

1958 CIERVA MEMORIAL PRIZE ESSAY COMPETITION  
PRIZE WINNING ESSAY



**Is Man-Powered Rotating-  
Wing Flight a Future  
Possibility**

by

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The Association is pleased to announce that the prize of £50 for the 1958 Cierva Essay Competition has been awarded to Mr E R Kendall

Mr Kendall, who is 25 years of age, is a Project Aerodynamicist with the Helicopter Division of Saunders-Roe Ltd. He served his apprenticeship with that Company and in 1955 was nominated by the S B A C for a two year course at the College of Aeronautics, where he specialised in helicopter aerodynamics. For his exceptional ability in technical study whilst at the College, he received the 1957 Alan Marsh Memorial Award which enabled him to take a short course of helicopter piloting instruction at Air Service Training. He is an Associate Member of the Association.

His prize winning essay is published herewith

(1) SUMMARY

The possibilities of man-powered rotating-wing flight are discussed on the basis of detailed performance and stability calculations which refer to a hovering rotor. This data is viewed together with that currently available from investigations concerned with a man-powered fixed-wing project to provide a more general discussion which gives mention to the "gyroplane," "cyclogiro" and the "convertiplane"

It is concluded that

- (1) Man-powered rotating-wing flight is a future possibility
- (2) The conditions which must prevail will be extremely difficult to achieve
- (3) Man-powered flight using a fixed-wing configuration is considerably easier from both the performance and stability viewpoint
- (4) A major advantage of the rotating-wing type is that full scale performance tests can be conducted and many adjustments made prior to the first free flight

## (2) INTRODUCTION

In recent years, a growing interest has been shown in the possibilities of man-powered flight. Revival of interest in this country began in 1955 with a paper <sup>(1)</sup> by Mr B S Shenstone.

Since then, more detailed theoretical and experimental work has been completed by Mr T R F Nonweiler <sup>(2, 3, 4)</sup> and we read <sup>(4)</sup> that a Committee has now been formed to promote the project.

All effort to date has been devoted to investigations concerned with a fixed-wing aircraft. This is not because of any definite advantage that it is known to possess, but rather because of the necessity to restrict the scope of research initially.

There is still a need for a comparison of the relative merits of various aircraft types, and clearly, this is not possible until detailed assessments of each type have been made.

In this essay, we are concerned with the possibilities of man-powered rotating-wing flight. All detailed work is limited to an assessment of the performance and stability of the "helicopter" type. The effect of blade weight, blade profile drag and ground cushion are assessed, and we are left with a fairly good idea of the values of these parameters which must be achieved before a man, using his own muscular efforts, can lift himself into the air on a rotor.

For much of the investigation, we are able to avoid the choice of a particular type of helicopter. We analyse first the performance of a rotor which is hovering in the ground cushion, and it is not until we begin to think of torque reaction, control and stability that we are forced to restrict the scope of our work. The types of helicopter which emerge are dictated mainly by the need to keep structure weight at a minimum. Assessments of the stability and control of the man-powered helicopter are limited to one of these configurations.

The detailed results, when viewed in conjunction with the design study of Ref 4, provide a broad but still incomplete background for an assessment of the difficulties of man-powered flight. Upon such a background, we attempt to place the "gyroplane," "cyclogiro" and "convertiplane" in perspective and so arrive at an assessment of the possibilities of man-powered rotating-wing flight.

## (3) PERFORMANCE

### 3.1 Introduction

In order to assess the performance possibilities of the man-powered helicopter, we choose as our yardstick the factor

$$\frac{W - W_B}{HP_{av}} \quad \text{where}$$

W = All-up Weight  
W<sub>B</sub> = Total Blade Weight  
HP<sub>av</sub> = Horsepower Available

When this parameter is multiplied by the horsepower output of a man and his weight is subtracted from the result, we are left with a weight margin

for fuselage, stabiliser, undercarriage, transmission system, controls, hub and blade attachments, etc

In this section we will attempt to show the effect of blade weight, blade profile drag and ground cushion on  $(W - W_B)/HP_{av}$  for a rotor which is hovering near the ground. Disc loading and rotor solidity will be treated as variables for most of the work

Using the results of research by Ursinus<sup>(5)</sup>, Wilkie and Nonweiler<sup>(3, 4)</sup> we will include human power output figures and so determine the weight margin mentioned above

### 3.2 Performance Equations

In Appendix I, the following expression is developed for a rotor which is hovering near the ground

$$\frac{W - W_B}{HP_{av}} = \frac{32.3 \left(1 - \frac{R_{\infty}}{W}\right)}{\omega^{1/2} \left[\frac{T_{\infty}}{T}\right]^{3/2} \left\{1 + 0.157 \left[\frac{C_D}{C_T^{1/2}}\right] \left[\frac{1}{C_{T_{\infty}/\sigma}}\right]\right\}} \quad \dots (1)$$

The ground cushion thrust factor  $(T/T_{\infty})$  is a function of  $(CT_{\infty}/\sigma)$  and  $(Z/D)$  as shown by Fig 5.13 of Ref 6

Throughout this investigation we use, instead of  $Z/D$ , the parameter  $\epsilon$  defined as

$$\epsilon = \sin^{-1} Z/R$$

This is the angle of tilt of a rotor having no coning, whose centre is at a height  $Z$  when its blade tip is touching the ground. Written so, we appreciate more readily the importance of achieving good stability characteristics if we have to place the rotor close to the ground in order to obtain the necessary performance

The lowest value of  $\epsilon$  considered will be  $5^\circ$  and the effect on performance of increasing the "permissible rotor tilt angle" will be shown

$(T/T_{\infty})$  is plotted against  $(CT_{\infty}/\sigma)$  and  $\epsilon$  in Fig 1

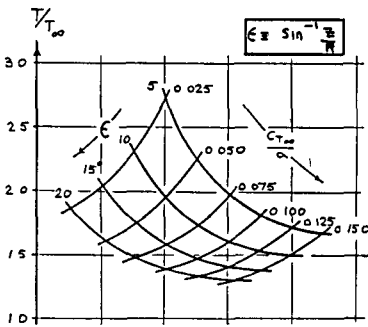


Fig 1 Ground cushion effect

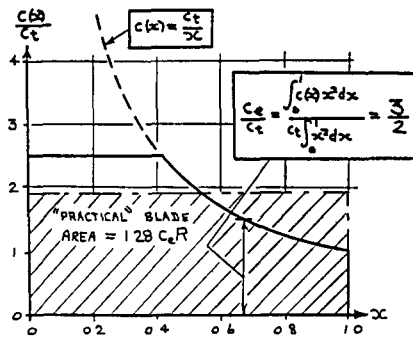


Fig 2 "Ideal" and "Practical" Rotor Blade Geometry

The equivalent specific blade weight ( $k$ ) is the blade weight per unit blade area multiplied by "blade area" solidity/equivalent thrust chord solidity ( $\sigma$ ). For blade weight considerations we choose a "practical" blade (see Fig 2) whose chord distribution is given by —

$$c = \text{constant} \quad \text{for } 0 \leq x \leq 0.4 \text{ say}$$

$$\text{and } c = \frac{2 c_e}{3 x} \quad \text{for } 0.4 \leq x \leq 1.0$$

This gives a mean blade chord based on area of  $1.28c_e$  and the equivalent specific blade weight ( $k$ ) is  $1.28 \times \text{Blade Weight/Unit Area}$

In the absence of more detailed work we cannot affix an accurate value to  $k$ , but because, from the quantitative point of view, the insertion of numerical values in equation (1) will only give an approximate performance assessment, we will choose a value for  $k$  which we think to be the minimum achievable at present

Perhaps we can be guided in our choice by the wing weights which have been achieved on some man-powered fixed-wing aircraft on the premise that a rotor blade is unlikely to be built to a lower specific weight

If the rotor is revolving slowly, the propeller moments and centrifugal loads should be fairly low. Construction difficulties brought about by the need to make the blade twisted and shaped in planform might cause the blade to be heavier than an equivalent wing but, by suitable design, bending stresses due to lift might be relieved by centrifugal forces

Ref 1 gives particulars of four such machines which took part in a competition in 1937. The main aim was to maintain height for as long as possible after assisted take-off. From the data given we obtain the wing weight per unit area for three of the aircraft, and this is summarized below, together with information from Ref 4

Wing loadings and aspect ratios are quoted for reference

<i>Aircraft</i>	<i>Wing Weight per Unit Area (lb/ft<sup>2</sup>)</i>	<i>Wing Loading (lb/ft<sup>2</sup>)</i>	<i>Aspect Ratio</i>
Bossi & Bonomi (Italian)	0.440	1.58	13.4
Seehase (German)	0.254	1.39	11.4
Russian	0.395	1.78	10.4
The Projected A/c of Ref 4	0.450	2.80	21.4

The Seehase wing comprised a doped silk covering over widely spaced ribs which were supported by magnesium tubes at their leading and trailing edges

The wing in Ref 4, on the other hand, will be covered with  $\frac{1}{32}$ " birch ply to maintain a rigid wing surface

The available data would seem to indicate that, at present, for a wing having a well supported skin, we cannot expect weights of less than 0.40 lb/ft<sup>2</sup>

The blade weight problem alone is an extremely difficult one, and a much more detailed investigation is required before a satisfactory answer can be given

The best that we can do at this stage is to take a range of values for blade weight starting at, say, 0.45 lb/ft<sup>2</sup>, and to show the effect of its variation on the performance of our man-powered machine

Stability-wise, of course, we are doing a bad thing by making the blades light—but this is another aspect which will receive consideration later

We must now study the blade sections available to obtain some indication of the drag of a section which would be used in this particular application. At first we tend to look for the highest lift/drag ratio until we note (see Fig 1) that ground cushion effect is increased for lightly loaded blades. A numerical analysis using equation (1) will give the optimum theoretical value

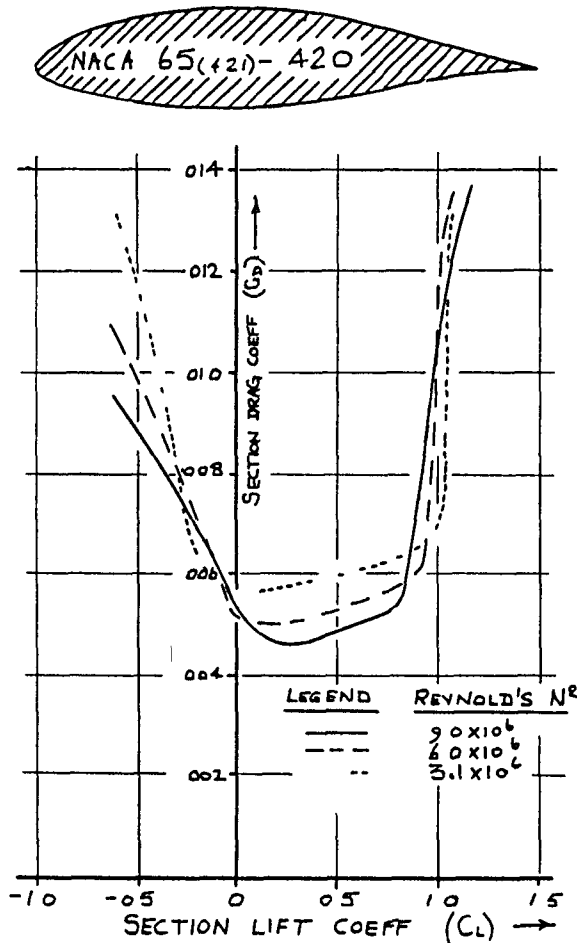


Fig 3  
Lift/ Drag Characteristics  
of NACA65(421)-420

of  $(C_{T\infty}/\sigma)$  and hence  $(C_L)$  for any given drag coefficient, and we must choose a section which has the lowest drag at the optimum incidence. We could iterate using equation (1) if we were interested in a precise theoretical value, but this would represent misuse of an expression which is only intended to give a fairly approximate performance assessment.

Instead, we consider sections for minimum drag over a lift coefficient range of about 0.2 to 0.8.

Fig. 9.11 of Ref. 6 shows that for the NACA 3-H-13.5 and NACA 8-H-12 sections, drag coefficients of below 0.006 might be achieved at a Reynold's Number of  $2.6 \times 10^6$ .

For sections like NACA 65(421) — 420, Ref. 7 shows values of less than 0.006 for the drag coefficient at a Reynold's Number of  $3.1 \times 10^6$  (see Fig. 3). These apply for aerodynamically smooth sections.

The mean blade Reynold's Number is likely to be much lower than those quoted above and, from the scanty low Reynold's Number data which is available, we must choose what we think might be the minimum possible drag coefficient achievable in this case.

Some available data is plotted in Fig. 4, and we use this as the basis for the suggestion that the drag coefficient of our rotor blade will not be less than 0.007.

We have now reached a stage where we can use equation (1) to evaluate  $(W - W_B)/HP_{av}$  over a range of values of disc loading, rotor solidity and blade loading, for what we consider to be the best achievable values of  $\epsilon$ ,  $k$  and  $C_D$ .

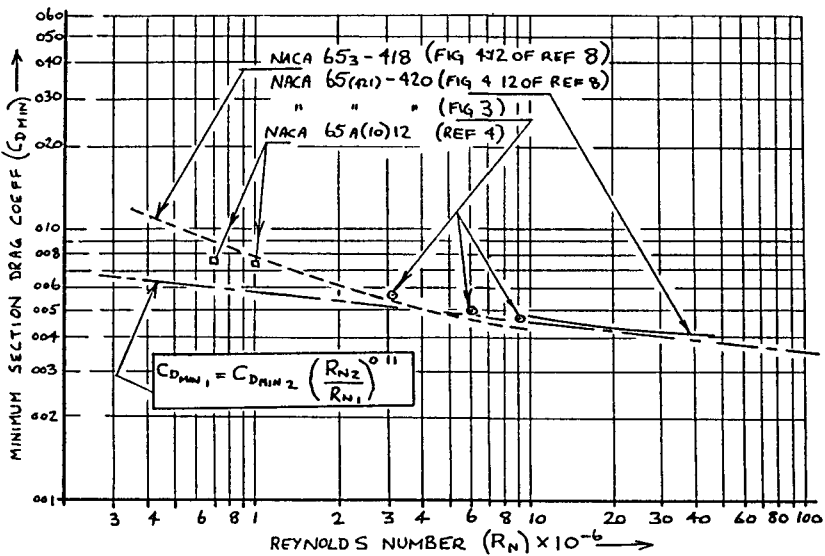


Fig. 4 Variation of  $C_{Dmin}$  with Reynold's No

The results are shown in Fig 5

It will be noted that

- (1) For each disc loading there is an optimum rotor solidity

APPLIES FOR -  $\eta = 0.85$

$E = 5^\circ$

$C_D = 0.007$

BLADE WEIGHT = 0.45 LB/FT<sup>2</sup>

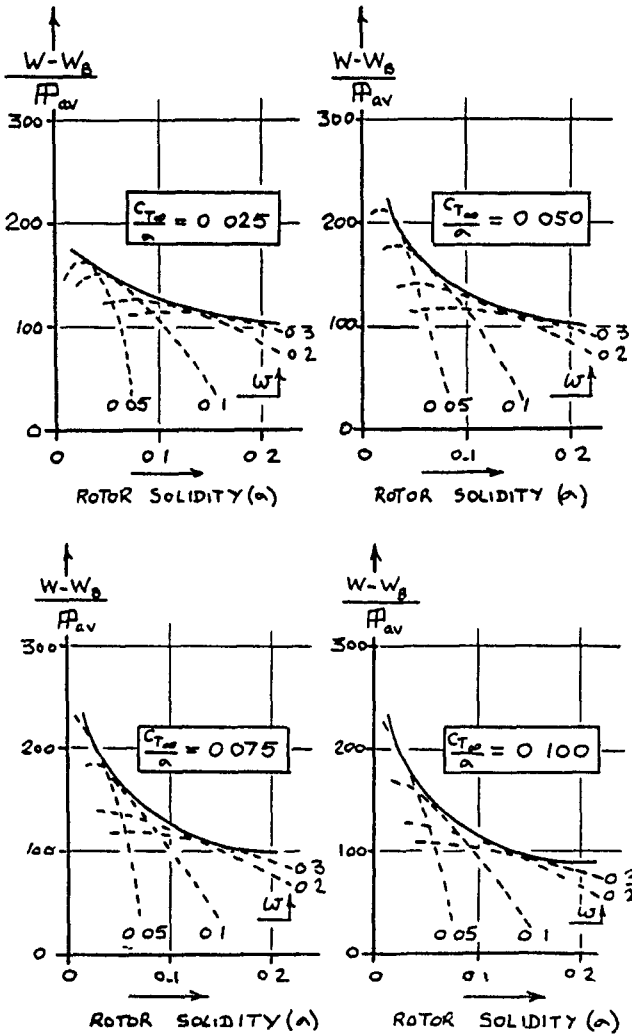


Fig 5  
Variation of  $(W-W_B)/P_{av}$  with  $\frac{C_{T_{00}}}{\sigma}$ ,  $w$ , and  $\sigma$

- (2) As the disc loading increases, the optimum solidity also increases
- (3)  $(W - W_B)/HP_{av}$  is improved as the disc loading is reduced
- (4) The blade loading factor  $(C_{T\infty}/\sigma)$  has an optimum value

The overall optimum conditions are found by plotting the upper extremes of Fig 5 against  $(C_{T\infty}/\sigma)$  and disc loading. The result is shown in Fig 6 and the optimum conditions are tabulated.

Because we know so little about the blade weight problem, we have worked in terms of a constant weight/unit area for all blades, and this has led to the result that the highest value of  $(W - W_B)/HP_{av}$  is when the disc

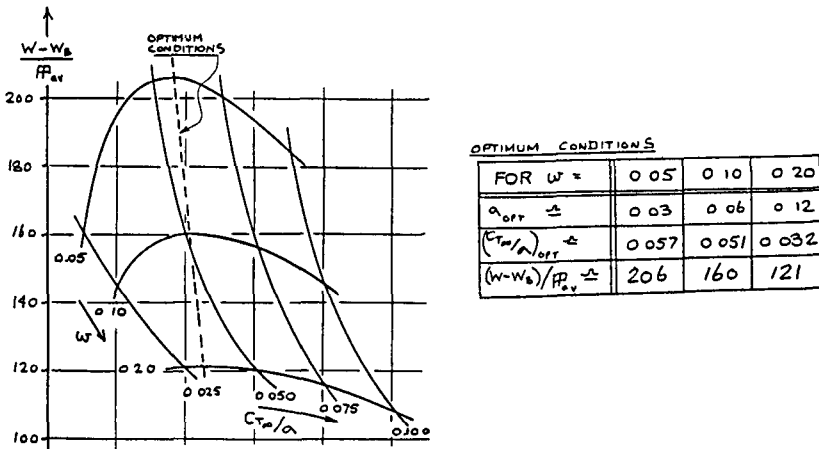


Fig 6 Determination of best  $(W - W_B)/HP_{av}$  from Fig 5

loading and solidity go to zero. There is obviously a practical limit to this trend, and our next task will be to impose a restriction on the lowest chord/radius ratio which can be achieved for the low blade weights already discussed. We can only hazard a guess and express the need for much further research on the problem of ultra-lightweight blade design.

In Fig 7, the blade of our chosen platform is drawn for a range of chord/radius ratios and, because for the optimum conditions tabulated in Fig 6 we can relate disc loading and solidity, we are able to plot chord/radius ratio against disc loading for several numbers of blades/rotor. We can also plot the optimum values of  $(W - W_B)/HP_{av}$  against disc loading. In the absence of detailed blade-weight data, we impose a lower limit on the values of  $(C_0/R)$  which can be achieved whilst still maintaining a blade specific weight of  $0.45 \text{ lb/ft}^2$ .

For a rotor having a given number of blades, this imposes a restriction on low disc loadings and high values of  $(W - W_B)/HP_{av}$ .

In full realisation of the importance of the decision to the results of this investigation, we choose, more by intuition than anything else, a lower limit to  $(C_0/R)$  of 0.10 for blades of  $0.45 \text{ lb/ft}^2$  specific weight.



This gives us the following values of best achievable  $(W - W_B) / H P_{av}$

No of Blades/Rotor	Lowest Disc Loading	Best $(W - W_B) / H P_{av}$
2	064	189
3	095	164
4	129	144

Now, in order to find the weight margin given by  $(W - W_B - W_{man})$ , we must find the power output ( $H P_{av}$ )

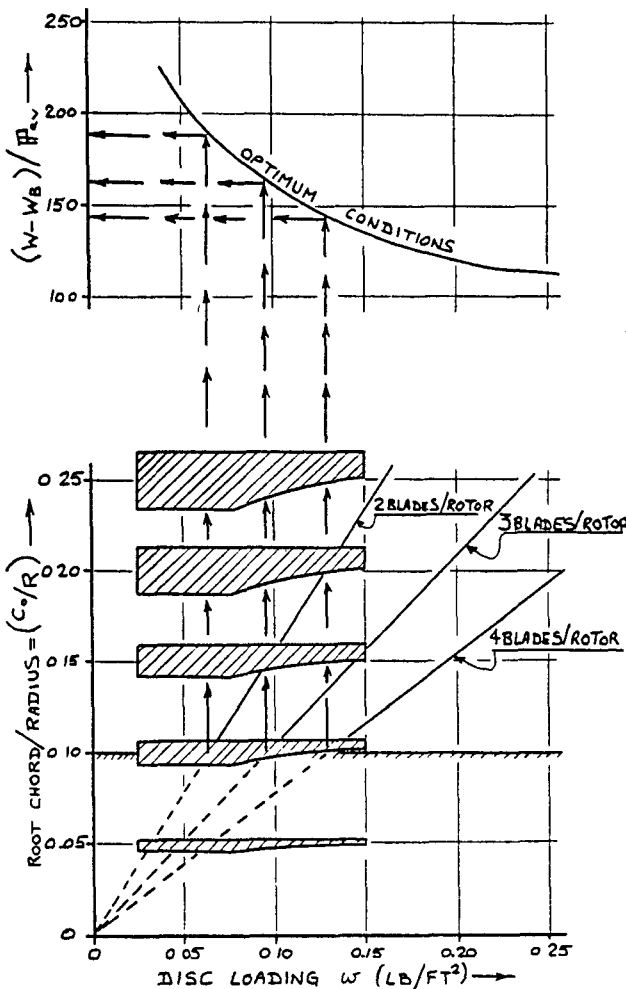


Fig 7  
Effect of No blades/  
rotor on the best  
 $(W - W_B) / H P_{av}$   
for various values of  
 $C_o/R$

### 3.3 Human Power Output

It is perhaps fortunate that we lag a few years behind the investigators of man-powered fixed-wing flight since they have already collected much information concerning human power output. A complete "engine brochure" is given in Ref. 4, and all we need to do at this stage is to present a brief summary of the information collected therein.

The persons most closely associated with methodical studies of the power generated by man are Ursinus, Wilkie and Nonweiler.

Dr Ursinus investigated the power output of his subject and gave us that data in Fig. 8 which refers to handcranking (arms only), cycling (legs only) and handcranking combined with cycling (arms and legs). He determined the best crank throw, the optimum speed for each duration of effort:

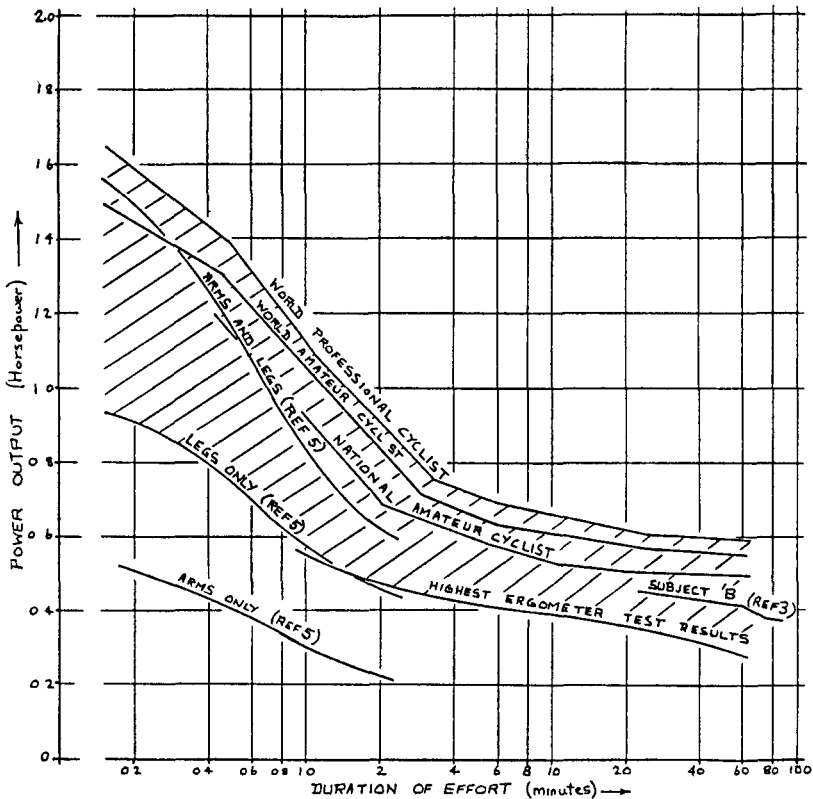


Fig. 8 Human Power Output Curves

and the correct phasing between arm and leg movement. He also investigated the effect of posture. We are told<sup>(4)</sup> that the subject of Ursinus' tests was not an athlete.

T R F Nonweiler<sup>(3)</sup> measured the air resistance of some amateur racing cyclists mounted on their bicycles in a closed section wind tunnel. Making allowance for rolling and mechanical resistance, he used the data so obtained in conjunction with "World Professional," "World Amateur" and "National Amateur" cycling records to compute an "average" power output. These results are also shown in Fig 8.

To complete the picture, we include data referring to the highest powers recorded in bicycle ergometer tests. These are due to a systematic search by Dr D R Wilkie through many references, and we understand that, although the data refers to trained cyclists, higher powers have been recorded in experiments on professional cyclists.

As we would expect, the power output decreases with increasing duration of effort. We see also that, for cranking motion of arms and/or legs, the power output of a trained man will almost certainly fall within a band, the upper limit of which is defined by the "World Professional Cyclist" results of Ref 3, the lower limit being defined by the "Legs Only" results of Ursinus for low durations and the "Highest Ergometer" results collected by Wilkie for durations over about  $1\frac{1}{2}$  mins.

For the remainder of this work, the upper limit will be called the "Absolute Maximum Output," and the lower limit will be known as a "Good Average Output."

### 3.4 Performance Results

It is now an easy matter to combine the results of Figs 7 and 8 to calculate  $(W - W_B)$  per man. This is shown in the upper part of Fig 9 for two and three blades/rotor and for both extremes of the power output band.

For the conditions implicit in its derivation, we see from the upper figure that a 150 lb man using only his own muscular effort cannot hover on a three-bladed machine if his power output is not above the "Good Average" rating. The weight margin for structure, etc (defined as all-up weight minus blade weight minus man weight) is zero at, and negative above, a duration of about ten seconds.

The lower part of Fig 9 applies for a 150 lb man generating 90% of the "Absolute Maximum Output" rating, and is based on the results given in the upper graph. We see that if the structure, undercarriage, transmission controls, etc, can be built for about 50 lb/man, we might expect hover durations in the order of 30 to 40 secs.

More for encouragement than any other reason, it is perhaps worthwhile to study the estimated weight breakdown of the man-powered fixed wing aircraft of Ref 4.

Wing	= 77 lb	= 0.45 lb/ft <sup>2</sup>
Fuselage-fin	= 20 lb/man	
Tailplane	= 5 lb/man	
Transmission	= 5 lb/man	
Wheels	= 5 lb/man	
Controls	= 3 lb/man	
Propeller	= 5 lb/man	
Contingency	= 4 lb/man	

We might say that the "weight margin" for this machine is 47 lb/man!

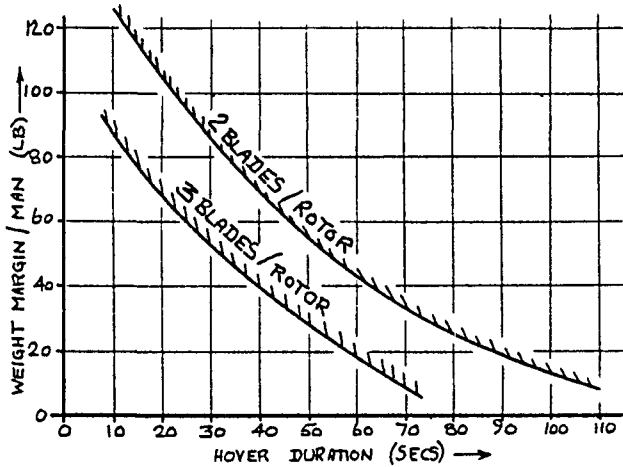
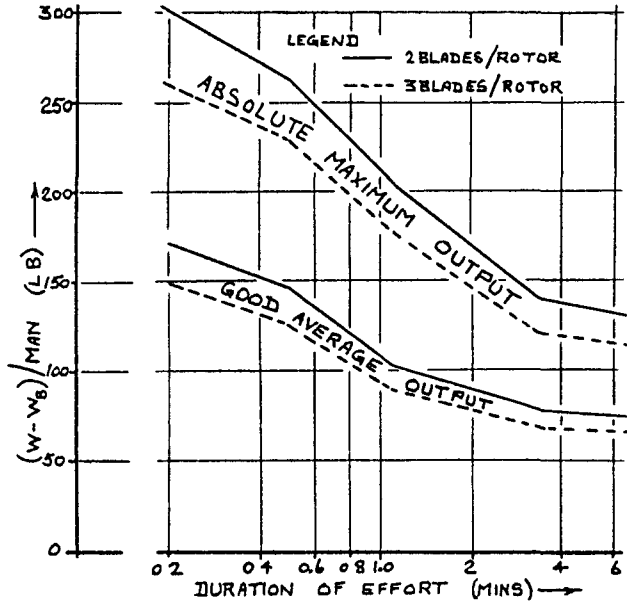


Fig 9  
 Hover  
 Performance  
 Graphs

The conclusion to be drawn from the results obtained so far is

“ If a helicopter can be built whose all-up weight less blades less occupant(s) is about 50 lb /occupant, and whose blade weight is 0.45 lb /ft<sup>2</sup>, then it can be hovered for about 30 seconds by the muscular efforts of the occupants provided that the blade drag coefficient is not appreciably greater than 0.007, and that the rotor tilt can be restricted to less than 5° ”

By repeating some of the calculations, we obtain an approximate assessment of the decrease in performance which results if we fall short of

achieving the conditions stipulated above They are as follows

- (1) An increase in blade specific weight of 0.01 lb/ft<sup>2</sup> results in a decrease in weight margin of about 2 lb/man
- (2) An increase in mean blade drag coefficient of 0.001 results in a decrease in weight margin of about 5 lb/man
- (3) An increase in the permissible tilt angle ( $\epsilon$ ) of 1° results in a decrease in weight margin of about 6 lb/man

Using these approximate performance derivatives, we see that the chances of achieving our aim are considerably less for a machine having the following characteristics

Specific blade weight	0.50 lb/ft <sup>2</sup>
Blade drag coefficient	0.010
Permissible tilt angle	10°

because weight margin is then reduced by as much as 55 lb/man

The above example is intended to demonstrate the importance of achieving the best possible conditions

#### (4) CONFIGURATION AND SIZE

In order to give some indication of how the size of the man-powered helicopter will be dependent on the configuration chosen, the rotor diameter for unit horsepower available is graphed in Fig. 10 against number of rotors. A single rotor helicopter and two types of double rotor machine are sketched in scale with a "one horsepower" man (Fig. 8 gives 1.4 HP for 30 secs as the absolute maximum, and the man drawn is approximately 4 ft, i.e., 6/1.4 ft). This is intended to give an impression of the size of machine envisaged, although it is appreciated that the scale is not directly applicable for other power outputs

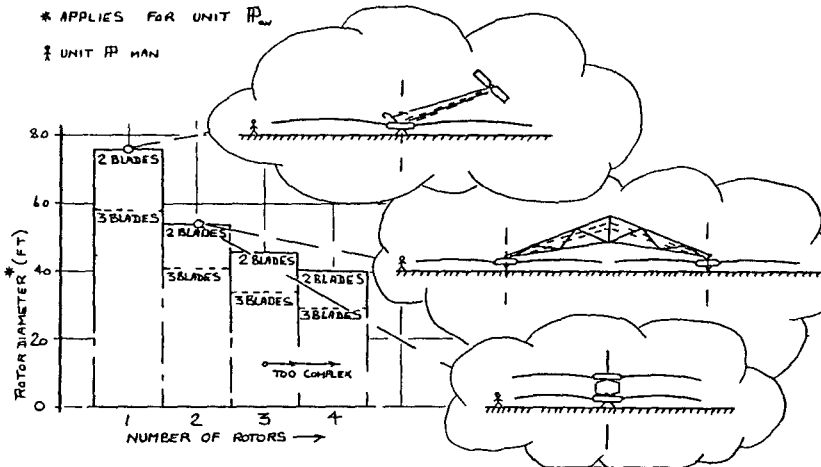


Fig. 10 Configuration and Size

It is felt that the side-by-side double rotor machine will require more than the estimated weight margin to build the interconnecting structure and transmission. Also, it is almost certain that more than two rotors will be prohibited by complexity. Without more detailed investigation it cannot be said whether the torque reaction power losses will be more on the single rotor machine than on the double rotor coaxial type which has its upper rotor further from the ground, and whose lower rotor must operate in a disturbed region of flow.

From Dr Focke's paper<sup>(9)</sup> we understand that prior to his twin rotor side-by-side helicopter, the Brequet helicopter (coaxial type) was the most successful of that period. Also, in his study of the various helicopter configurations, Dr Focke eliminates the single rotor machine with tail rotor at an early stage on the basis of prohibitive torque reaction power losses. Successful helicopters of today are nearly all of this type, but that is only because we can compromise performance to some extent for compactness and relative simplicity. Fortunately, these considerations need not arise in our case. We are not able to compromise the performance of our machine for any other factor (except perhaps for stability) because we are worse off with our "engine" of about 0.01 power-weight ratio than even the very early investigators in the rotating wing field. On the other hand, we cannot let the early Brequet success influence us in our choice of configuration between single rotor or coaxial type because of the ground cushion influence. Once more we must be content to express the need for further investigation.

## (5) STABILITY AND CONTROL

### 5.1 Introduction

Every pilot uses some degree of concentration to control his aircraft, especially when he is flying close to the ground. It is generally appreciated that the hovering helicopter is very difficult to stabilise and that the underpowered rotor can easily get out of hand if not controlled with extreme care.

Our pilot has virtually no time for the control and stabilisation of his underpowered rotor which will be damaged if it tilts more than 5°. If much attention is required for control, a decrease in his power output will almost certainly result. (The egg and spoon race is perhaps the slowest of all running events.)

These thoughts, following so closely on the very marginal performance results, make the overall problem seem insuperable—but let us not become too despondent. For the moment we will pretend that the performance problem is not too difficult and that the *only* barrier to man-powered rotating-wing flight is the lack of inherent stability. In this frame of mind we approach the problem more readily.

The three outstanding questions which must be answered are

- 1 How bad is the stability of our man-powered helicopter likely to be ?
- 2 Will we be able to adjust it ?
- 3 Can we possibly achieve very good stability ?

For guidance in the formulation of our answers, we must write some equations, insert some numbers and study the trends of our results.

## 5.2 The Uncontrolled Aircraft

We get the impression from Fig 10 that the aircraft centre of gravity will almost certainly be above the rotor centre and at a distance from it which is small in comparison with the rotor radius. A coaxial layout similar to that sketched in Fig 11 will have its c.g. between the two rotors and very close to the resultant thrust vector.

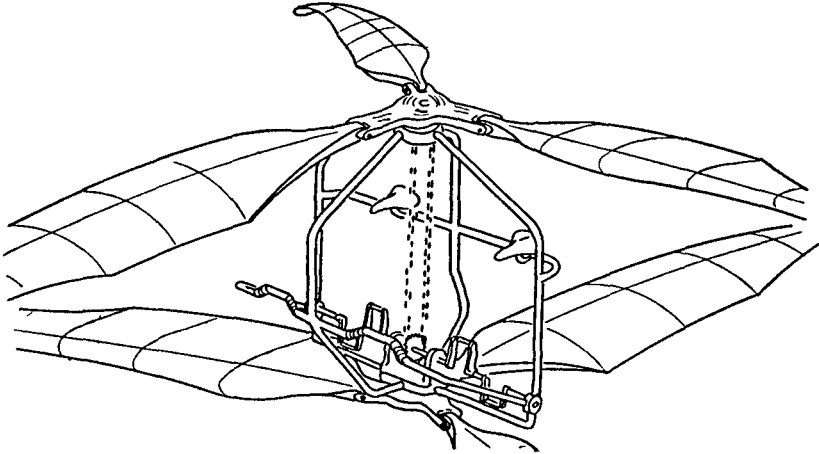


Fig 11 A Coaxial Layout

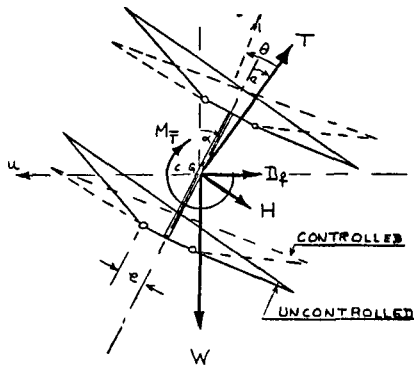


Fig 12 Diagram of Forces

In the work which follows, we will consider a helicopter with offset flapping hinges and its c.g. on the equivalent rotor centre

Under the usual assumption of small disturbances from equilibrium and the separation of longitudinal and lateral motions we get (See also Fig 12)

Horizontal Force Equation

$$T(a_1 + \alpha) + H + D_f + \frac{W}{g} u = 0 \quad \text{--- (2)}$$

Pitching Moment Equation

$$M_{\bar{T}} - I \alpha = 0 \quad \text{--- (3)}$$

Putting  $T = W,$

$$a_1 = a_{1u} u - a_{1q} \alpha$$

$$H = H_u u$$

$$D_f = D_{fu} u = 0 \text{ in hover,}$$

$$\text{and } M_{\bar{T}} = M_{\bar{T}a_1} a_1$$

$$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{--- (4)}$$

gives

$$\left[ \left( a_{1u} + \frac{H_u}{W} \right) + \frac{D}{g} \right] u + [1 - a_{1q} I] \alpha = 0 \quad \text{--- (5)}$$

$$[M_{\bar{T}a_1} \cdot a_{1u}] u - [M_{\bar{T}a_1} \cdot a_{1q} I + I I^2] \alpha = 0 \quad \text{--- (6)}$$

which in turn leads to the following characteristic equation

$$\sum_{i=0}^{i=3} \bar{A}_i p^i = 0 \quad \text{--- (7)}$$



The coefficients  $\bar{A}_i$  are

$$\left. \begin{aligned} \bar{A}_3 &= 1 \\ \bar{A}_2 &= g \left( a_{1u} + \frac{H_u}{W} \right) + \frac{M\bar{\Gamma}a_1}{I} a_{1q} \\ \bar{A}_1 &= g \frac{M\bar{\Gamma}a_1}{I} a_{1q} - \frac{H_u}{W} \\ \text{and } \bar{A}_0 &= g \frac{M\bar{\Gamma}a_1}{I} a_{1u} \end{aligned} \right\} \text{---(8)}$$

Routh's criterion states that the system is stable when

$$\left. \begin{aligned} \bar{A}_i &> 0 \text{ for all } i \\ \text{and when } \bar{A}_2 \bar{A}_1 &> \bar{A}_0 \end{aligned} \right\} \text{---(9)}$$

Because the blade geometry, size, specific weight and rotational speed are all determined by performance requirements in this case, we have very little control over the derivatives  $a_{1u}$ ,  $H_u/W$  and  $a_{1q}$ . Also, the aircraft moment of inertia cannot be easily controlled. Zbrozek<sup>(10)</sup> finds that “ a substantial decrease in helicopter moment of inertia is beneficial for oscillatory motion and values of  $\lambda_1$  ” He goes on to mention that “ From this point of view, the compact design of the helicopter, without rotor torque compensation devices seems to be advisable ”

The moment,  $M\bar{\Gamma}$ , for a rotor having blades of a given weight and rotational speed can be chosen within limits by a proper choice of flapping hinge offset ( $e$ ) but, of course, its use will necessitate more than two blades/rotor

In order to show that the use of  $M\bar{\Gamma}$  alone is insufficient to give stability, an example man-powered helicopter will be chosen and the coefficients ( $\bar{A}_i$ ) of the stability cubic will be calculated. Already, from the expression for  $\bar{A}_0$  we can see that flapping hinge offset is certainly a necessity if we are to achieve stick-fixed static stability, and it is clear that when the aircraft CG is on the rotor centre, hinge offset is required to effect control by rotor tilt

The example man-powered helicopter is described below

Configuration	Coaxial	
No of Men	2	
No of Blades/Rotor	3	
Disc Loading	0.095	(Fig 7)
Rotor Diameter for Unit HP <sub>av</sub>	41 ft	(Fig 10)
Rotor Area for Unit HP <sub>av</sub>	1,320 ft <sup>2</sup>	
HP <sub>av</sub> = 2 × 0.90 Abs Max for 30 secs	2.46	(Fig 8)
Total Rotor Area	6,500 ft <sup>2</sup>	

A U W = 0 095 × 6,500 = 616 lb and is made up as follows	
Man Weight = 2 × 150 =	300 lb
Weight Margin for 30 secs duration (See Fig 9)	
= 50 lb/man =	100 lb
Blade Weight	
= 1 28 × 0 45 × Total Blade Area	
= 0 576 × $\sigma_{opt}$ × Rotor Area =	216 lb
Total =	616 lb

The rotor tip speed is found to be 80 ft /sec and the mean blade lift coefficient is 0 69

The stability derivatives have been calculated using the above data in conjunction with the equations listed in Table I, some of which are only very approximate

As a result, the answers obtained are not likely to be numerically correct but it is thought that they will give a fair indication of the problems which will be encountered in the stabilisation of the man-powered helicopter

The calculated stability parameters are given in Table I for a range of the flapping hinge offset ratio ( $e/R$ )

It is found from equations (8) and (9) that the condition for neutral dynamic stability leads to the following quadratic in  $a_{1q}$

$$\frac{M_{\bar{F}a_1}}{I a_{1u}} \frac{H_u}{W} (a_{1q})^2 + \frac{g}{a_{1u}} \frac{H_u}{W} (a_{1u} + \frac{H_u}{W}) a_{1q} - 1 = 0 \text{ -----(10)}$$

The solutions are tabulated below together with the  $a_{1q}$  values of Table I

Also, in order to demonstrate that no worthwhile improvement in stability can be gained by careful blade design with respect to radial weight distribution or even by the use of blade tip weights, the increase in  $a_{1q}$  due to a 10 lb mass at each blade tip (*i.e.*, a 60 lb weight penalty in our case) is shown

$e/R$	0	0 1	0 2	0 3	0 4
$a_{1q}$ for neutral stability must be greater than	304	10 8	7 9	6 7	6 2
" Natural " $a_{1q}$ (Table I)	0515	0456	0406	0356	0295
$a_{1q}$ with blade tip weights of 10 lb each	106	090	076	063	049

It is seen that for all hinge offset ratios considered, a much increased value of  $a_{1q}$  is required, and we might conclude that from the stability

viewpoint alone, man-powered rotating wing flight will not be possible unless some form of automatic stabilisation is used

Here, we have revealed the need for yet another investigation, a comprehensive treatment of which would involve a study of such devices as the Bell, Hiller, Squire and Willmer gyrary systems, when used in conjunction with the man-powered rotor

Clearly, the full work cannot be undertaken in a paper such as this and yet we are unable to give a satisfactory answer to its title unless we have a few facts on which to base a conclusion

Therefore, we must try for form an initial assessment of the situation on the basis of a very brief analysis

### 5.3 The Controlled Aircraft

Dr G J Sissingh<sup>(11)</sup> has shown that "For the rapid subsidence of the disturbance of a dynamically unstable helicopter, periodic control displacements in phase with the attitude and the angular velocity of the helicopter are required"

M A P Willmer<sup>(12)</sup> has obtained results which "are sufficient to indicate how mechanical apparatus should be designed for practical application of the principle"

He reviews the shortcomings of those systems which are currently employed and proposes a "second-order" system which permits greater flexibility of choice of the constants  $\theta_\alpha$  and  $\theta_x$  in the controlling term

$$\theta = \theta_\alpha \alpha + \theta_x \dot{\alpha} \quad \text{--- (11)}$$

If we re-write the equations of motion including this term, the coefficients of the characteristic equation become

$$\left. \begin{aligned} \bar{A}_{z_c} &= 1 \\ \bar{A}_{z_c} &= g(a_{1u} + \frac{H_u}{W}) + \frac{M\bar{I}a_1}{I} (a_{1q} + \theta_\alpha) \\ \bar{A}_{1_c} &= \frac{M\bar{I}a_1}{I} \left[ g \frac{H_u}{W} (a_{1q} + \theta_\alpha) + \theta_\alpha \right] \\ \bar{A}_{o_c} &= g \frac{M\bar{I}a_1}{I} (a_{1u} + \frac{H_u}{W} \theta_\alpha) \end{aligned} \right\} \begin{array}{l} \text{Note - Suffix 'c'} \\ \text{refers to the} \\ \text{'controlled' aircraft} \\ \text{--- (12)} \end{array}$$

From these we see that

- (1)  $\theta_\alpha$  effectively increases  $a_{1q}$
- (2)  $\theta_\alpha$  and  $\theta_x$  increase the value of  $\bar{A}_1$
- (3) Neither  $\theta_\alpha$  nor  $\theta_x$  can have any effect without flapping hinge offset

\* Sissingh uses  $\theta = \theta_0 + \theta_s \sin \psi$  and  $\theta_s = (\theta_\alpha \alpha + \theta_x \dot{\alpha})$ . Our sign convention (Fig 12) shows the disc to be tilted against the disturbance ( $\alpha$ ) with respect to the uncontrolled disc

For the Bell, Hiller and Squire (dumb-bell) systems, it is shown in Ref 11 that the "in-phase" control constants are given approximately by

$$\left. \begin{aligned} \theta_\alpha &= \frac{\bar{v}^2}{K^2 + \bar{v}^2} \\ \text{and } \theta_\alpha \Omega &= \frac{K}{K^2 + \bar{v}^2} \end{aligned} \right\} \text{----- (13)}$$

and for Willmer's second order system they are

$$\left. \begin{aligned} \theta_\alpha / G &= \frac{L \bar{v}^2}{M + N \bar{v}^2} \\ \text{and } \theta_\alpha \Omega / G &= \frac{P + Q \bar{v}^2}{M + N \bar{v}^2} \end{aligned} \right\} \text{----- (14)}$$

In order to assess (as briefly as possible) their relative merits in this particular application we will treat the helicopter with its control device as a servomechanism (see Fig 13) and use the frequency response method of analysis

The feedback term is

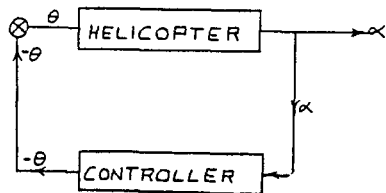
$$\theta(j\bar{v}) = -(\theta_\alpha + j\theta_\alpha \Omega \bar{v})\alpha(j\bar{v}) \text{----- (15)}$$

where  $\theta_\alpha$  and  $\theta_\alpha \Omega$  are already defined in the proper terminology by equations (13) and (14), and the helicopter transfer function is found to be

$$\alpha(j\bar{v}) = \frac{M\bar{T}\alpha_1 \left[ g \frac{H_u}{W} + j\Omega \bar{v} \right] \theta(j\bar{v})}{\sum_{i=0}^{i=3} \bar{A}_i (j\Omega \bar{v})^i} \text{----- (16)}$$

Using equations (13) to (16) the separate transfer function loci are plotted in Fig 14. The control defined by equations (13) and (15) is plotted for a value of  $K = 0.03$  since this is considered to be the minimum practical value

Fig 13  
The Helicopter/Controller  
Closed-Loop System



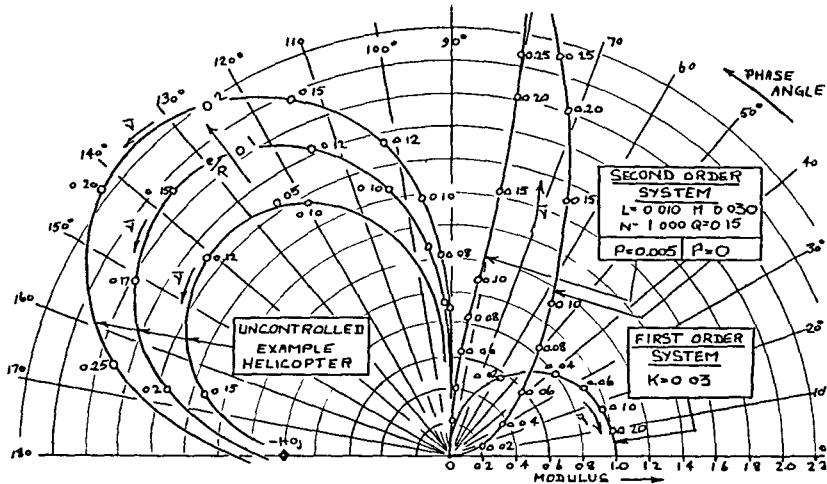


Fig 14 Helicopter and Controller Transfer Functions

From Fig 21 of Ref 12 we note that for  $\tan \psi_1 = 0.34$  and  $\tan \psi_2 = 0.5$  a second order system having only mechanical damping is in an "available-region" when  $A_{11} = 0.30$  and  $A_{12} = 0.35$ . Using these values we obtain the following approximate values for the controlling parameters

$$\left. \begin{aligned} L &= 0.010 \\ M &= 0.030 \\ N &= 1.000 \\ P &= 0.005 \\ Q &= 0.150 \end{aligned} \right\} \text{---(17)}$$

Many other combinations are possible if the specific damping is varied and if spring restraint and aerodynamic damping are included.

The transfer function locus for the Willmer bar has been plotted in Fig 14 using the values defined by (17). The same system but with  $P = 0$  is plotted for reference.

Nyquist's criterion for the stability of a closed-loop system states that the open-loop transfer function locus must not include the point  $-1 + j0$  in the complex plane.

From Fig 14 we see that the aircraft transfer function locus must be advanced in phase if the stability criterion is to be satisfied. We also note that the form of Willmer bar considered gives a greater phase advance than the first order system at the high values of  $\bar{v}$ .

The open loop transfer function locus for the helicopter with "first-order" control is shown in Fig 15 for several values of hinge offset, and the locus with "second-order" control is shown in Fig 16 for  $e/R = 0.1$  and several values of the controller gain factor ( $G$ ). Although we have by no means exhausted the possibilities of either system it appears that the

man-powered helicopter, with its C G on the rotor centre and having offset flapping hinges might be stabilised, and that we will be more likely to achieve good stability if we use a Willmer system. Whether we can design for the required "tee-bar" moment and stabilisation without using too much of the permissible weight margin, is a problem which requires a more detailed assessment than can be given at this stage.

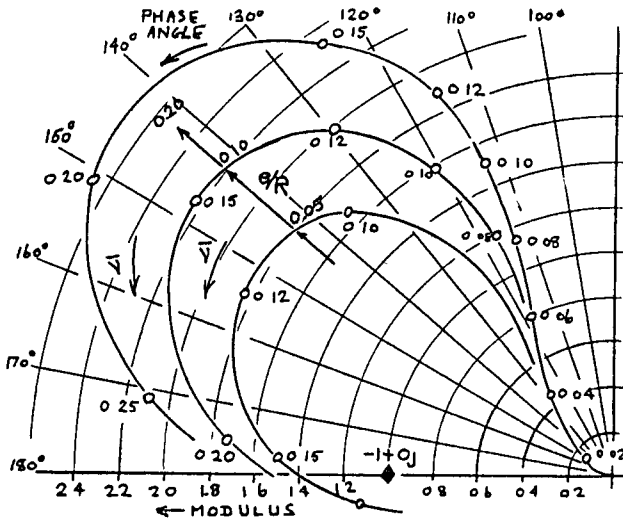


Fig 15 Transfer Function Locus of Helicopter with First Order Control

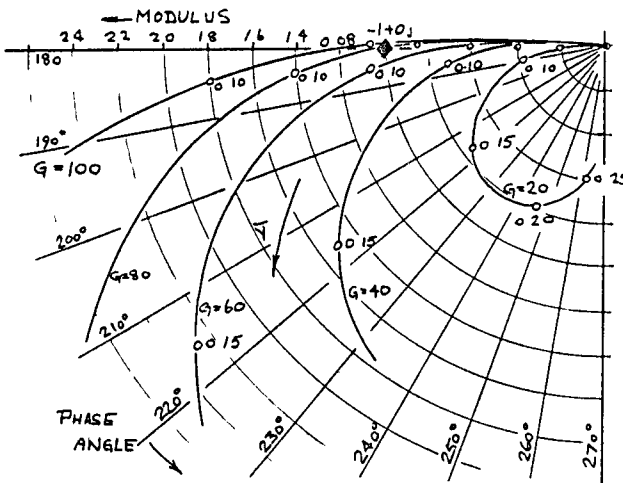


Fig 16 Transfer Function Locus of Helicopter with Willmer Control

There is, however, one very critical factor which has not been mentioned in the work so far. It has far reaching effects on both performance and stability, and has been intentionally omitted until now since it was thought that the consequences would be more readily appreciated when all other factors had been discussed.

It is that the blades of our machine, if allowed to flap freely, will assume a coning angle of about  $65^\circ$ !!!

Performance considerations dictate that we must restrict this. The "plan disc" area would be about one-fifth of the unconed area, and the ground cushion effect would be almost non-existent.

From the stability viewpoint, it would seem that if we are forced to

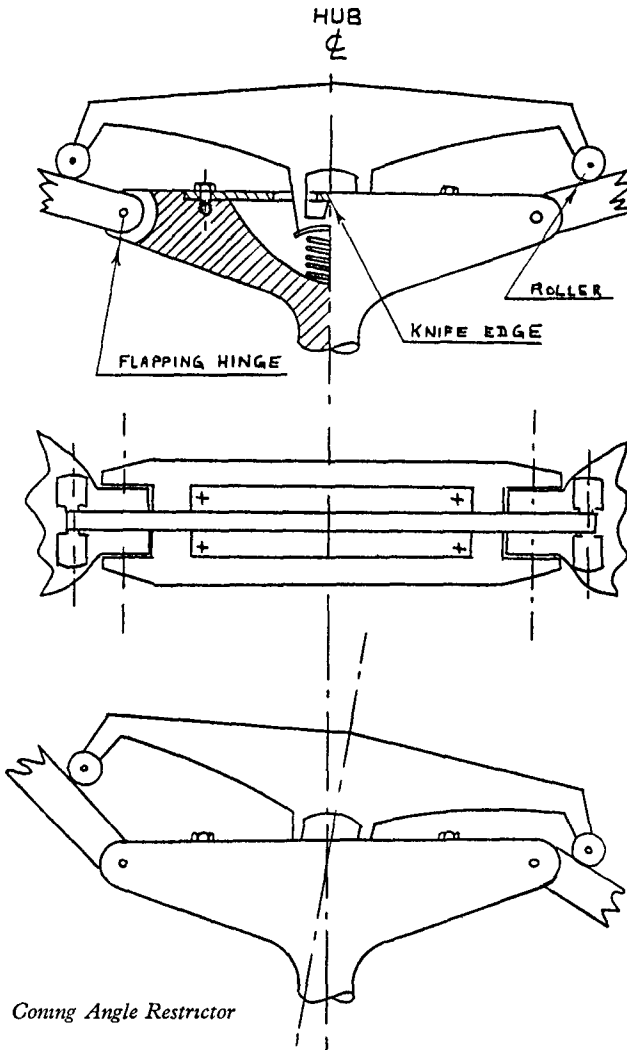


Fig 17 Coning Angle Restrictor

abandon the apparent benefits of flapping blades on offset hinges, we will experience considerable difficulty in achieving good stability by any other means

A device to permit *free* flapping about a *restricted* coning angle is required if the above performance and stability requirements are to be satisfied. Such a mechanism is shown in Fig 17. The sketch is only intended to suggest a principle and is drawn for a two-bladed rotor for ease of presentation. It is equally applicable for more than two blades when the central pivot is replaced by a "ball and cup" or point contact bearing. Again, a weight penalty is involved due to the device itself and to the resulting blade stresses.

## (6) DISCUSSION

In this paper we have compiled sufficient information to form only an initial assessment of the possibilities of man-powered hovering flight. The answers lead us to believe that such a feat is not impossible, even though many practical difficulties exist. It is almost certain that 50 years ago such a proposition would have been dismissed out of hand, owing to the practical difficulties alone. Basic aerodynamic relationships have not changed, of course, but our knowledge of engineering has increased considerably mainly due to efforts which have been made to master almost insurmountable problems such as this one. We have only to extrapolate the trend a little beyond the "point" provided by this investigation to conclude that man-powered rotating-wing flight *is a future possibility*.

We might ease the performance problem by considering the possibility of translational rotating-wing flight, but this would almost certainly lead to difficult control problems. Care must be taken not to dismiss this possibility on the basis of such an argument, however, since clearly a separate investigation is called for if a representative assessment is to be obtained.

This applies equally to other rotating-wing types such as the gyroplane, cyclogiro and convertiplane.

The gyroplane can fly at lower flight speeds than the fixed-wing aircraft, but a calculated comparison by Schrenk (given in Ref 9) shows that the minimum power required to fly is considerably greater for the gyroplane than for the corresponding fixed-wing type.

Mr Nonweiler has a good chance of improving the comparison from his point of view, since he has greater flexibility of choice concerning aspect ratio and more chance of attaining lower parasite drag values than those which would be available to the designer of a man-powered gyroplane.

It is difficult to fit the cyclogiro into our general theme. Shapiro<sup>(13)</sup> includes the counter cyclogiro in his list of "defensible lifting rotors," but mentions at a later stage that "One of the practical difficulties of the cyclogiro which has so far prevented its introduction as a lifting or controlling rotor is the fact that its efficiency critically depends on the embodiment of a rather complicated law of periodic pitch change."

It would seem that either we reject it on this basis alone, or we assume that the practical difficulty can be overcome by careful design, in which case, its other parameters of importance to the achievement of man-powered rotating wing flight must be investigated.



It is anticipated that the convertiplane which, for commercial uses, is designed to incorporate the advantages of both the helicopter and the fixed-wing aircraft, would in this application only magnify the disadvantages of each type

Assuming that in the above discussion no type has been unfairly assessed, and that the results for the hovering man-powered rotor are representative of what might be achieved in practice, we might conclude that

- (1) If the achievement of man-powered flight is the sole aim, then the fixed-wing configuration offers the greatest chance of success
- (2) If we are to enter into healthy competition with the fixed-wing designers for the achievement of the first man-powered flight (without assisted take-off or unrestricted power storage) then from the complete range of rotating-wing configurations we should choose the helicopter rotor, designed only for hovering in the ground cushion

For such a type, we would be able to conduct full scale tests on the basic machine and so make many adjustments prior to the first free flight. In fact, if the meaning of man-powered flight is not rigidly defined with respect to control and stability requirements, we might even claim the initial achievement with no stability problems at all by mounting our machine on a vertical pole through its mechanical axis so that only its vertical motion is unrestricted.

#### (7) CONCLUSIONS

(1) If a helicopter can be built for an all-up-weight less blades less occupant(s) of about 50 lb/occupant and for a blade weight of 0.45 lb/ft<sup>2</sup>, then it can be hovered for about 30 secs by the muscular efforts of the occupants, provided that the blade drag coefficient is not appreciably greater than 0.007, and that the rotor tilt can be restricted to less than 5°

(2) A serious decrease in performance will result if we fall far short of conditions similar to those described above

(3) Automatic stabilisation will be required

(4) Provided that the arrangement can be built within the weight margin, a Willmer system used in conjunction with flapping hinge offset and a coning restrictor can be used to stabilise the configuration chosen for the example calculations

(5) A major advantage of the helicopter type is that full scale tests can be conducted and many adjustments made prior to the first free flight

(6) Further investigations are required to assess the problems associated with (a) lightweight blade design, (b) choice of helicopter type, (c) forward flight of the man-powered helicopter, (d) the man-powered gyroplane, (e) the man-powered cyclogiro

#### (8) REFERENCES

<i>Ref No</i>	<i>Author</i>	<i>Title</i>
1	B S SHENSTONE	"The Problem of the Very Light Weight Highly Efficient Aeroplane" <i>Canadian Aeronautical Journal</i> , March, 1956
2	T NONWEILER	"The Man-Powered Aircraft—A Preliminary Assessment" C o A Note No 45, March, 1956

Ref No	Author	Title
3	T NONWEILER	"The Air Resistance of Racing Cyclists" C o A Report No 106, October, 1956
4	T NONWEILER	"The Man-Powered Aircraft—A Design Study" <i>R Ae Soc Journal</i> , October, 1958
5	O URSINUS	"Mitteilungen des Muskelflug—Instituts" Nos 1-6, "Flugsport," 1936 and 1937
6	Gessow & Myers	"Aerodynamics of the Helicopter" Macmillan, N Y, 1952
7	ABBOTT, DOENHOFF AND STIVERS	"Summary of Aerofoil Data" NACA Report No 824, 1945
8	DOMMASCH, SHERBY AND CONNOLLY	"Airplane Aerodynamics" Pitman, 1951
9	H FOCKE	"Autogiro and Helicopter Problems" <i>Schriften der Deutsch, Akad d Luftg</i> , Vol 22, 1937
10	J K ZBROZEK	"Introduction to Dynamic Longitudinal Stability of Single-Rotor Helicopter with Hinged Blades" R A E Report No Aero 2248, 1948
11	G J SISSINGH	"Investigations on Automatic Stabilisation of the Helicopter" R A E Report No Aero 2277, 1948
12	M A P WILLMER	"On the Generalised Simple System for Automatic Stabilisation of a Helicopter in Hovering Flight" <i>Journal of the Helicopter Association</i> , Vol 10, No 2, October, 1956
13	J SHAPIRO	"Principles of Helicopter Engineering" Temple Press, 1955

(9) LIST OF SYMBOLS

Symbol	Quantity	Units
a	Lift curve slope	
a <sub>1</sub>	Fore and aft flapping due to forward velocity and rate of pitch	
	$= a_{1u} u - a_{1q} \dot{\alpha}$	
a <sub>1u</sub>	$= \partial a_1 / \partial u$	ft <sup>-1</sup> sec
a <sub>1q</sub>	$= -\partial a_1 / \partial \dot{\alpha}$	sec
$\frac{A}{A_1}$	Total disc area	ft
b	Coefficients of characteristic equation	
b	Total number of blades	
C(x)	Blade chord at station x	ft
C <sub>D</sub>	Profile drag coefficient	
C <sub>c</sub>	Equivalent thrust chord	ft
	$= \frac{\int_0^1 C(x) x^2 dx}{\int_0^1 c^2 dx}$	
C <sub>L</sub>	Sectional lift coefficient	
C <sub>0</sub>	Blade root chord	ft
C <sub>T∞</sub>	Rotor free air thrust coefficient	
D	Rotor diameter	ft
Df	Fuselage drag	lb

<i>Symbol</i>	<i>Quantity</i>	<i>Units</i>
$Df_u$	$= \partial Df / \partial u$	lb ft <sup>-1</sup> sec
$e$	Flapping hinge offset	ft
$g$	Acceleration due to gravity	ft sec <sup>-2</sup>
$H$	Longitudinal force in disc plane	lb
$H_u$	$= \partial H / \partial u$	lb ft <sup>-1</sup> sec
$I$	Moment of inertia of helicopter	slugs ft <sup>2</sup>
$I_1$	Moment of inertia of one blade about its flapping hinge	slugs ft <sup>2</sup>
$k$	Specific blade weight	lb ft <sup>-2</sup>
$K$	Specific damping of stabiliser	
$M_{\bar{T}}$	"tee bar" moment followed by equations	lb ft
$M_{\bar{T}}$	$= \frac{1}{2} S e a_1$ or $\frac{1}{2} S e (a_1 - \theta)$	
$M_{\bar{T} a_1}$	$= \partial M_{\bar{T}} / \partial a_1 = \partial M_{\bar{T}} / \partial (a_1 - \theta) = \frac{1}{2} S e$	lb ft.
$n$	Stabiliser linkage ratio = 1 for "dumb-bell"	
$P_1$	Induced power	lb ft sec <sup>-1</sup>
$P_0$	Profile power	lb ft sec <sup>-1</sup>
$P_{av}$	Power available	lb ft sec <sup>-1</sup>
$H P_{av}$	Housepower available	
$r$	Radial distance along a blade from the rotor centre	ft
$R$	Rotor radius	ft
$R_N$	Reynold's Number	
$S$	Total centrifugal force of all blades	lb
$T$	Rotor thrust in ground cushion	lb
$T_{\infty}$	Rotor thrust in free air	lb
$T_H$	Time to half amplitude	sec
$u$	Horizontal velocity increment	ft sec <sup>-1</sup>
$V_T$	Rotor tip speed	ft sec <sup>-1</sup>
$w$	disc loading	lb ft <sup>-2</sup>
$W$	All-up weight	lb
$W_B$	Blade weight	lb
$x$	$= r/R$	
$Z$	Rotor height above ground	ft
$\alpha$	Disturbance in pitch angle	
$I_0$	Blade inertia number	
$E$	Ratio of inertia forces	
	$= \frac{\text{moment of inertia of a blade in pitch}}{\text{moment of inertia of stabiliser}}$	
$\epsilon$	permissible tilt angle	
	$= \sin^{-1} Z/R$	
$\eta$	Efficiency factor	
$\theta$	Blade pitch angle	
$\theta_0$	Mean pitch setting of rotor blade	
$\theta_s$	Fore and aft cyclic pitch	
$\theta_a$	$= \partial \theta / \partial a$	
$\theta_a$	$= \partial \theta / \partial a$	
$\theta_a$	$= \partial \theta / \partial a$	
$\lambda_1$	First root in stability equation	
$\nu \Omega$	Frequency of oscillation	sec <sup>-1</sup>
$\bar{\nu}$	$= \gamma / \Omega$	
$\rho$	Density of air	slugs ft <sup>-3</sup>
$\sigma$	$= \frac{b c_e R}{A}$	
$\psi$	Azimuth angle	
$\psi_1$	Phase angle between blade and stabiliser	
$\Omega$	Rotor speed	sec <sup>-1</sup>

(10) APPENDIX I

Derivation of the Performance Equation

For steady hovering flight,

$$\eta \frac{P_{av}}{W} = \frac{P_i}{W} + \frac{P_o}{W} \quad \text{--- --- --- --- --- } A(1)$$

From the momentum theory assuming constant inflow over the disc we have,

$$P_i = \left[ \frac{T_{\infty}^3}{2\rho A} \right]^{1/2} = \left[ \frac{T^3}{2\rho A} \right]^{1/2} \left[ \frac{T_{\infty}}{T} \right]^{3/2}$$

and for a machine which is hovering in the ground cushion  $T = W$

$$\therefore \frac{P_i}{W} = \left[ \frac{W}{2\rho} \right]^{1/2} \left[ \frac{T_{\infty}}{T} \right]^{3/2} \quad \text{--- --- --- --- --- } A(2)$$

The profile power  $P_o$  is given by

$$P_o = \frac{1}{2} \rho V_T^3 b R \int_0^1 c_D c x^3 dx$$

and if we consider a blade which is twisted and shaped in planform to give constant induced velocity and angle of attack distribution along its radius we may write

$$\text{and } C = \frac{2}{3} \frac{C_e}{x} \quad (Neglecting Reynold's Number Effects)$$

The expression for profile power then becomes

$$P_o = \frac{1}{9} c_D \rho V_T^3 a A.$$

If we now express the tip speed in terms of the free air thrust coefficient and the ground cushion thrust factor, we get

$$V_T^3 = \left[ \frac{T}{\rho A c_{T_{\infty}}} \right]^{3/2} \left[ \frac{T_{\infty}}{T} \right]^{3/2}$$

Then the profile power required per pound of weight lifted is

$$\frac{P_o}{W} = \frac{1}{9} \left[ \frac{W}{\rho} \right]^{1/2} \left[ \frac{c_D}{c_{T_{\infty}}^{1/2}} \right] \left[ \frac{1}{c_{T_{\infty}/a}} \right] \left[ \frac{T_{\infty}}{T} \right]^{3/2} \quad \text{--- --- --- --- --- } A(3)$$

Substituting in equation A (1) from equations A (2) and A (3) gives

$$\eta \frac{P_{av}}{W} = \left[ \frac{\omega r^{\frac{3}{2}}}{\rho} \right] \left[ \frac{T_{\infty}}{T} \right]^{3/2} \left\{ \frac{1}{\sqrt{2}} + \frac{1}{9} \left[ \frac{C_D}{C_{T_{\infty}}^{1/2}} \right] \left[ \frac{1}{C_{T_{\infty}}/\alpha} \right] \right\} \text{----- A (4)}$$

In practice, we should be able to achieve a machine which is 85% efficient. A typical power loss breakdown might be

Induced Losses	2%
Tip Losses	2%
Transmission	5%
Torque Reaction	6%

Putting  $\eta = 0.85$ ,  $\rho = 0.00238$  and  $P_{av} = 550 \text{ HP}_{av}$ , equation A (4) becomes

$$\frac{W}{P_{av}} = \frac{323}{\omega^{1/2} \left[ \frac{T_{\infty}}{T} \right]^{3/2} \left\{ 1 + 0.157 \left[ \frac{C_D}{C_{T_{\infty}}^{1/2}} \right] \left[ \frac{1}{C_{T_{\infty}}/\alpha} \right] \right\}} \text{----- A (5)}$$

An expression for the total blade weight  $W_B$  in terms of its equivalent weight/unit area is

$$W_B = k \sigma A \text{----- A (6)}$$

Where  $k$  accounts for the difference between the "blade area" solidity and the "equivalent thrust chord" solidity ( $\sigma$ )

Equation A (6) can be re-written as

$$\frac{W_B}{P_{av}} = \frac{k\alpha}{\omega} \text{----- A (7)}$$

Combining equations A (5) and A (7) gives

$$\frac{W - W_B}{P_{av}} = \frac{323 \left( 1 - \frac{k\alpha}{\omega} \right)}{\omega^{1/2} \left[ \frac{T_{\infty}}{T} \right]^{3/2} \left\{ 1 + 0.157 \left[ \frac{C_D}{C_{T_{\infty}}^{1/2}} \right] \left[ \frac{1}{C_{T_{\infty}}/\alpha} \right] \right\}} \text{----- A (8)}$$

which is the equation given in Section 3.2

The equations used for the estimation of the stability parameters are as follows

$$a_{1u} = \frac{2}{\pi R} \left( \frac{4}{3} \theta_0 - \lambda \right)$$

$$H_u/W \approx \frac{C_D}{4} \frac{\rho \alpha}{\omega} \pi R$$

$a_{1q}$  includes the "Amer" effect and has been taken as

$$a_{1q} = \frac{16}{\delta_0 \Omega} \cdot \frac{3}{2} \left( 1 - 0.29 \frac{\theta_0}{c_T/\alpha} \right)$$

$$\approx \frac{12}{\delta_0 \Omega}$$

and since  $I$  will be greater than  $bx I_1$  ( $c/R = 0$ ) it has been taken as

$$I \approx \frac{6 I_1 (c/R=0)}{0.9}$$

TABLE I—ESTIMATED STABILITY PARAMETERS FOR THE EXAMPLE HELICOPTER

$c/R$	0	0.1	0.2	0.3	0.4
$a_{1u} \times 10^3$	5.3	5.3	5.3	5.3	5.3
$H_u/W \times 10^3$	0.1	0.1	0.1	0.1	0.1
$I_1$	299	265	237	207	172
$\delta_0$	94	106	119	136	164
$a_{1q}$	0515	0456	0406	0356	0295
$I$	2000	2000	2000	2000	2000
$M_{Ta1}$	0	886	1680	2320	2690
$\bar{A}_3$	1	1	1	1	1
$\bar{A}_2$	174	194	208	215	204
$\bar{A}_1 \times 10^4$	0	661	1 100	1 330	1 280
$\bar{A}_0$	0	076	144	199	231