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THE BARGAINING SET OF A REINSURANCE MARKET

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1. OBJECTIONS AND COUNTER-OBJECTIONS: THE BASIC MECHANISMS OF THE BARGAINING SET

This paper uses the same notations and some of the results of BATON and LEMAIRE (1981). The reader is referred to that work for more details about the classical risk exchange model, which will not be recalled here. The main result of that former paper was to characterize the core of the market in the case of exponential utilities, and to show that it is never empty. Since the core always exists, and since it is such an intuitive notion, one might wonder why we introduce here a much more complicated concept. The reason is that the core is presently subject to a heavy fire of criticisms–both experimental and theoretical–from leading researchers in game theory; they claim that the core is much too static, that it does not take into account the real dynamics of the bargaining process, that it does not introduce the full spectrum of negotiation threats of the traders. Indeed, experimental data consistently produce final payoffs that lie outside the core, but within the bargaining set (abbreviated: b.s.). We shall attempt to illustrate those criticisms in 4. We shall define the b.s. in a general non transferable game in 2, and characterize it in the special case of a 3-company reinsurance market in 3, but first of all we would like to explain intuitively the basic mechanisms of the b.s. by means of a simple example (with transferable utilities).

The basic difficulty in modelling a negotiation process is to express what is the purpose of the game. Certainly, the objective is not just to get the maximal amount of profits, because if everyone demands the highest payoff he can obtain in a coalition, no agreement will be reached; the goal of the process is to attain some state of stability, to which the bargainers should agree if they want any agreement to be enforced. This stability should reflect in some way the power of each player, which results from the rules of the game, his initial situation, his attitude towards risk, . . .

A bargaining process is a multi-criteria situation, in which the players certainly attempt to maximize their payoffs, but also try to enter into a "safe" or "stable" coalition. Very often, a player might be willing to give up some of his profits in order to enter a coalition that he thinks has less chances to fall apart. If people do not feel safe enough, they often do not enter a coalition even if they can win more in it. This demand for safety is usually considered legitimate and a sound way to convince the partners to get a smaller amount of profit in order that no one in the coalition will feel deprived. There is an incentive for "fair play". Most people will accept that "if all things are equal", the benefits of cooperation should be shared equally. If "things are not equal", people will still be happy with their coalition if they agree that the "stronger" players get more. During the negotiations, each player will consequently try to convince his partners that in some sense he is strong; this can be done in various ways, among which an important factor is his ability to show that he has other, perhaps better alternatives. His partners, besides pointing out their own alternatives, may argue in return that even without his help they can keep their proposed shares. It is this dynamic process of "threats" and "counter-threats" that the b.s. attempts to formulate mathematically; according to this theory, stability is reached if all objections can be answered by counter-objections.

Consider the following 3-person game:

$$v(1) = v(2) = v(3) = v(123) = 0$$
$$v(12) = v(13) = 100$$
$$v(23) = 50,$$

where $v(S)$ is the monetary amount the members of coalition S may share, providing of course this coalition forms. When no confusion is possible, we shall simply write e.g. 12 for $\{12\}$, the coalition between players 1 and 2. According to core theory, there is no stable payoff: the core is empty; for instance the players will not agree upon a payoff like (75, 25, 0) (75 for player 1, 25 for player 2, 0 for player 3) because it is dominated by (76, 0, 24). b.s. theory, on the other hand, claims that such a payoff is stable; if player 1 threatens 2 of a solution (76, 0, 24), this objection can be met with the counter-objection (0, 25, 25): player 2 shows that, without the help of player 1, he can protect himself and keep his payoff at the level 25, player 3 receiving more in the counter-objection than in the objection. In the same way, a counter-objection of (75, 0, 25) can destroy the effectiveness of the objection (0, 27, 33) of player 2. So if a proposal (75, 25, 0) arises during the bargaining process, it is probable that it will be the final payoff, since any objection, by either 1 or 2, can be countered by the other partner.

Thus, in addition to all the undominated solutions (the core), the b.s. also contains all payoffs against which there exists objections, providing they can be met by counter-objections. The b.s. for the example consists of the 4 points

(0, 0, 0)
(75, 0, 25)
(75, 25, 0)
(0, 25, 25).

A proposal like (80, 20, 0) is unstable; player 2 can object that he and player 3 will get more in (0, 21, 29); player 1 has no counter-objection, because he cannot keep his 80 while offering player 3 at least 29.

2. The bargaining set of a non transferable game (Peleg (1969))

Let $N = \{1, \ldots, n\}$ be the set of all the players, and $S \subset N$ any sub-coalition. Denote $\bar{y}^S = \{y_i,\ i \in S\},\ \forall\ S$; y_i will here represent the payoff (or utility) to player i. Let $v(S)$ be the set of all the payoffs that can be achieved by coalition S. $\bar{y}^S \in v(S)$ is Pareto-optimal (for S) if there is no $\bar{y}'^S \in v(S)$ such that $y'_i \geqslant y_i,\ \forall\ i \in S$, with at least one strict inequality.

Definition 1

A coalition structure is any partition s of N into coalitions: $s = \{S_1, \ldots, S_p\}$.

Definition 2

A payoff configuration (abbreviated p.c.) is a couple $(\bar{y}, s)$, where s is a coalition structure and $\bar{y} = (y_1, \ldots, y_n)$ a payoff vector such that $\bar{y}^S$ is Pareto-optimal for each S of s. It is a possible outcome of the game, a sharing of the profits that satisfy the rules of the game, once the coalition structure has been decided. Note that the coalition structure is supposed to be given and fixed. The problem is to share among the members of each coalition the total payoff that this coalition can achieve on its own.

Definition 3

Let $(\bar{y}, s)$ be *a* p.c. If $T \in s$, and if $S \subset T$ $(S \neq T,\ S \neq \phi)$, $\bar{z}^Q$ is an objection of S against $T \setminus S$ with respect to $(\bar{y}, s)$ if

$$\begin{cases} Q \neq \phi \\ Q \subset N \\ Q \cap T = S \\ \bar{z}^Q \in v(Q) \\ z_i > y_i \quad \forall\ i \in Q \end{cases}$$

Interpretation

A p.c. is proposed. Some of the players of a coalition $(S \subset T)$ are dissatisfied with their payoff and threaten to destroy the existing structure by creating a new group Q. This threat is credible since the dissidents propose to their new potential partners more than now $(z_i > y_i,\ \forall\ i \in Q)$, and are in a position to keep their promises $(\bar{z}^Q \in v(Q))$.

Definition 4

Suppose the grand coalition N has decided to form. The *core* is the set of all payoffs against which there exists no objection.

Definition 5

Let j_0 be a member of $T \setminus S$. $\bar{t}^R$ is a counter-objection of type I of j_0 against $\bar{z}^Q$ with respect to $(\bar{y}, s)$ if

$$\begin{cases} R \subset N \\ R \cap Q = \phi \\ j_0 \in R \\ \bar{t}^R \in v(R) \\ t_i \geqslant y_i \quad \forall\ i \in R \end{cases}$$

Interpretation

A player j_0, threatened by the objection $\bar{z}^Q$, claims that, without the help of the dissidents $(R \cap Q = \phi)$, he can protect himself by forming a new coalition R, that is in position $(\bar{t}^R \in v(R))$ to give to each of its members at least its former payoff $(t_i \geqslant y_i)$.

Definition 6

$\bar{t}^R$ is a counter-objection of type II of j_0 against $\bar{z}^Q$ with respect to $(\bar{y}, s)$ if

$$\begin{cases} R \subset N \\ R \cap Q \neq \phi \\ S \setminus R \neq \phi \\ j_0 \in R \\ \bar{t}^R \in v(R) \\ t_i \geqslant y_i \quad \forall\ i \in R \\ \exists\ k \in R \cap Q \text{ such that } t_k \geqslant z_k \end{cases}$$

Interpretation

j_0 is in a slightly more difficult situation here, since in order to protect his payoff he has to "divide and rule" and to dynamite coalition Q. He claims he can form a coalition R, that contains some (but not all: $S \setminus R \neq \phi$) of the members of Q $(R \cap Q \neq \phi)$. This new coalition can offer $(\bar{t}^R \in v(R))$ to each of its members (including j_0) at least its initial payoff $(t_i \geqslant y_i)$ and can break Q apart by offering to (at least) one of its members at least what he was promised in Q $(t_k \geqslant z_k)$.

Definition 7

An objection $\bar{z}^Q$ is strongly justified if

(*i*) no $j \in T \setminus S$ has a counter-objection of type II
(*ii*) $\exists\ j_0 \in T \setminus S$ that has no counter-objection of type I.

Interpretation

z^Q is a powerful threat, since none of the players left aside by Q can break it apart, and at least one of those will suffer, since he cannot protect his initial payoff.

Definition 8

A p.c. is weakly stable if no strongly justified objection against it exists.

Definition 9

The *bargaining set* of the game is the set of all the weakly stable p.c. It is the set of the all payoffs against which there exists no strongly justified objection.

PELEG (1969) has shown that the b.s. is always non-empty. A look at definitions 4 and 9 shows that the core is included in the b.s.

The core is the set of payoffs against which there exists no objection. The b.s. is the set of payoffs against which there exists no strongly justified objection.

3. THE BARGAINING SET OF THE 3-COMPANY REINSURANCE MARKET WITH EXPONENTIAL UTILITIES

We shall explicitly compute the b.s. of a general 3-company reinsurance market, where all companies use exponential utility functions (with parameters $1/\alpha_i$). The characterization of the b.s. of larger markets would not introduce many theoretical complications, but even for a 4-company economy it would take several pages only to list all the conditions. We shall state without proofs our theorems; the results are intuitive, their proofs are of the same nature as in BATON and LEMAIRE (1981), but much more lengthy. We know that the core of the market is non void. As the b.s. always includes the core, we can state without using PELEG's theorem:

Theorem 1

The b.s. of an n-company reinsurance market is never empty.

Since the b.s. for any other coalition structure than $\{N\}$ is obvious, we shall only characterize the b.s. for that grand coalition. We shall use the abbreviation $\bar{y}$ instead of $(\bar{y}, \{123\})$.

Since we assume exponential utilities, it is well known that the Pareto-optimal treaties take the form

$$y_i = \frac{\alpha_i}{\sum_{j \in S} \alpha_j} (\sum_{j \in S} x_i) + y_i(0) \;\; \forall \; S, \; \forall \; i \in S, \text{ with } \sum_{i \in S} y_i(0) = 0.$$

Instead of characterizing the b.s. by the utilities resulting from those treaties, it is equivalent to work with the side-payments or fees to the pool $y_i(0)$.

Theorem 2

No company has an objection against the other two, with respect to a p.c. $\bar{y} = (y_1, y_2, y_3)$.

This result is an obvious consequence of the individual rationality condition.

Theorem 3

(z_2, z_3) is an objection of $\{23\}$ against $\{1\}$ with respect to $\bar{y}$ iff

$$\begin{cases} z_i = \dfrac{\alpha_i}{\alpha_2+\alpha_3}(x_2 + x_3) + z_i(0) \qquad i = 2,3 \\ z_2(0) + z_3(0) = 0 \\ z_2(0) < y_2(0) - P_2^{23} + P_2^{123} \\ z_3(0) < y_3(0) - P_3^{23} + P_3^{123} \end{cases}$$

Recall that P_2^{123} for instance denotes the exponential utility premium of company 2, if it agrees to form a 3-player coalition with 1 and 3, see BATON-LEMAIRE (1981).

Note

The two last conditions are equivalent to

$$-y_3(0) + P_3^{23} - P_3^{123} < z_2(0) < y_2(0) - P_2^{23} + P_2^{123}$$

Interpretation

We know from our former paper that exponential utility premiums play the role of certainty equivalents: everything happens as if each company evaluates its portfolio by the corresponding exponential utility premium. So P_2^{23} - P_2^{123} is the certainty equivalent of the (positive or negative) profit company 2 enjoys from participating to the global pool $N = \{123\}$ instead of playing with 3 only. 2 and 3 have an objection iff their fixed contribution is too high, i.e. iff it is possible for them to secede from N, with a resulting side-payment $z_i(0)$ $(i = 2,3)$ lower than the former fee $y_i(0)$ less the "secession cost" P_i^{23} - P_i^{123}.

Corollary

$\{23\}$ has no objection against $\{1\}$ with respect to $\bar{y}$ iff

$$y_1(0) \geqslant P_2^{123} + P_3^{123} - P_2^{23} - P_3^{23}$$

Interpretation

The contribution of company 1 is already so big that it cannot be objected against it.

Theorem 4

Faced to an objection, no company can react with a counter-objection of type I.

This result again derives from the individual rationality condition.

Theorem 5

Let (z_2, z_3) be an objection of $\{23\}$ against $\{1\}$. (t_1, t_2) is a counter-objection of type 2 of $\{1\}$ iff

$$\begin{cases} t_i = \dfrac{\alpha_i}{\alpha_1+\alpha_2}(x_1+x_2) + t_i(0) & i=1,2 \\ t_1(0) + t_2(0) = 0 \\ t_1(0) \leqslant y_1(0) - P_1^{12} + P_1^{123} \\ t_2(0) \leqslant z_2(0) - P_2^{12} + P_2^{23} \end{cases}$$

Note

The two last conditions are equivalent to

$$-z_2(0) + P_2^{12} - P_2^{23} \leqq t_1(0) \leqq y_1(0) - P_1^{12} + P_1^{123}$$

Interpretation

It is possible for company 1 to entice company 2 to break coalition {23} apart; the last condition states that 2 will get more by playing with 1 than with 3; the next-to-last condition states that 1 will receive at least what he had before the objection.

Theorem 6

A p.c. belongs to the b.s. of the market iff

		Interpretation
(1)	$y_i = \dfrac{\alpha_i}{\alpha_1+\alpha_2+\alpha_3}(x_1+x_2+x_3) + y_i(0)$	Pareto-optimality
(2)	$y_1(0) + y_2(0) + y_3(0) = 0$	Admissibility
(3)	$y_i(0) \leqslant P_i^i - P_i^{123}$	Individual rationality
(4)	$y_1(0) \geqslant P_2^{123} + P_3^{123} - P_2^{23} - P_3^{23}$	{23} has no objection against {1}
	or	
(5)	$y_1(0) < P_2^{123} + P_3^{123} - P_2^{23} - P_3^{23}$	{23} has an objection against {1} ...
	and	
(6)	$2y_1(0) + y_2(0) \geqslant P_1^{12} + P_2^{12} + P_3^{123} - P_1^{123} - P_2^{23} - P_3^{23}$	.. but {1} can counter-object with {2}
	or	
(7)	$-y_1(0) + y_2(0) \leqslant P_2^{23} + P_3^{23} + P_1^{123} - P_2^{123} - P_1^{13} - P_3^{13}$	.. but {1} can counter-object with {3}
	and	
(8)	$y_2(0) \geqslant P_1^{123} + P_3^{123} - P_1^{13} - P_3^{13}$	{13} has no objection against {3}
	or	
(9)	$y_2(0) < P_1^{123} + P_3^{123} - P_1^{13} - P_3^{13}$	{13} has an objection against {2} ...
	and	
(10)	$y_1(0) + 2y_2(0) \geqslant P_1^{12} + P_2^{12} + P_3^{123} - P_2^{123} - P_1^{13} - P_3^{13}$	.. but {2} can counter-object with {1}
	or	
(11)	$y_1(0) - y_2(0) \leqslant P_1^{13} + P_3^{13} + P_2^{123} - P_1^{123} - P_2^{23} - P_3^{23}$	.. but {2} can counter-object with {3}
	and	

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(12)	$y_1(0) + y_2(0) \leqslant P_1^{12} + P_2^{12} - P_1^{123} - P_2^{123}$	$\{12\}$ has no objection against $\{3\}$
	or	
(13)	$y_1(0) + y_2(0) > P_1^{12} + P_2^{12} - P_1^{123} - P_2^{123}$	$\{12\}$ has an objection against $\{3\}$ . . .
	and	
(14)	$2y_1(0) + y_2(0) \leqslant P_1^{12} + P_2^{12} + P_3^{123} - P_1^{123} - P_2^{23} - P_3^{23}$	. . but $\{3\}$ can counter-object with $\{2\}$
	or	
(15)	$y_1(0) + 2y_2(0) \leqslant P_1^{12} + P_2^{12} + P_3^{123} - P_2^{123} - P_1^{13} - P_3^{13}$	. . but $\{3\}$ can counter-object with $\{1\}$

Sketch of proof

The b.s. is the set of payoffs against which there exists no strongly justified objection. By theorem 2, objections can only arise from $\{12\}$, $\{13\}$ or $\{23\}$. Let us for instance establish the necessary and sufficient conditions such that there exists no strongly justified objection from $\{23\}$. Two possibilities arise

- either there are no objections from $\{23\}$ at all (condition 4)
- or there are objections (condition 5), but they can be countered by 1 . By theorem 4, 1 has to break $\{23\}$ apart if he wants to counter-object. He can do so either with 2 or with 3.

By theorem 3, an objection amounts to find a number a that belongs to the interval

$$A =]-y_3(0) + P_3^{23} - P_3^{123};\ y_2(0) - P_2^{23} + P_2^{123}[$$

By theorem 5, a counter-objection of 1, using $\{12\}$, amounts to find a number b in the interval

$$B = [-a + P_2^{12} - P_2^{23};\ y_1(0) - P_1^{12} + P_1^{123}].$$

So, any objection of $\{23\}$ has a type 2 counter-objection from $\{12\}$ iff B is non void for each a $\in A$, in other words iff

$$P_2^{12} - P_2^{23} \leqq a + y_1(0) - P_1^{12} + P_1^{123} \qquad \forall a \in A$$

Using the following

Lemma

Let C, D, E, F be 4 real numbers. $E \leqq z + F \quad \forall z \in]C, D[$ iff $F \leqq C + F$ this condition amounts to

$$P_2^{12} - P_2^{23} \leqq -y_3(0) + P_3^{23} - P_3^{123} + y_1(0) - P_1^{12} + P_1^{123}$$

or $y_3(0) - y_1(0) \leqq P_2^{23} + P_3^{23} + P_1^{123} - P_3^{123} - P_1^{12} - P_2^{23}$

or $2y_1(0) + y_2(0) \geqq P_1^{12} + P_2^{12} + P_3^{123} - P_1^{123} - P_2^{23} - P_3^{23}$

i.e. condition 6.

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The b.s. of this market can thus be represented as the set of solutions of a conjunctive system of linear inequalities involving the side-payments as unknowns. As it seems a priori difficult to analyse the different sets of conditions, we shall represent them by an "electrical circuit". Consider a source of power D, and an exit F. We then construct the following graph, where each condition is represented by a switch that contains its number. The switch is open iff the condition is satisfied, and a p.c. belongs to the b.s. iff current can flow from D to F.

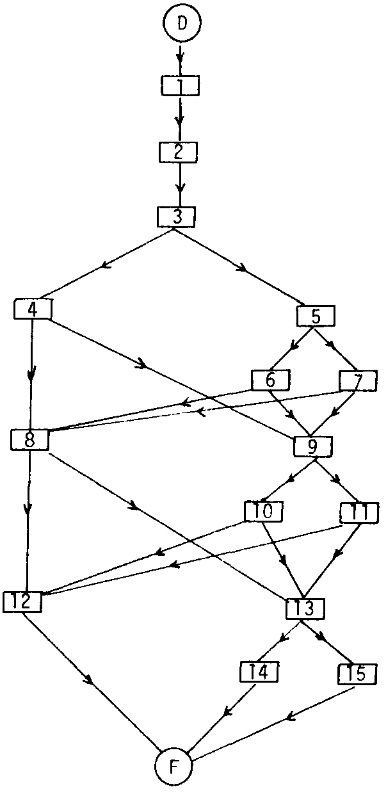

Fig 1

One may count 27 ways to go from D to F. Fortunately, a logical analysis of those 27 sets of conditions shows that most are not compatible. For instance, one may prove that

Lemma

If a pair of companies has no objection against the third one, it can never react by a counter-objection.

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So, for example, conditions (4), (9) and (11) are not compatible (this can easily be seen by adding (9) and (11)).

After deletion of all the unnecessary paths, only 4 remain, so that we have

Theorem 7

The b.s. of a 3-company risk exchange market with exponential utilities is the union of the 4 following sets of p.c.

(*i*) the core of the market

(*ii*) $\{\bar{y} \mid y_1(0) < P_2^{123} + P_3^{123} - P_2^{23} - P_3^{23}$ and $2y_1(0) + y_2(0) = P_1^{12} + P_2^{12} + P_3^{123} - P_1^{123} - P_2^{23} - P_3^{23}\}$

(*iii*) $\{\bar{y} \mid y_1(0) < P_2^{123} + P_3^{123} - P_2^{23} - P_3^{23}$ and $y_1(0) - y_2(0) = P_1^{13} + P_2^{123} + P_3^{13} - P_1^{123} - P_2^{23} - P_3^{23}\}$

(*iv*) $\{\bar{y} \mid y_2(0) < P_1^{123} + P_3^{123} - P_1^{13} - P_3^{13}$ and $y_1(0) + 2y_2(0) = P_1^{12} + P_2^{12} + P_3^{123} - P_1^{13} - P_2^{123} - P_3^{13}\}$

providing such p.c. are individually rational.

Interpretation

Let us for instance interpret (*ii*), which consists of all p.c. that satisfy conditions (5), (6) and (14). This looks asymmetric, but one may show that (5), (6) and (14) together imply condition (13). So (*ii*) is the set of all p.c. such that

{23} has an objection against {1}, but {1} can counter-object with {2}
and
{12} has an objection against {3}, but {3} can counter-object with {2}.

Coalitions {12} and {23} play completely antagonistic roles in the "objection-counter-objection" mechanism: they neutralize each other.

It remains to show by an example that the b.s. may be strictly larger than the core. Fig. 2 presents the b.s. of the market described in Baton and Lemaire (1981); it consists of the core (hachured area) and the (thick) straight line $2y_1(0) + y_2(0) = 1.84$, that represents treaties that are stable due to the mutual neutralization of the two antagonistic coalitions {12} and {23}. The b.s. is less generous than the core towards player 2; it proves that 1 and 3 are not completely helpless, since they can counter-object to all objections against them.

4. What is wrong with the core?

It is customary to argue that the b.s. has an advantage over the core, because the core is empty in many cases, whereas the b.s. never is. But even when one knows for sure that the core is not empty (like in our reinsurance market), the b.s. is presently considered to better reflect the real behaviour of the economic agents. The first inclination is to claim that the core is a superior solution concept because it a priori contains "safer" outcomes than the b.s.;

indeed, a p.c. may seem safer when there are no objections to it that when there are objections that can be countered. But this argument is not quite convincing: participants in a game may be willing to adopt outcomes which are less safe, providing they yield higher payoffs.

The glamour of the core was attacked for the first time in 1973, when AUMANN (1973) presented an example of an exchange economy with a continuum of

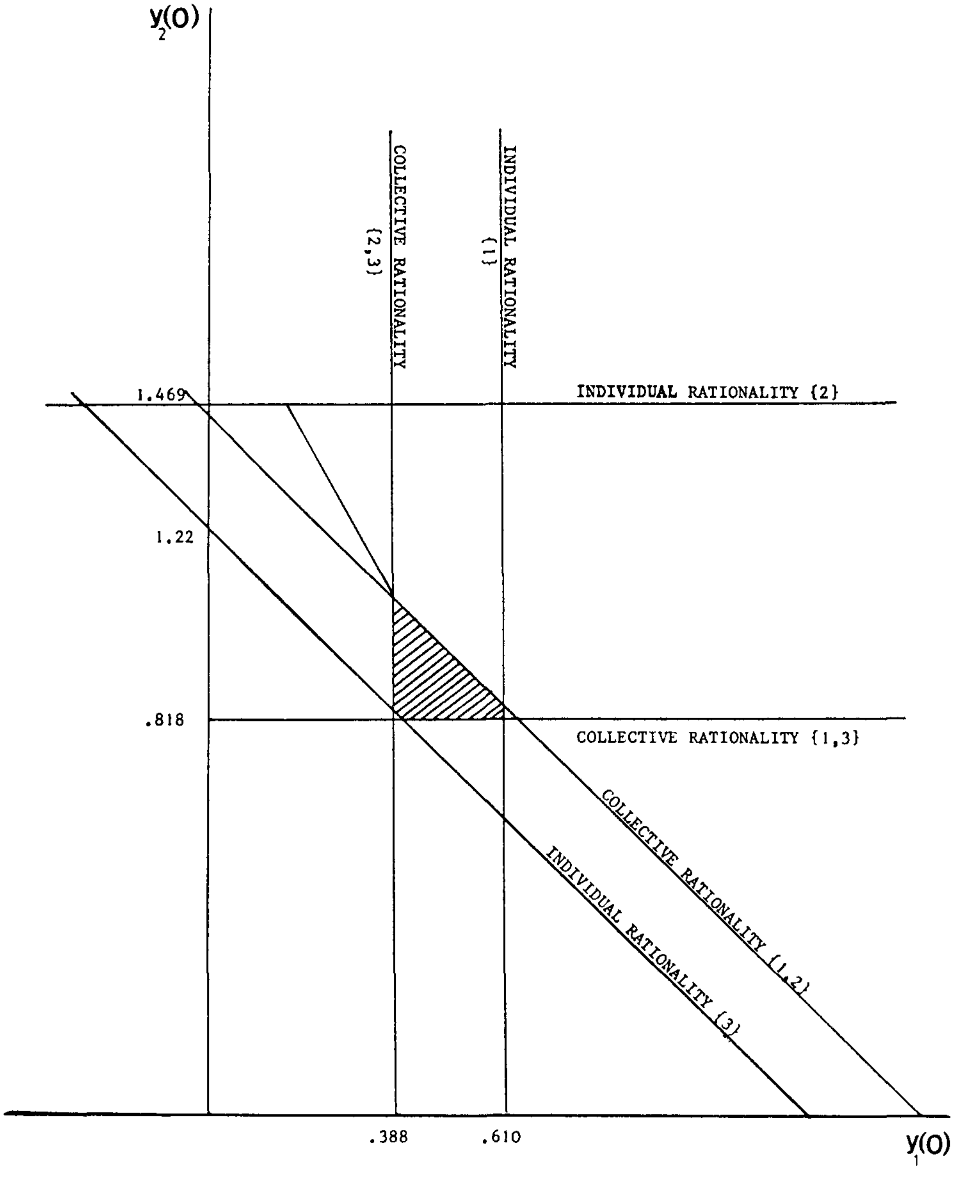

Fig. 2

traders, where it is to the disadvantage of some players to syndicate themselves, even if coalition puts them in the role of a monopoly. The reason for this very curious result lies in the fact that after syndication the core of the original game widens to include outcomes that are worse for the members of the monopolistic syndicate. In some of the examples only outcomes that are worse for the coalition are added. Since this phenomenon is unintuitive, contradicts economic experience and theory, Aumann concluded that the core is not the proper solution concept for studying syndication.

Let us discuss a simpler example of a 5-trader economy (MASCHLER (1975)): each of two manufacturers (players 1 and 2) owns two machines that can be operated by skilled workers. There are exactly 3 available workers (players 3, 4 and 5), each of which willing to work at most 8 hours a day. When a worker operates a machine during 8 hours he produces an item that can be sold at a net profit of 3.000 Francs (net profits being computed here before paying the wages to the workers). How should those profits be distributed among manufacturers and workers? In other words, what is a "fair" salary for 8 hours of work?

Core theory

The core of the game consists of the unique point (0, 0, 3000, 3000, 3000): the workers receive the full profit, the owners of the machines get nothing.

Note that the same paradoxical result is obtained when there are 500 owners (= 1000 machines) and 999 workers; the game is practically symmetric, and all the profits go to the workforce.

According to core theory, this inintuitive solution results from the oversupply of machines: intense competition will develop between players 1 and 2 in the determination of the terms of exchange with the workers; this competition drives the payoff to 1 and 2 down to zero; any attempt by either player to get more will lead to the other one forming a coalition with 2 of the 3 workers and "underselling" him.

This reasoning can certainly be criticized. Whereas it is true that there is a threat to form a coalition {234} it is not at all clear that this threat will drive the payment to 1 down, and eventually down to zero. Clearly 1 is not helpless: he knows that without him the rest of the traders will share together only 6,000 Francs. In order to reach the Pareto-optimal total income of 9,000 Francs they need his cooperation. Is it reasonable to expect that with all of those arguments and this bargaining power he will feel compelled to cut his payoff down to zero?

Let us examine this process of "underselling" more closely. Start, for example, with a payoff of (1500, 1500, 2000, 2000, 2000) which may result if the manufacturers decide to cooperate, put a price of 1000 on the rental of each machine, and divide the profits equally. Core theory tells us that this outcome (and this price) is not stable. Indeed, if player 1 decides to cut his price to, say, 875, he may attract workers 3 and 4, who will rent his two machines, with a resulting outcome of (1750, 2125, 2125) to {134}. But even if this threat is carried out successfully, agents 2 and 5 still can share 3000 francs. Will this put the price of the rental of the machines of 2 down to zero? According to core theory, yes: 2 will have to cut his price below 875; this will in turn lower 1's price, . . . until both prices reach zero. But is the following bargaining behaviour totally unrealistic? If I were owner 2, I would on the contrary tell worker 5 that I *raise* my price to 1500, on the ground that an outcome of (1500, 1500) looks fine to me. Once {134} have seceded, the game is completely symmetric between 5 and me, and everybody agrees that a symmetric game should have a symmetric solution. After all, it is not my fault that trader 5 turned out to be excluded from the first coalition. And 5 has not possible answer if he believes that I shall carry out my threat. If one concedes that this "bargaining-story" is possible, one has to admit that there is also a possibility of an intense competition among the members of {345} which drives *their* payments down. We therefore have to conclude that, although a threat to form {134} is possible, with 1 cutting his price, it does not determine who should lose, owner 2 or worker 5 if the threat is carried out. If one admits that the negotiation behaviour of owner 2 described above may exist and succeed, one must concede that the core is a completely useless tool to analyse this game.

A lesson that could be drawn from those considerations is that it is not sufficient to consider threat capabilities; one also has to study how the traders can react when faced with such threats. But such considerations from the spirit of the b.s.

Bargaining set theory

In this case the b.s. is quite large, since it consists of all the points of the straigth line segment

$$\{(\alpha, \alpha, \beta, \beta, \beta) \mid 0 \leqslant \alpha \leqslant 4500;\ 2\alpha + 3\beta = 9000\}$$

So every (symmetric) outcome that arises by assigning a price $0 \leqslant P \leqslant 3000$ to each machine belongs to the b.s.

Monopolistic syndication

Suppose all workers decide to form a trade-union and act as a single player. The core of this 3-person game is of course unchanged, but the bargaining set limits the value of α to 3000. So it is more advantageous to the workers: a syndication of players is never disadvantageous to them, an important property that the core does *not* possess.

REFERENCES

AUMANN, R., (1973). Disadvantageous monopolies. *Journal of Economic Theory* **6**, 1-11.

AUMANN, R., and MASCHLER, M., (1964). The bargaining set for cooperative games. Advances in game theory. *Annals of Math. Studies,* **52**, 443-476. Princeton Univ. Press.

BATON, B. and LEMAIRE, J., (1981). The core of a reinsurance market. *Astin Bulletin,* **12**, 57-71.

MASCHLER, M., (1975). *An advantage of the bargaining set over the core.* Research Memorandum n° 5. Center for research in mathematical economics and game theory. Hebrew University of Jerusalem.

PELEG, B., (1969). *The extended bargaining set for cooperative games without side payments.* Research Memorandum n° 44. Center for Research in mathematical economics and game theory. Hebrew University of Jerusalem.