A Case Study in the Application of Mathematics to Physics: Descartes' PRINCIPLES OF PHILOSOPHY, Part II[1]

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The question of how and why the application of mathematics to physical reality is possible has occupied philosophers for many centuries. In contemporary discussions, Philip Kitcher's attack on a priorist approaches to the question is particularly interesting, for it suggests that there is no global answer (Kitcher 1983, Chapters 1-4). In this essay, I would like to develop his insight by arguing, first, that the problem of how mathematics relates to physical reality should be addressed by an appeal to the history of mathematics and the sciences in the form of case studies which analyze the peculiar and changing ways in which these domains interact. Generalizations about such interaction should be made carefully, in a kind of local and piecemeal retrospective; I believe that the historical record will show that the grounds of justification and the form of applied mathematics changes dramatically from one historical era to the next. Second, I offer a critical account of Descartes' application of geometry to physics in Book II of the Principles of Philosophy (Descartes 1644), to illustrate one version of the seventeenth century problematic, and the difficulty of relating the unities of geometry to physically individuated objects.

1. Res Extensa and Euclidean Space

What account of the unity of physical things does Descartes offer in the Principles of Philosophy? Though there are certain passages in the Principles (Part II, §25-§32; Descartes 1644, pp. 51-55) which seem to offer a straightforward answer to this question, it is not really easy to answer, and in fact has generated controversy among commentators, when it has not been avoided altogether. André Gombray (1983) concludes a comment on an essay by F.C.T. Moore (1983) on the mind-body problem in Descartes with this observation: "It forces us to confront a question which has been strangely neglected by commentators, and yet whose answer must be crucial to understanding Descartes on mind and body, since after all he persisted in holding them to be in some sense one: what is it to be one, for Descartes?" (p. 277). S.V. Keeling asserts that the only

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unity attributable to res extensa is the world taken as a whole, tri-dimensionally extended (Keeling 1968). And Thomas Lennon disagrees with him, claiming that extension is determined as individual extended things which support qualities (Lennon 1974).

This question becomes especially difficult when we consider that in one sense for Descartes, res cogitans, thinking substance, is the paradigm of unity. In his arguments for the distinctness and immortality of the soul, Descartes takes over the initially Platonic argument that thought, being unextended and therefore simple (having no parts), must then be indestructible, an identity and a unity (Mijuskovic 1974). In light of this argument and the metaphysical assumptions it expresses, res extensa, partes extra partes, seems like an unlikely candidate for having any kind of unity at all. If the surest way of exhibiting unity is to have no parts, what kind of unity can be granted to something characterized as partes extra partes?

In a letter to Henry More, Descartes writes, "I hold that matter left to itself and receiving no external impulse would be perfectly quiescent." (Descartes 1649, p. 258). This passage indicates that for Descartes there is at least an analytic moment in the concept of the substance res extensa where we can consider it independent of God's injection of motion into it. And the cosmogony in Part III of the Principles yields a genetic version, as if there were a stage in the career of matter temporally prior to God's injection of motion: "Let us therefore suppose, if you please, that God, in the beginning, divided all the matter of which he formed the visible world into parts as equal as possible and of medium size, that is to say that their size was the average of all the various sizes of the parts which now compose the heavens and the stars. And let us suppose that he endowed them collectively with exactly the amount of motion which is still in the world at present." (Descartes 1644, p. 106). (Although Descartes proffers this cosmogony merely as an hypothesis, he clearly means it to be taken seriously.)

In Descartes' metaphysics, the duality res cogitans/res extensa is characterized by the duality active/passive. In this cosmological context, the true seat of all activity or force is God; matter is taken to be passive and inert, as it is in itself, apart from God's action upon it. Motion is a mode of matter, but the cause and origin of motion is God, for activity must be referred to spirit. Thus we should look closely at the stage of res extensa when it is 'perfectly quiescent', for this stage will reveal important aspects and difficulties involved in the concept of res extensa.

Descartes takes quiescent res extensa to be a plenum of homogeneous stuff, three dimensional, and indefinite in extent (Descartes 1644, pp. 40, 46, 49 and 50). (These features are explicated in sections §4-§23 of Part II of the Principles, which treat res extensa in itself; the explication of motion begins at §23.) Each feature is asserted on analogy with three dimensional Euclidean space, which has no holes, is perfectly symmetric and isotropic, and has no boundaries. As is well known, Descartes

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wanted to make the analogy an equation, though even his assertion of the equation is qualified by an admission of difference.

> §11 That space does not in fact differ from natural substance. Further, if we concentrate on the idea which we have of some body, for example a stone, and remove from that idea everything which we know is not essential to the nature of body; we shall easily understand that the same extension which constitutes the nature of body also constitutes the nature of space, and that these two things differ only in the way that the genus or species differs from that of the individual. (Descartes 1644, p. 44).

Descartes asserts that the difference is only that between genus or species, and its individual instantiation; thus, quiescent res extensa is a material instantiation of three-dimensional Euclidean space which exhibits exactly the same features as its "genus". What kind of unity does it have? What kind of internal organizaton, if any, does it have? Is it a possible object of knowledge?

In order to answer these questions in the case of res extensa, we need to look at the answers to them in the case of (three-dimensional) Euclidean space. Though it does not have any external boundaries, 3-d Euclidean space has an internal articulation which stems from the way in which points bound lines, lines bound surfaces, and surfaces bound volumes. Euclid[2] introduces the objects of geometry, points, lines, surfaces, and volumes and the peculiar integrity of each object, in terms of these bounding relationships (Euclid, Vol. I, pp. 153-155). And it is the study of these objects, these unities, which reveal the features of Euclidean space: its lack of holes, homogeneity, and indefinite extent. (One might also add, in topological terms, its connectedness: Euclidean space is all of a piece, and does not fall apart into two or more distinct segments.) Points, lines, areas and volumes are the parts of Euclidean space and also, as boundaries, determine the parts of space; and this internal articulation of Euclidean space into parts both determines, and is determined by, the features of Euclidean space as a whole.

In one sense, 3-d Euclidean space as a whole does not have parts: one region is not distinguishable from another precisely because Euclidean space is symmetric and isotropic, does not have holes or boundaries, and is not disconnected. Thus, like 17th century spirit, it has unity because it does not have parts.[3] Moreover, Euclidean space as a whole does <u>not</u> have the kind of unity conferred by shape, that of a cube or sphere, for example; it is "indefinite in extent". And if it did have the unity of shape, of course, it would also have parts in the sense of distinguishable regions. The kind of unity which Euclidean space as a whole has requires partlessness and boundlessness, and so is quite unlike the unity of shape, which depends on the way in which some geometric objects bound and serve as components of others.

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In another sense, 3-d Euclidean space does have parts, the internal articulation and organization provided by points and bounded lines, surfaces and volumes. These parts have a unity in a strong sense: on Euclid's account, what makes a triangle a whole cannot be reduced to its components (angles, area or sides) nor explained by reference to more generic matters (like compass and ruler construction). Rather, the integrity of a triangle as a bounded and shaped surface is exhibited through relations of similarity and congruence, and thus a triangle, while a unity, is not an individual but an equivalence class (Euclid, Vol. I, pp. 241-369). This internal articulation into parts (which themselves have a characteristic unity) both conditions and is conditioned by the sort of partless unity, absence of distinction among regions, which Euclidean space as a whole exhibits. Euclidean space is a possible object of knowledge because of the mutual determination of its internal articulation into (integral) parts and its (partless or regionless) unity as a whole.

Now, what about the case of res extensa? Quiescent res extensa does seem quite like 3-d Euclidean space insofar as it also seems to exhibit a (partless or regionless) unity as a whole: isotropic, symmetric, connected, no holes or external boundaries. However, it has no internal articulation, because in quiescent res extensa there is no physical analogue of a boundary, nor, consequently, of a point, bounded line, surface or volume. There are no articulating parts with the integrity of shape. But if the internal articulation into parts of Euclidean space is the condition for the (partless or regionless) unity as a whole which it exhibits, one must wonder on what grounds Descartes asserts the (partless or regionless) unity of res extensa as a whole. Without an internal articulation or organization, res extensa is not a possible object of knowledge and has no structure; it is a surd and Descartes has no warrant to make any claims about its characteristics.

Another difficulty arises here. What Descartes needs for his physics are individuated objects. How can the mere unities of geometry, which are not individuals but equivalence classes, serve as principles of individuation? In fact, as we shall see, he is quite aware that they cannot play this role.

2. The Ambiguous Status of Geometry

My intention is not to accuse Descartes of inconsistency, but to explore an important ambiguity in his doctrine of how res extensa is individuated into unified substances, which in turn hinges on ambiguities in his conception both of res extensa itself and of geometry. Does res extensa, apart from the activity of God, have geometric, and therefore cognitive, structure? Is the kind of unity which physical things have, individuated res extensa, geometric unity (the unity of shape)? Is the source of geometric unity (the unity of shape) God's activity?

Philosophers have always been puzzled about the ontological status of mathematical objects, and in particular, of Euclidean space and its internal articulations. Descartes is no exception,

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but his puzzlement takes on a distinctive character in the context of his other systematic concerns. Descartes is doctrinally a dualist, but he has the instincts of a monist, as his methodological presuppositions reveal: the demand for homogeneity, for ampliative but truth-preserving procedures, and so forth.[4] Moreover, any dualist cannot leave his polar terms wholly sundered if he is going to address the problem of knowledge, or the nature of the human self. There must be some way to assimilate mind and matter, soul and body. I want to argue that for Descartes, geometry is a middle term which links res cogitans and res extensa. It does not clearly belong to either category, and its very ambiguity as to its ontological classification allows Descartes to use it as a bridge.

On the one hand, it seems to belong to the category of spirit. At the beginning of the Meditations (Descartes 1641, pp. 149-185), the self in its first-person perspective is furnished with the ideas of consciousness (the cogito), God, and mathematics, including geometry. These ideas are secured for it independent of whether or not the material world, res extensa, exists. On the other hand, it seems to belong to the category of matter, for, as Descartes claims in the Principles, Part II, §11, space, Euclidean space, does not in fact differ from material substance, or at least "only in the way that the nature of genus or species differs from that of the individual." (Descartes 1644, p. 44). Thus geometry, a purely intelligible structure which is nonetheless instantiated by matter, links mind to matter and supplies the possibility of our knowledge of the external world: this is the lesson of Meditation VI (Descartes 1641, pp. 185-199).

Geometry also performs a function which Descartes probably would not have acknowledged. It materializes spirit in the sense that the cogito with the idea of extension is, as I have argued elsewhere (Grosholz 1986/7), already implicated in an external, spatially articulated world. And it spiritualizes matter, for res extensa which instantiates geometry has already been organized by spirit (God). Thus, the presence of geometry presupposes and expresses a prior assimilation of spirit to matter and matter to spirit.

This ambiguity in the status of geometry generates the question, is geometric structure to be attributed to res extensa per se, or only in virtue of the activity of God? (This is a version of the question: Is the category for geometry mind or matter?) Since, as we have seen, Descartes' treatment of the status of geometry is highly ambiguous, we should not be surprised if the former question proves difficult to answer. And indeed, Descartes has reasons for wanting to both assert and deny that quiescent res extensa has geometrical structure. His ambivalence on this point is one of the most striking features of Part II of the Principles.

3. The Principle of Unity for Physical Objects

In Part II, §23-§64, the discussion of motion and its laws in the Principles (Descartes 1644, pp. 50-77), Descartes disqualifies shape as a principle of unity in physics. Instead, physical unity, whose seat is the activity of God, is defined in kinematic terms. Physics requires specifically physical principles of individuation which geometry cannot provide. For the items of geometry, while unities, are not individuals but equivalence classes, constructed by means which make no appeal to motion.

Descartes makes it quite clear that matter is individuated, and boundaries formed, within the monolith of quiescent res extensa in virtue of motion whose cause is God's activity or force. In II, §23 (Descartes 1644, p. 51), he writes that movement is "the transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous with it and considered as at rest, into the vicinity of some others." Therefore the unity of a physical body is the common motion of its parts, which is distinguished in a uniform way from the motion of contiguous bodies: "By one body, or one part of matter, I here understand everything which is simultaneously transported." He adds, "I also say that it is a transference, not the force or action which transfers, in order to show that this motion is always in the moving body and not in the thing which moves it." In other words, God's force or action is the cause of motion, but motion is a mode of res extensa; Descartes is here guarding against the accusation of conflating God with nature. This transference of motion "is only a mode [of the moving body] and not a substance, just as shape is a mode of the thing shaped, and rest, of the thing at rest."

The implication of this last remark is best seen in light of II §43 and §55 (Descartes 1644, pp. 63 and 70) where Descartes invokes the first law of motion (Descartes 1644, p. 59) in combination with II, §23 to explain why bodies which are one persist as one.[5] In §43 he says that since each thing strives, as far as is in its power, to remain in the same state, it follows that "a body which is joined to another has some force to resist being separated from it, while a body which is separate has some force to remain separate." Thus, he claims,

> §55 That the parts of solid bodies are not joined by any other bonds than their own rest (relative to each other). Furthermore, our reason certainly cannot discover any bond which could join the particles of solid bodies more firmly together than does their own rest. For what could this bond be? It could not be a substance, because there is no reason why these particles, which are substances, should be joined by any substance other than themselves. Nor is it a mode different from rest; for no other mode can be more opposed to the movement which would separate these particles than is their own rest. Yet, besides substances and modes, we know no other kinds of things.

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We can conclude from this passage that geometric shape (which is a mode of physical bodies, II, §23) cannot be what accounts for the persistence of a bond between the parts of one body; the principle of unity is common motion. Note that Descartes calls individuated physical bodies 'substances'; this is the strongest possible term he could use to underscore their unity.

However, with this definition of the unity of a physical body as common motion of parts, Descartes has unwittingly revealed the troublesome ambiguity at the heart of his account. For he assumes that parts of matter are already available for this account of individuation; but parts must themselves be individuated. Either these parts are individuated by geometric principles, by shape, an alternative he wants to deny; or they are kinematically individuated. But then he has presupposed the very individuation he sought to explain.

Indeed, Descartes' definition of motion itself involves an appeal to the parts of matter as already available. Then it appears that, while motion requires available parts (certainly the monolith of inert res extensa cannot be moved), the availability of parts (qua individuated matter) requires motion. All that can save Descartes from this circularity is individuation of res extensa by geometric principles alone. Sometimes in fact Descartes writes as if this kind of individuation were possible, especially in passages prior to II, §23. In such passages, not surprisingly, he assimilates res extensa most closely to mathematical extension.

> Nor in fact does space, or internal place, differ from corporeal substance conceived in it, except in the way in which we are accustomed to conceive of them. For in fact the extension in length, breadth and depth which constitutes the space occupied by a body, is exactly the same as that which constitutes the body. The difference consists in the fact that, in the body, we consider its extension as if it were an individual thing, and think that it is always changed whenever the body changes. However, we attribute a generic unity to the extension of the space. (Descartes 1644, pp. 43-44).

Thus, Descartes wants to both deny and assert an internal articulation of inert res extensa prior to the divine injection of motion which organizes, activates, and individuates it. The reasons for his dilemma are not far to seek. On the one hand, his radical dualism demands that res extensa be shorn of all cognitive structure: unity and intelligibility must be referred to spirit. Thus the activity of God should account for all the ways in which matter is accessible to the human mind. But in that case, res extensa becomes such a surd that it is unclear how it could figure in a philosophical system, or lend itself to God's organizing activity. So on the other hand, Descartes is tempted to say that in itself, res extensa does have a low-grade, purely geometrical articulation which God can then actualize through motion.

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The foregoing passages from the Principles, Part II, are only one stage of Descartes' systematic attempt to align geometry and physics. But the difficulties which arise in them point to certain of his presuppositions as deserving further scrutiny: that mathematics serves as a middle term between res extensa and res cogitans; that res extensa is radically opposed to res cogitans, the locus of unity and intelligibility; that quiescent res extensa is a homogeneous, unarticulated continuum.

And it illustrates the general lesson that understanding the application of mathematics to physical reality is never easy because at some level of the analysis we must suppose that physical reality "already" exhibits mathematical structure. Descartes the philosophical physicist gets caught in this circularity. One way for the philosopher of science to escape the danger of circularity is to cast the problem in terms of the history of science. In any given period (for example, the seventeenth century), physics will already have been mathematized to a certain extent, and indeed mathematics itself will have been adjusted to suit the demands which physics imposes upon it. Then the problem of applied mathematics is just to see how this mutual assimilation of the two domains is taken one step further. Such an analysis would reveal interesting advances and sidesteps in Descartes' own program; but that is the subject of another essay.

Notes

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[2]I am indebted in my discussion of Euclid to Smigelskis (1985).

[3]This may be part of the reason why certain seventeenth century thinkers like Newton assimilated space to the mind of God.

[4]I argue this point at length in Grosholz (1986/7).

[5]The first law of nature: that each thing, as far as is in its power, always remains in the same state; and that consequently, when it is once moved, it always continues to move. This law, in combination with the second law (Descartes 1644, p. 60) that all motion is, of itself, along straight lines, yields the law of inertia.

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