

Consequences of Rapid Rotation on Mode Identification

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The present work deals with the effect of rotation on the adiabatic oscillation frequencies of stellar models. Here, rotation, as described in a perturbation theory, is said to be rapid in the sense that second order perturbation effects have to be taken into account. Rotation has two second order effects on the oscillation frequencies: departure from equidistance within a given multiplet split by rotation and an overall shift of the multiplet itself. The first effect has been investigated for realistic models of δ Scuti stars in uniform rotation (Dziembowski & Goode, 1992), whereas H. Saio's study includes both effects for a polytrope of index 3 in uniform rotation (Saio 1981). Both effects are investigated for realistic models of δ Scuti stars. Results for the overall shift for a $2 M_{\odot}$ model in rapid uniform rotation (v up to ~ 200 km/s) are outlined below. The rotation rate corresponding to this surface velocity is $\Omega = 1.44 \cdot 10^{-4}$ rad/s.

The relevant eigensystem of linear adiabatic equations is expanded with respect to the parameter Ω/ω_0 with ω_0 the eigenfrequency. In the notation used by H. Saio, the second order correction $\delta\omega_{n,l,m}$ to the eigenfrequency $\omega_{n,l,m}$ can be written as:

$$\delta\omega_{n,l,m} = \underbrace{\omega_{0n,l} \left(\frac{\Omega}{\omega_{0n,l}} \right)^2 (X_1 + X_2 + Z)}_{\text{overall shift}} + \omega_{0n,l} \left(\frac{\Omega}{\omega_{0n,l}} \right)^2 m^2 (Y_1 + Y_2)$$

where n, l, m are the radial order, the degree, and the azimuthal order of the eigenmode respectively. All perturbative frequencies depend on the eigenmode (n, l) and on the associated eigenfrequency $\omega_{0n,l}$ of the non rotating star. The second order correction which is induced by the centrifugal force is :

$$\delta\omega_{\text{centrifug},n,l,m} = \omega_{0n,l} \left(\frac{\Omega}{\omega_{0n,l}} \right)^2 [(X_2 + Z) + m^2 Y_2]$$

The Z contribution comes from structure perturbations induced by the spherical part of the centrifugal force while X_2 and Y_2 are due to those induced by the non spherical part of the same force. Z and X_2 appear in the correction which is independent of m and therefore leads to an overall shift of each given n, l multiplet split by rotation (first order effect).

A Chandrasekhar-Milne expansion (Tassoul, 1978) can be used to obtain an analytical expression for the Z contribution in terms of the structural quantities of the non rotating star. The results thereby obtained for a polytrope of

index 3 are in good agreement with those obtained by H. Saio (1981). However this method applied to realistic models of stars does not take into account the effect of rotation on the evolution of the star, and leads to values of Z which significantly differ from numerical Z values directly computed from a 'pseudo' rotating model, i.e., a model which is evolved with the spherically symmetric part of the centrifugal force included from the start. Therefore, in what follows the overall shift of the oscillation frequencies is computed with numerical values of Z . As an illustration, resulting $m = 0$ eigenfrequencies in a given frequency range is displayed in Figure 1 for a $2 M_{\odot}$ model having burned 32% of its central hydrogen.

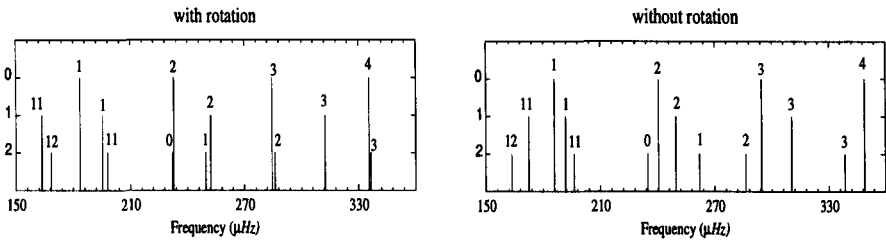


Figure 1. Effect of rotation on the oscillation frequencies. Numbers refer to radial order n , different arbitrary amplitudes are assigned to modes of different degree, which is specified in ordinate.

The overall shift clearly leads to a frequency spectrum for a rotating star which greatly differs from the one obtained when rotation is disregarded: for instance, the order of appearance of the frequencies $(n, 2)$ and $(n, 0)$ is reversed. Also pairs of very close frequencies appear much more often for the rotating model. Quantitatively here, the smallest separation between two frequencies is $0.6 \mu\text{Hz}$, most of the time it is larger than $1 \mu\text{Hz}$. A three weeks campaign of observation gives a resolution in frequencies which is about $0.55 \mu\text{Hz}$ so that these patterns are resolved. Close pairs are indeed often seen in observed spectra (E. Michel, private communication). The present investigation leads therefore to a new understanding of observational frequency spectra as it provides a possible explanation for the closest observed pairs otherwise difficult to explain with nonrotating models.

As a conclusion, not taking into account rotation effects for most rapid δ Scuti stars could lead to a misleading mode identification. Detailed discussion and results will be found in a paper that is to be submitted. Note further that the above frequency range includes not only eigenfrequencies of pure p-modes but also those of mixed modes, i.e., modes which have amplitude both near the convective core and at the surface of the star. As the shift should be different for these modes if the core is rotating faster than the surface, our next step will be to investigate depth dependent rotation effect.

References

- Dziembowski, W., & Goode, P. 1992, ApJ, 394, 670
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 Tassoul, J.-L. 1978, Theory of rotating stars (Princeton Univ. Press, Princeton)