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Letter to the Editor

Magnetization of Rydberg plasmas by electromagnetic waves

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Abstract. It is shown that the ponderomotive force of a large-amplitude electromagnetic wave in Rydberg plasmas can generate quasi-stationary magnetic fields. The present result can account for the origin of seed magnetic fields in the ultracold Rydberg plasmas when they are irradiated by the high-frequency electromagnetic wave.

1. Introduction

The ultracold plasmas (UCPs) are created in laboratory by direct photo-ionization of laser-cooled magneto-optically ultracold gas [1] or by spontaneous evolution from neutral to ionized gas when the atoms are in highly excited Rydberg quantum states [2]. The UCPs have an electron temperature as low as 100 mK, an ion temperature as low as 10 μK and the electron number density as high as 2×10^9 cm^{-3}. Collective plasma behaviour occurs because of the trapping of electrons by the positive ion cloud when the UCP Debye screening length becomes smaller than the sample size. In the UCPs both electrons and ions are strongly coupled.

Recently, there has been a great deal of interest [3–11] in investigating various aspects of the UCPs, including a stable electron plasma wave [3–5], an instability [8]

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arising from the coupling between the electron drift wave and a harmonic of the electron gyrofrequency, expansion in a magnetic field [9] and a new electromagnetic mode [11].

In this 'brief communication', we demonstrate the generation of dc magnetic fields in the UCPs in the presence of a large-amplitude electromagnetic waves. The magnetization of the UCPs occurs because of a new form of the refractive index of Rydberg plasmas that has been recently provided in [11]. DC magnetic fields in the classical plasma may arise because of colliding electron clouds [12] and because of the ponderomotive forces [13–17] of the high-frequency electromagnetic wave.

2. Basic equations

We consider the propagation of the high-frequency electromagnetic wave, with the electric field $\mathbf{E}(\mathbf{r},t) = (1/2)\hat{\mathbf{x}}E_0(x,t)\exp(-i\omega t + ikz)$ + c.c., in the UCPs, where $\hat{\mathbf{x}}$ is the unit vector along the x-axis in a Cartesian coordinate system. The ions are immobile. Here, $\hat{\mathbf{x}}E_0(x,t)$ is the envelope of the electromagnetic field at the position x and time t, and 'c.c.' stands for complex conjugate.

The evolution of the wave field in this medium can be described by the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0\frac{\partial^2\mathbf{P}}{\partial t^2} + \mu_0\frac{\partial \mathbf{J}}{\partial t}, \tag{1}$$

where $\mathbf{J} = -en_0\mathbf{v}_e$ is the electron plasma current and $\mathbf{P}$ is the polarization vector associated with the neutral atoms. Here, e is the electron charge; n_0 is the mean electron density; and $\mathbf{v}_e$ the electron velocity. We know that for transverse waves, the amplitude of density perturbations is equal to zero. In this composite medium, it can easily be shown that the frequency ω and the wave vector $\mathbf{k} = k\hat{\mathbf{z}}$ are related by [11]

$$\frac{k^2c^2}{\omega^2} = N = 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 + \beta\frac{\omega^2}{(\omega_a - \omega)}\right], \tag{2}$$

where c is the speed of light in vacuum; N is the index of refraction; $\omega_{pe} = (4\pi n_0e^2/m_e)^{1/2}$ is the electron plasma frequency; n_0 is the electron number density; e is the magnitude of the electron charge; m_e is the electron mass, $\beta = f_aN_aD/n_0$ has the dimension of time; f_a is the oscillator strength [11]; N_a is the density of the neutral atoms; D is the population coefficient; and ω_a is the radiation transition frequency. Equation (1) has been simply obtained from the Maxwell equations including the electron current and the contribution to the Maxwell equation because of the polarization vector, which equals the product of the atomic susceptibility, the neutral number density and the electromagnetic wave electric field.

The high-frequency electromagnetic wave exerts a ponderomotive force $\mathbf{F}_p = \mathbf{F}_{ps} + \mathbf{F}_{pt}$ on the plasma electrons, where the stationary and non-stationary ponderomotive forces [13] are, respectively,

$$\mathbf{F}_{ps} = \frac{(N-1)}{16\pi}\nabla|E_0|^2 \tag{3}$$

and

$$\mathbf{F}_{pt} = \frac{1}{16\pi}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}\frac{\partial|E_0|^2}{\partial t}. \tag{4}$$

Notice that in a pure plasma ($\beta = 0$), this non-stationary ponderomotive force is absent, $\mathbf{F}_{\mathrm{pt}} = 0$, and the stationary term will be reduced to the familiar expression

$$\mathbf{F}_{\mathrm{ps}} = -\frac{1}{16\pi}\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\nabla|E_0|^2 = -\frac{m_{\mathrm{e}}n_0}{4}\nabla|U_0|^2, \tag{5}$$

where $\vec{U}_0$ is the amplitude of the electron velocity oscillations associated with the wave field.

3. Quasi-static magnetic fields

The low-frequency ponderomotive force pushes the electrons locally and creates the slowly varying space charge electric field

$$\mathbf{E}_{\mathrm{s}} = \frac{1}{n_0 e}\mathbf{F}_{\mathrm{p}} \equiv \frac{(N-1)}{16\pi n_0 e}\nabla|\mathbf{E}_0|^2 + \frac{1}{16\pi n_0 e}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}\frac{\partial|E_0|^2}{\partial t}. \tag{6}$$

The induced slowly varying magnetic field $\mathbf{B}_{\mathrm{s}}$ is then determined from Faraday's law

$$\frac{\partial\mathbf{B}_{\mathrm{s}}}{\partial t} = -c\nabla\times\mathbf{E}_{\mathrm{s}}, \tag{7}$$

which, together with (6), yields

$$\mathbf{B}_{\mathrm{s}} = -\frac{c}{16\pi n_0 e\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}\nabla\times\left(\mathbf{k}|E_0|^2\right). \tag{8}$$

Noting that

$$\frac{\partial[\omega^2(N-1)]}{\partial\omega} = \frac{\omega_{\mathrm{pe}}^2\beta\omega(\omega-2\omega_{\mathrm{a}})}{(\omega-\omega_{\mathrm{a}})^2}, \tag{9}$$

where N has been replaced by (1), we can express the magnitude of the magnetic field as

$$|\mathbf{B}_{\mathrm{s}}| = \frac{eck\beta(2\omega_{\mathrm{a}}-\omega)|E_0|^2}{4m_{\mathrm{e}}L\omega(\omega-\omega_{\mathrm{a}})^2}, \tag{10}$$

where L is scale length of the envelope $|E_0|^2$. Equations (10) reveals that the magnetic field strength is proportional to β, which depends on the atomic processes in Rydberg plasmas. Furthermore, there is a resonant enhancement of the magnetic field when $\omega \sim \omega_{\mathrm{a}}$.

The electron gyrofrequency Ω_{ce} is

$$\Omega_{\mathrm{ce}} = \frac{e|\mathbf{B}_{\mathrm{s}}|}{m_{\mathrm{e}}c} = \frac{k\beta\omega(2\omega_{\mathrm{a}}-\omega)U_0^2}{4L(\omega-\omega_{\mathrm{a}})^2}, \tag{11}$$

where $U_0 = e|E_0|/m_{\mathrm{e}}\omega$ is the electron quiver velocity in the electromagnetic field. The resonant character of this effect will be limited by the existence of a spontaneous lifetime of the transition radiation at the frequency ω_{a}. By assuming a finite value of the spontaneous bandwidth γ, we can transform (10) into

$$|\mathbf{B}_{\mathrm{s}}| = \frac{eck\beta(2\omega_{\mathrm{a}}-\omega)(\omega-\omega_{\mathrm{a}})|E_0|^2}{4m_{\mathrm{e}}L\omega[(\omega-\omega_{\mathrm{a}})^2+\gamma^2]}, \tag{12a}$$

where the field singularity is no longer present. From here we can estimate the maximum expected value for the quasi-static magnetic field. This is illustrated in

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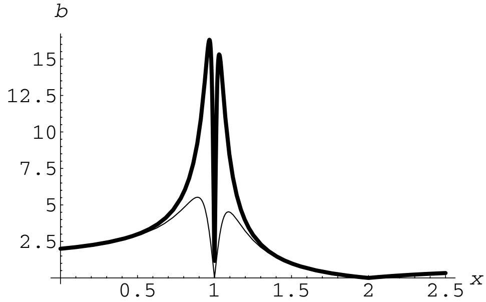

Figure 1. Dimensionless magnetic field b, as a function of the relative wave frequency x, for two different values of the natural bandwidth parameter $g = \gamma^2/\omega_a^2$, for $g = 0.01$ and $g = 0.001$ (bold).

Fig. 1, where the dimensionless quantity

$$b = \frac{4m_e L\omega|\mathbf{B}_s|}{eck\beta|E_0|^2} \tag{12b}$$

is represented as a function of the relative frequency $x = \omega/\omega_a$.

4. Conclusions

To summarize, we have shown that quasi-stationary magnetic fields in Rydberg plasmas can be generated by the ponderomotive force of a high-frequency electromagnetic wave. Specifically, the non-stationary ponderomotive force of the high-frequency electromagnetic wave pushes the electrons locally and creates slowly varying space charge electric field and the dc magnetic field. The latter can govern the UCP expansion and the transport of the electrons. An important question, not considered here, concerns the possible physical scenarios in which such quasi-static magnetic field could eventually be observed. At present, the dimensions of Rydberg plasmas that can be created in magneto-optical traps are still very small. This implies the existence of a quite severe lower wavelength cut-off, given by the size of the plasma medium, below which wave propagation cannot be assumed. However, we expect that in the future, large volumes of the UCPs will eventually be produced.

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