# SPHERICITY EFFECTS OF EXTENDED ATMOSPHERES OF LATE-TYPE GIANTS IN THE HR DIAGRAM

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## 1. THEORETICAL AND EMPIRICAL HR DIAGRAMS

In the late phases of stellar evolution, evolutionary tracks of stars with different masses come together along the Hayashi line in the HR diagram. The theoretical HR diagram (log L, log $T_{eff}$) is accordingly partially degenerate in the domain of late-type giants and supergiants, with respect to the third parameter, the stellar mass M. The stellar radius, R, being determined by log L and log $T_{eff}$, the mass determines the surface gravity log g at the radius R. These parameters enable us to transform a point in the theoretical HR diagram to the corresponding point in the empirical HR diagram $M_V$, (R-I) or spectral type. This transformation is conventionally carried out within the framework of the plane-parallel approximation in stellar atmospheres, and the parameters for the abscissa of the empirical HR diagram are dependant upon $T_{eff}$ and log g alone, irrespective of the mass itself. In this case, the parameter M indirectly affects the observable quantities through log g, but the effects of a variation by $\Delta$log g= $\pm$0.5, corresponding to $\Delta$log M=$\pm$0.5, are almost insignificant (*cf.* Tsuji 1976). The transformation between the theoretical and the empirical HR diagram is, therefore, almost one-to-one, within the framework of the plane-parallel approximation. Late-type giants and supergiants, however, have moderately extended atmospheres in general (*cf.* Schmid-Burgk and Scholz 1975), and their photometric colors and spectra are expected to be influenced by the sphericity of the atmospheric structure. Consequently, in comparing empirical HR diagrams with theoretical ones, it is important to know how atmospheric sphericity affects the transformation in the degenerate domains of the theoretical diagram. The sphericity can be represented, for instance, by the atmospheric extent $\eta$ relative to the



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stellar radius;

$$\eta = \frac{(\text{geometrical thickness between } \tau = 10^{-3} \text{ and } 1)}{(\text{stellar radius defined at } \tau = 1)} ,$$

where $\tau$ is the radial optical depth for the Rossland mean opacity.

## 2. SPHERICITY EFFECTS IN THE HR DIAGRAM

Spherical model atmospheres (static, non-grey, LTE, solar composition) were constructed for late-type giants and supergiants, by a revised method analogous to that proposed by Hundt, Kodaira, Schmid-Burgk and Scholz (1975). The line blocking and blanketing effects of the CO, CN, OH, $H_2O$ and TiO molecules were taken into account by an approximation which is realistic enough to yield the magnitude of the sphericity effects; details of the model calculations will be published elsewhere (Watanabe and Kodaira 1977). Model parameters were chosen roughly along theoretical evolutionary tracks computed or suggested by different authors [Refsdal and Weigert (1970), Robertson (1972), Paczynski (1970, 1971); for massive stars, see the references in Stothers (1972)], and are given in Table I. The relative extents, $\eta$, in the table were determined from the spherical model atmospheres. The models in the A series have the same values of $T_{eff}$ and log g. Consequently, they would have the same colors and spectrum if the plane-parallel atmospheric model are applied. The models in the series B, C, D, or E, have common values of L and $T_{eff}$, but differing values of $\eta$ corresponding to M. By examining the spherical model atmospheres in the A series, we found that the sphericity effect primarily appears as a dilution of the radiation field, leading to a general cooling of the outer layers. This cooling, however,

TABLE I. PARAMETERS FOR SPHERICAL MODEL ATMOSPHERES

| Model | | log $L/L_\odot$ | log $R/R_\odot$ | $m/m_\odot$ | log g | log $T_{eff}$ | $\eta$(%) | $M_V$ | (R-I) | (J-K) |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 1 | 5.207 | 3.12 | 20 | -0.50 | 3.505 | 5.0 | $-4^m.45$ | $2^m.16$ | $1^m.05$ |
| | 2 | 4.384 | 2.71 | 3 | | | 12.0 | -1.98 | 2.24 | 1.04 |
| | 3 | 3.906 | 2.47 | 1 | | | 23.0 | -0.93 | 2.27 | 1.04 |
| B | 1 | 4.822 | 2.78 | 15 | 0.05 | 3.580 | 3.0 | -7.28 | 0.91 | 0.87 |
| | 2 | | | 4.2 | -0.50 | | 12.0 | -6.97 | 1.00 | 0.85 |
| | 3 | | | 1.5 | -0.95 | | 54.0 | -5.19 | 1.34 | 0.89 |
| C | 1 | 2.800 | 1.84 | 3 | 1.24 | 3.544 | 1.3 | -0.50 | 1.55 | 0.96 |
| | 2 | | | 1 | 0.76 | | 4.6 | -0.17 | 1.62 | 0.96 |
| | 3 | | | 0.3 | 0.24 | | 17.4 | +0.60 | 1.77 | 0.97 |
| D | 1 | 3.700 | 2.36 | 10 | 0.71 | 3.505 | 1.6 | -1.35 | 2.02 | 1.00 |
| | 2 | | | 3 | 0.18 | | 5.5 | -0.87 | 2.13 | 1.03 |
| | 3 | | | 1 | -0.29 | | 16.6 | -0.34 | 2.22 | 1.01 |
| E | 1 | 4.600 | 2.93 | 15 | -0.25 | 3.447 | 2.6 | -3.16 | 2.21 | 0.93 |
| | 2 | | | 7 | -0.58 | | 6.1 | -2.93 | 2.26 | 1.00 |
| | 3 | | | 3 | -0.95 | | 17.1 | -2.39 | 2.38 | 0.99 |

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enhances the formation of the temperature-sensitive molecules such as TiO and $H_2O$, and the blanketing effects of these molecules further modify the atmospheric structure in a complicated manner. Among the models in series B through E, a decrease in the stellar mass leads to a lowering of the surface gravity but, contrary to the plane-parallel cases, enhances the formation of TiO and $H_2O$, owing to the temperature decrease caused by the increased dilution effect in the extended atmospheres.

The distribution of emergent energy flux is slightly shifted towards longer wavelength, reflecting the general cooling of the surface layers. The most striking effect is the enhancement of the molecular absorption bands of TiO: Strengths of TiO bands increase with the increasing atmospheric extension, and strongly decrease the emerging flux in the photometric passband V. The effect of the enhanced formation of $H_2O$ is partially cancelled by the general shift of the continuous energy distribution, and the colors in the near-infrared region are less affected by the sphericity effects than the colors in the red to blue region. The resulting effects in the absolute visual magnitude $M_V$ and colors (R-I) and (J-K) are given in the last three columns of Table I. Since no special effort was made to bring the photometric colors precisely into the Johnson system, the absolute values are subject to an uncertainty of about $\pm 0\overset{m}{.}2$, but the relative values, which are important in estimating the sphericity effects, are estimated to have an accuracy of about $\pm 0\overset{m}{.}02$. The values of $M_V$ also should be interpreted differentially. According to the present result, when L and $T_{eff}$ are fixed, a decrease of stellar mass by $\Delta\log M \simeq 1$ causes lowering of the absolute visual magnitude by $\Delta M_V \simeq 1^m$, and reddening of $\Delta(R-I) \simeq 0\overset{m}{.}2$ which approximately corresponds to the difference for one spectral sub-class in the MK classification (cf. Lee 1970).

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DISCUSSION

*FOY:* I have two comments. Firstly, I think that the sphericity could be related to the microturbulence variations which I have described this morning if microturbulence is induced by overshooting. Indeed, the sphericity obviously depends on the stellar radius, and the overshooting scale height is a function of the thickness of the convective zone and consequently of the stellar radius, too.

Secondly, I wish to mention a new work by Labeyrie and his group and myself. New speckle interferometer observations at Palomar in narrow spectral bands show abrupt variations in the diameter of Mira and R Leonis as a function of wavelength. The diameter is at least twice as large in strong TiO bands as outside them. From Tsuji's models we have interpreted these variations to be due to the large opacity of TiO. In the strong TiO features the optically thick region of the atmosphere extends several astronomical units outwards from the continuum formation layer (up to $\tau_{cont} \simeq 10^{-5}$). Since the stellar radius in the continuum is smaller than one astronomical unit, the plane parallel approximation is surely not valid in the intense TiO bands. This supports your arguments.

*KODAIRA:* I agree with you on the first point. As for the second point, I am afraid that you may have to invoke dynamical extended atmospheres in order to interpret the large radii observed. I hope that you can obtain a qualitative insight into sphericity effects from our model calculations.

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