irrevocable in the middle of page 42, differently than at the bottom of page 84, and intervene at the top of page 169. The article the is frequently omitted as in Theorem 6 on page 168.

J. C. Burkill

A Short Account of the History of Mathematics, by W. W. Rouse Ball. Dover Publications Inc., New York, 1960. xxiv + 522 pages. $2.00

The History of the Calculus and its Conceptual Development, by Carl B. Boyer. Dover Publications Inc., New York, 1959. 346 pages. $2.00.

The Dover series of inexpensive reprints of important books in mathematics, the sciences, and many other fields of learning, has long won widespread recognition by students and scholars alike. While some of the reprinted works are of recent origin, others date back as far as the middle of the 19th century.

In mathematics, the prospective buyer of a treatise such as, say, Cantor's "Transfinite Numbers" or Knopp's "Theory of Functions" usually knows what he is about to acquire: not a volume filled with the latest results of mathematical research, but a classic in its field. But when it comes to the history of science and mathematics, the situation is altogether different. Apart from perhaps a handful of specialists, books in these fields will be sold mostly to students, teachers, and interested laymen, who will use them as an important - or even as the only - source of information. Unfortunately, there seems to be a widespread belief that our knowledge in the history of the sciences, and particularly of mathematics, is rather static, so that a book written a generation ago will be able to inform us just as well as one published more recently. This is, of course, quite untrue. One need, for instance, only think of the most remarkable findings of the last 30 years concerning pre-Greek and medieval mathematics.

On these grounds this reviewer concludes that it would have been better not to reprint Ball's "History of Mathematics". When the 4th edition, which is reproduced here, was published in 1908, Ball remarked in a note: "No material changes have been made since the issue of the second edition in 1893." - There is no point in elaborating on the shortcomings of the book when compared with the present state of knowledge in this field.

The "History of the Calculus" by Professor Boyer was first published in 1939. A second printing in which, according to the author, "a few minor errors in the text have been corrected", appeared in 1949.

The present Dover publication contains the statement: "This new Dover edition first published in 1959 is an unabridged and unaltered republication of the work first published in 1949 under the title ≫The Concepts of the Calculus, A Critical and Historical Discussion of the Derivative and the Integral≪."

It is most regrettable that the book now offered is, thus, essentially, that of 1939. Had the author revised his very valuable account in the light of the findings of more recent historical research, he could have given us a truly excellent and up-to-date history of the conceptual development of the calculus. (To mention only one point: we have now detailed studies on the early contributions of Newton and Leibniz (based on much unpublished material) which would have rendered the citation of several of the sources quoted in Ch. V superfluous.) It would have been desirable at least to include the most important publications since 1939 in the extensive bibliography.

Apart from these critical remarks, the book is well suited to introduce the reader to the numerous mathematical concepts which had to be clarified and to come together before the calculus could be given its rigorous form in which we use it today. Many of these ideas are only too familiar to us to become aware of the immense obstacles that stood in the way of a sufficiently precise definition. Professor Boyer traces this struggle for clear concepts back to the very origins in Greek mathematics. He untangles for us the maze of geometrical, mechanical, and arithmetical ideas to be found in the treatment of indivisibles, infinitesimals, fluxions, etc. through the centuries until finally, based on the precise formulation of the limit concept, satisfactory constructions for derivative and integral could be given.

In the Chapter on "Antiquity" we find, apart from the well-known anticipations of the calculus by Archimedes, a discussion of the basic problem of the continuum whose clarification was to be so vital for the later development. It is in the following Chapter that the author offers some of the very interesting results of his research on "Medieval Contributions", especially on the Liber Calculationum (ca. 1340) of Suiseth, the Calculator. He, Bradwardine, Occam, Oresmes and others belonged to the famous schools of Oxford and Paris where in the late Scholastic time in the theory of the "formlatitudes" a good deal of later mathematical development had been foreshadowed. Only fairly recently has some of the material of this period been closely studied; Professor Boyer's examination from the mathematical point of view is therefore most informative.

Almost one-third of the text is devoted to the "Century of Anticipation" - not surprising to anyone who knows to what extent particular procedures were already used by such men as Stevin, Kepler, Galileo, Cavalieri, Torricelli, Tacquet, Roberval, Pascal, Fermat, Descartes, Wallis, Barrow, Gregory and others. Following the Chapter on "Newton and Leibniz" are one on the 18th century ("Period of Indecision") and one on the 19th century ("Rigorous Formulation"), which together contain comments on an equally impressive list of names. The conclusion summarizes both the ideas which have proven fruitful and those which have hampered the development. But, as the author emphasizes: "Each [notation] is to be considered in the light of the mathematical and scientific milieu of the period in which it appeared."

To prevent misunderstandings let it be said again that the book under review is not a history of all aspects and implications of the calculus. Within its restrictions to the fundamental concepts of derivative and integral it presents a clear and painstaking examination of the development basic to all further extensions and additions. It can be highly recommended to every student of mathematics, for he will find in it what is rarely combined to such a degree in a book on the history of mathematics: historical information and clarification of important basic concepts.

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<u>Modern Probability Theory and its Applications</u>, by E. Parzen. John Wiley and Sons, Inc., New York and London. 464 pages. $10.75.

This pleasing book contains a large variety of theoretical and applied results in probability. Nevertheless it is so readable that one is surprised, when one has finished it, to realize how much has been covered. The book starts with elementary set theory and occupancy problems, and ends with inversion theorems for characteristic functions, and Lyapunov's condition for the central limit theorem. Obviously many students could read the beginning who could not get through to the end, but the plan of the book is that elementary calculus should be all that one needs until near the end, and more advanced techniques are only used in the last two chapters.

The plan is good but there may be argument about the consistency and success with which it is carried out. The author introduces probability in a continuum as early as possible. This greatly enriches the collection of applied problems, but it means that he must face well-known difficulties in developing the theory, and he has tackled these with ingenuity: In Chapter 4 he introduces "Numerical-Valued Random Phenomena" but apart from a brief reference and a provisional definition he defers the notion of a random variable for another hundred pages. From Chapter 4 onwards, however, he presents many interesting results and problems on continuous distributions and this is where his difficulties begin. His development of the theory involves mentioning