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COHOMOLOGY OF FILIFORM LIE ALGEBRAS OVER FIELDS OF CHARACTERISTIC TWO

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My thesis focuses on the cohomology of filiform Lie algebras over a field of characteristic two. It is surprising how little we know about these algebras, although they are 'the most structured subclass of nilpotent Lie algebras' as described in [1].

In the first chapter, we start by giving basic definitions. In more detail, we examine a theorem, due to Michèle Vergne [5], which gives a description of the structure of filiform Lie algebras over a field of characteristic zero. We observe that this theorem is no longer true for filiform Lie algebras over a field of characteristic two and we give examples on that. Then we define two families of Lie algebras over a field of characteristic two, Lie algebras of Vergne and of suspended Vergne type. We make some remarks on the structure of such algebras and we lay the groundwork for the study of their cohomology.

In Figure 1 we see all Lie algebras of Vergne type up to dimension 12. The edges of this graph connect two algebras where the algebra on the lower end is a central extension of the algebra on the upper end. The key point is that we observe some symmetry: the left-hand side of this graph is the same as the right-hand side.

This observation is further explored in the second and last chapter of the thesis, where we introduce the cohomology with trivial coefficients of Lie algebras over a field of characteristic two. This chapter contains most of our original results and is divided into two parts. In the first half we study the cohomology of finite Lie algebras of Vergne type over a field of characteristic two. First, we obtain Theorem 1 which states that the Lie algebras $\mathfrak{m}_0(n)$ and $\mathfrak{m}_2(n)$ have the same Betti numbers over fields of characteristic two.

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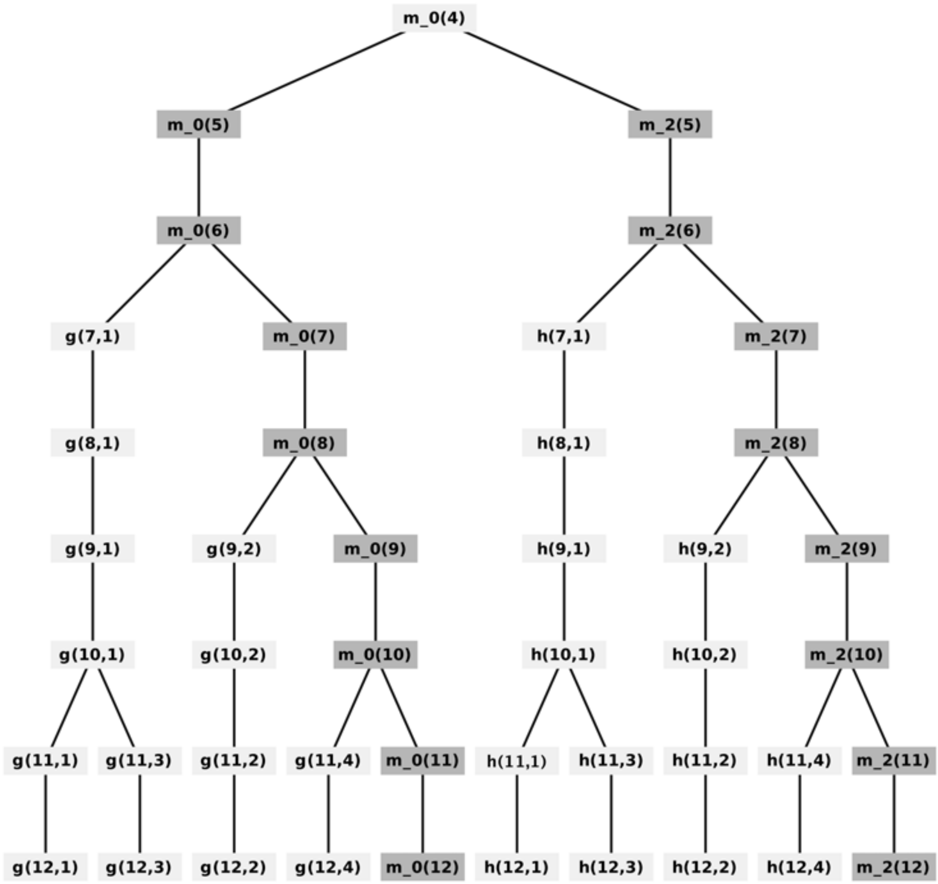

FIGURE 1. Lie algebras of Vergne type of dimension $n \leq 12$.

THEOREM 1. *The cohomology rings $H^*(\mathfrak{m}_0)$ and $H^*(\mathfrak{m}_2)$ over the field $\mathbb{Z}_2$ are isomorphic. The respective cohomology classes of the cocycles*

$$e^1,\ e^2,\ F(e^{i_1} \wedge e^{i_2} \wedge \cdots \wedge e^{i_q}, e^{i_q}), \tag{0.1}$$

where $q \geq 1,\ 2 \leq i_1 < i_2 < \ldots < i_q$, form a basis for $H^*(\mathfrak{m}_0)$ and for $H^*(\mathfrak{m}_2)$, respectively.

The fact that $H^*(\mathfrak{m}_0)$ and $H^*(\mathfrak{m}_2)$ over $\mathbb{Z}_2$ are isomorphic (note that $\mathfrak{m}_0$ and $\mathfrak{m}_2$ are not isomorphic over any ground field) is specific to the $\mathbb{Z}_2$ case: over a field of characteristic zero, $H^*(\mathfrak{m}_2)$ is very different [2, Theorem 5.5].

Then we prove Theorem 2, which can be stated as follows.

THEOREM 2. *For any nilpotent Lie algebra of Vergne type of dimension at least* 5 *over a field of characteristic two, there exists a nonisomorphic Lie algebra of Vergne type having the same Betti numbers.*

The second half is devoted to the computation of the cohomology of the infinite dimensional Lie algebras $\mathfrak{m}_0$ and $\mathfrak{m}_2$ over $\mathbb{Z}_2$. We also calculate the first three Betti numbers for the finite Lie algebras $\mathfrak{m}_0(n)$ and $\mathfrak{m}_2(n)$ over $\mathbb{Z}_2$. Most of these results can be found in [3, 4].

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