ESTIMATION OF THE PARAMETERS OF THE EARTH'S POLAR MOTION

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The parameters of the Earth's polar motion have been estimated by a method which accounts for observational error and which treats two possible statistical models for the polar motion excitation process. Both a 100 year and a 78 year long polar motion series have been used to estimate the Chandler wobble frequency, F_c, and quality factor, Q_c; the noise level in the polar motion data; and the magnitude of the excitation process. Discussion of the annual component of polar motion has been omitted because its frequency (1 cycle per year (cpy)) and cause (annual motion of air and water) are presumed known.

1. INTRODUCTION

A traditional approach to estimating the polar motion parameters, particularly F_c and Q_c, has been to use the periodogram method of fitting sines and cosines of various frequencies to the polar motion data. However, in 1940 Sir Harold Jeffreys suggested that since the wobble excitation process is likely to be random, a statistical estimation method ought to be superior to periodogram analysis, because a randomly excited damped oscillator will not display purely sinusoidal behavior. The estimation method used here is based upon such a statistical approach.

If the polar motion excitation behaved as an independent random process, then its Fourier power spectrum would be white, containing the same variance at all frequencies. One would then expect the polar motion power spectrum to show a symmetric peak at the Chandler frequency, like the dotted line in Figure 1. The solid line in Figure 1 shows that the spectrum of the polar motion data (for the years 1901-1970, with annual term removed) is similar to the dotted curve near the Chandler frequency, but much larger elsewhere. Apart from the Chandler frequency peak, the spectrum of the data is best described as "red" since it rises toward zero frequency. It appears that the data is contaminated by a substantial amount of "red" noise. Rather than introducing separate corrections to account for the presence of this noise, I will estimate noise and polar motion parameters simultaneously, thereby hopefully

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eliminating any noise bias.

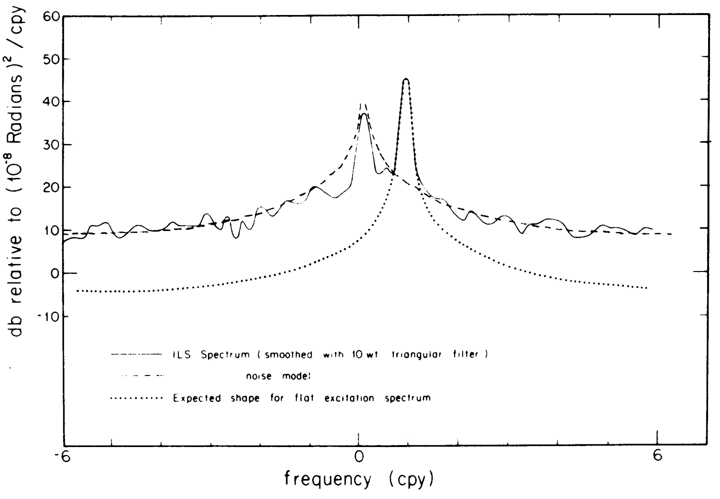

Fig. 1. The power spectrum of the 1901-1970 ILS series suggests that a substantial amount of red noise is contaminating the data.

2. THE DATA

The polar motion time series used in this study has been compiled from the following sources for the periods indicated: 1878-1977 data were taken from the tabulation by Rykhlova (1969). A gap of several years duration in the component along 90° east longitude was interpolated by hand using the Greenwich component as a guide. 1890-1899 data were taken from Stoyko (1972, Tables 7a, 7b). 1900-1967 data were the ILS pole positions reported by Vicente and Yumi (1969, 1970). 1968-1977 data were the 5-day raw pole positions given in the BIH annual reports and Circular D. The 5-day data were smoothed using a 7 weight triangular filter in preparation for the interpolation described below.

Adjustments were made to the means of the 1878-1899 and 1890-1899 series to give a smooth transition where they were joined with each other and with the ILS series. A cubic spline interpolator was applied separately to the two components of the 1878-1977 series to resample them at uniform intervals of 1/12 year, resulting in approximately mid-month samples. Annual and higher harmonic components were removed by subtracting the means of the same-named months from the monthly samples.

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3. ESTIMATION OF THE PARAMETERS FROM THE DATA

3.1. Notation and Fundamental Equations

Let M_t be the complex valued time series of true polar motion (uncontaminated by noise), and let X_t be the complex valued time series of the position of the excitation axis. Variation of the real part of M_t or X_t represents motion along the Greenwich meridian, while variation of the complex part represents motion along the 90° East meridian. The relationship between M_t and X_t is

$$M_t = -\alpha X_t + e^{\alpha} M_{t-1}, \tag{1}$$

where $\alpha = 2\pi i F_c T(1+i/2Q_c)$, $i = \sqrt{-1}$, and T is the sample interval. $Z = M_t + N_t$ are the data which are contaminated by the noise N_t. I will assume that N_t satisfies the equation

$$N_t = N\eta_t + N_{t-1}, \tag{2}$$

where N is a real-valued constant and η_t is a unit variance, complex-valued white noise process whose real and imaginary parts are independent, identically distributed, zero mean, Gaussian random variables. The dashed line in Figure 1 shows the spectral shape of a time series obeying (2) and demonstrates the similarity with the ILS spectrum apart from the Chandler frequency peak.

Letting the first difference of the data be $D_t = Z_t - Z_{t-1}$ and using (2) to describe N_t, D_t satisfies the equation

$$D_t = -\alpha X_t + \alpha X_{t-1} + N\eta_t - e^{\alpha} N\eta_{t-1} + e^{\alpha} D_{t-1}. \tag{3}$$

3.2. Two Models for the Excitation Process

Model 1: The simplest assumption that might be made about X_t is that it is described by the equation

$$X_t = X\varepsilon_t, \qquad \text{(Model 1)} \tag{4}$$

where ε_t is a unit variance white noise process and X is a real-valued constant. Wilson and Haubrich (1976) show that Model 1 is an appropriate assumption if atmospheric variation is the main contributor to X_t.

Model 2: If earthquakes are a major contributor to X_t, as suggested by O'Connell and Dziewonski (1976), then the power spectrum of X_t is probably red. A simple model for X_t in this event is that

$$X_t = X\varepsilon_t + X_{t-1}, \qquad \text{(Model 2)} \tag{5}$$

where X is a real-valued constant (but with a value different from that of Model 1) and ε_t is again a unit variance white noise process.

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3.3. Estimating Parameters and Confidence Intervals

Box and Jenkins (1970, p. 122) show that (3) has a statistically equivalent description as

$$U_t = D_t - e^{\alpha} D_{t-1} - (A/V) U_{t-1}, \tag{6}$$

where A and V are chosen to make the autocovariances of D_t the same in both (3) and (5), and where U_t is a white noise process with variance V.

For Model 1: $A = -\left(|\alpha|^2 X^2 + e^{\alpha} N\right);$

$V = .5[h_1 + (h_1{}^2 - 4A^2)^{\frac{1}{2}}]$, where $h_1 = (2|\alpha|^2 X^2 + N^2 + |e^{\alpha}|^2 N^2)$.

For Model 2: $A = -(e^{\alpha} N^2);$

$V = .5[h_2 + (h_2{}^2 - 4A^2)^{\frac{1}{2}}]$, where $h_2 = (|\alpha|^2 X^2 + N^2 + |e^{\alpha}|^2 N^2)$.

For given values of the polar motion parameters, U_t is generated recursively from the data using equation (6) with a starting value U_o of zero. By searching with a computer, one may find parameter values which minimize the variance of U_t and hence are the least squares and maximum likelihood estimates. The search is not difficult because Q_c and F_c are already fairly well known, and because there is only one additional parameter (X/N) contained in (A/V) in (6). X and N are determined separately from the final minimum variance of U_t. The minimum variance of U_t measures how well the assumed model fits the data, with a smaller value indicating a better fit.

Monte Carlo experiments with artificial data were used to demonstrate that Model 2 estimates of all parameters are unbiased. Estimates of Q_c, F_c, and N made using Model 1 were also unbiased, but Model 1 consistently estimated X to be one quarter of its true value. The values in Table 1 have been corrected for this bias. Intervals of confidence were obtained using Box and Jenkins' method (1970, p. 229) which examines changes in the variance of U_t as a function of Q_c, F_c, and X/N.

4. RESULTS AND CONCLUSIONS

Table 1 shows estimates of the various parameters obtained from the entire 1878-1977 series and from the 1900-1977 portion, and also gives power spectral densities of X_t at the frequency, F_c, denoted by $S_x(F_c)$. Estimates of Q_c, F_c, N, and $S_x(F_c)$ obtained from both time series and both models are approximately the same.

Model 1 was found to fit both time series slightly better than Model 2, suggesting that X_t has a white rather than a red power spectrum. Both

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Table 1. Estimates and Intervals of 90% Confidence.

Data	X 10^{-8} Radians	$S_x(F_c)$ (10^{-8} Radians^2/cpy)	Q_c Dimensionless	F_c cpy	N 10^{-8} Radians
Model 1 - White Excitation Spectrum					
1879-1977	15.3 (13.1-18.5)	19.5 (14.3-28.6)	59 (26->1000)	.848 (.841-.857)	19.6 (19.3-19.9)
1900-1977	13.6 (11.2-16.4)	15.5 (10.5-22.3)	61 (27->1000)	.845 (.837-.853)	18.4 (18.2-18.7)
Meteorological Variation (Wilson and Haubrich, 1976)	7-10	5-8			
Model 2 - Red Excitation Spectrum					
1878-1977	6.9 (5.9-8.2)	20.7 (15.1-29.2)	60 (27->1000)	.849 (.841-.857)	19.7 (19.4-20.0)
1900-1977	6.1 (5.2-7.2)	16.3 (11.6-22.7)	62 (27->1000)	.845 (.837-.853)	18.5 (18.3-18.8)
Earthquakes (O'Connell and Dziewonski, 1976)	2	2			

Models 1 and 2 fit the 1900-1977 series considerably better than the 1878-1977 series, suggesting that the pre-1900 data do not have the same statistical description as the later data.

Estimates of F_c shown in Table 1 are slightly larger than the one obtained by Jeffreys (1972), perhaps because Jeffreys did not use precisely the same data set. The difference may also be due to the fact that red noise in the data would tend to make Jeffreys' estimates of F_c too small, while the estimates in Table 1 are presumably not biased in this way.

Estimates of Q_c are nearly the same as Jeffreys' (1972) value. As shown in Table 1, the upper confidence limit for Q_c exceeds 1000 for both models and both time series. However, Q_c is probably not much larger than 1000, due to dissipation in the mantle and oceans. Thus, it appears that physical reasoning may provide a more stringent upper bound on Q_c than does the polar motion data. For Model 1, Q is less well determined for the longer time series, since the confidence interval is slightly wider. This suggests that the pre-1900 data has not been used properly and that it needs to be analyzed in some other way.

From the Model 2 results, the earthquake effect is too small by a factor of about 3 in amplitude (X) or a factor of 9 in power ($S_X(F_c)$) to account for the Chandler wobble excitation. From the Model 1 results, the estimated meteorological effect is too small by a factor of roughly 2 or 3 in power, or $\sqrt{2}$ to $\sqrt{3}$ in amplitude to account for the Chandler wobble excitation. Since there is some evidence for correlation between atmospheric variation and polar motion, as shown by Wilson and Haubrich (1976), the discrepancy in amplitude is perhaps the best measure of how much of the Chandler wobble excitation remains to be explained.

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