant source of dissipation.

3. SOLID FRICTION

The local dissipation rate at a point r in a body is proportional to the stress energy. If only shear contributes to dissipation, then the appropriate expression for a periodic deformation is

$$\frac{dE}{dt} = \sigma\, Q^{-1}(r, \sigma)\, E_\mu\,(r, \sigma, Q) \qquad (3)$$

where μ is the rigidity and

$$E_\mu = \frac{1}{2}\,\mu \sum \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad (4)$$

In the tidal band, E_μ is nearly independent of frequency and $Q(r,\sigma)$ can be calculated using the elastic structure parameters. For a Maxwell body, $Q = (1 + x^2)/x$ where $x = \rho\eta\sigma/\mu$. More complex models have been proposed in which dissipation observed at seismic frequencies is extended to the tidal band. Instead, we assume that the structure and frequency dependence of Q (r, σ) is unconstrained. A bound on core deformation mean Q_c is obtained through a comparison of estimates of the solid tide mean Q_t observed for the diurnal and semi-diurnal band.

First define a dimensionless stress energy function W(r) in which E_μ has been averaged over the surface of radius r.

$$W = a \frac{\int_S dS \ E_\mu}{\int_V dV \ E_\mu} \tag{5}$$

The mean $\overline{Q}$ averaged over the mantle is therefore

$$\overline{Q}^{-1}(\sigma) = a^{-1} \int_b^a dr \ W(r) \ Q^{-1}(r,\sigma). \tag{6}$$

The best estimate of the mean tidal Q_t comes from comparison of the observed tidal gravity field obtained from analysis of tidal perturbations in Lageos' orbit (Christodoulidis et al., 1986) with contributions predicted from ocean tidal models of Parke (Yoder, 1982) and Schwiderski (Melbourne et al., 1983). A lower bound of about 60 with mean of ~100 for Q_t can be deduced from this kind of analysis. An

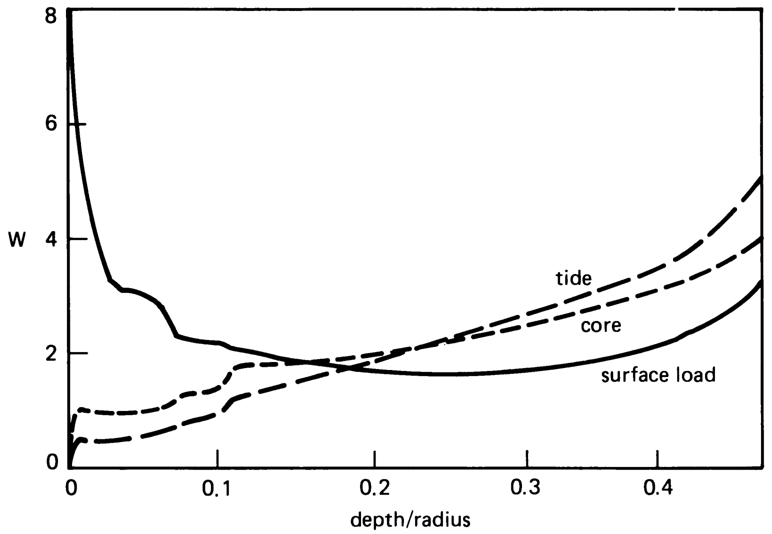

Figure 2: Stress energy distribution functions W.

upper bound is more difficult to establish, given uncertainties in the ocean models. Possibly the best estimate is to appeal to the observed Q for Mars inferred from the tidal acceleration of Phobos' orbit. Duxbury and Callahan (1981) find Mars' $Q \approx 90$. It seems implausible that earth's Q_t could be much larger than Mars' Q_t: adopt $60 \leq Q_t \leq 120$ as a reasonable range. The mean Q_c due to core deformation could be different if the local dissipation has structure and the weighting function W for earth tides and core deformation are dissimilar.

The W functions appropriate to the following second harmonic deformations have been calculated using the 1066B model: 1) surface load, 2) tidal deformation and 3) core nutation. Obviously, the surface load W function is conspicuously different in that stress energy is concentrated in the upper mantle. Tides and core nutation W functions are largest at the core-mantle boundary and have similar distributions.

In order to demonstrate the effect of W on $\overline{Q}$, consider the case where Q(r) is constant in either the upper third or lower third of the mantle and infinite elsewhere. If Q(r) is adjusted so that the tidal $Q_t = 100$, the $\overline{Q}$ obtained for the other types of deformation are shown below.

	$\overline{Q}$	
Type	Upper Mantle	Lower Mantle
tidal	100	100
core	65	115
surface	29	163

We conclude that Q_c is within 30% of the tidal Q_t. From the bounds established for Q_t, we find $40 \leq Q_c \leq 140$. Thus the expected out-of-phase component of the annual signature from solid friction is 0.08 to 0.26 mas compared to the observed value of 0.2 to 0.3 mas, (Himwich and Harder, 1988). Wahr and Bergen (1986) have obtained similar results for the effect of solid friction.

4. SUMMARY

Solid friction may account for a significant fraction, if not all of the observed phase shift in the annual nutation. A more precise estimate of Q_c depends in part on narrowing the bounds on the tidal Q_t. Introduction of a layer at the CMB does significantly change core deformation β and gravimetric δ_1 factors. However, constraints on δ_1 (Neuberg et al., 1986) restrict changes in β to less than $\cong 2 \times 10^{-5}$. The interpretation that the shift in core free nutation period to ~433d is caused by a non-hydrostatic $e_d \cong 1.2 \times 10^{-4}$ seems secure. However, the nutation equations are presently being critically reexamined to determine if any important physical mechanism has been inadvertently omitted which might affect estimates of e_d.

5. ACKNOWLEDGEMENT

The research in this publication was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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DISCUSSION

Dickey: You suggested a model with an extra “layer”. Can this be supported by seismic tomography?

Reply by Yoder: I strongly doubt it. In any case, the point of introducing the hypothetical layer was to demonstrate that mechanisms which alter the CMB ellipticity, inferred from nutations, also are likely to change some other observable, in this case, surface gravity.