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Polarized radiative transfer equation in some nontrivial coordinate systems

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Abstract. Explicit expressions for the differential operator of stationary quasi-monochromatic polarized radiative transfer equation in Euclidean space with piecewise homogeneous real part of the effective refractive index are obtained in circular cylindrical, prolate spheroidal, elliptic conical, classic toroidal and simple toroidal coordinate system.

Keywords. Polarization, radiative transfer, methods: analytical

1. Introduction

AGB stars, post-AGB objects, protoplanetary nebulae and young planetary nebulae often have gas-dust shells of complex morphology and nontrivial geometry. Conical, toroidal, maybe ellipsoidal and other structures are often observed. Radiation coming from these objects is often highly polarized.

While modeling such objects generally the radiative transfer equation (RTE) should be written and solved in such a coordinate system which reflects the symmetry of the physical problem – at least the symmetry of matter distribution; the symmetry of radiation field is usually somewhat lower. A general method how to write down RTE in different coordinate systems was developed recently by Freimanis 2011a.

Let us consider homogeneous isotropic host medium in Euclidean space with polydisperse scatterers (e.g. dust) of volume concentration n_0 in it. The conditions of validity of stationary quasi-monochromatic RTE (Mishchenko *et al.* 2006), paragraphs 8.11 and 8.15 (Mishchenko 2008a, b) are satisfied, with possible addition of internal primary radiation sources as in Freimanis 2011a. We assume that the birefringence can be neglected, and the real part of the effective refractive index is piecewise homogeneous, so that the radiation propagates along straight lines.

Denoting the Stokes 4-vector in the point of observation $\mathbf{r}$ by $\mathbf{I}(\mathbf{r}, \vartheta, \varphi)$, where the spherical angles (ϑ, φ) describe the direction of propagation with respect to spatial basis vectors, the imaginary part of wavenumber in host medium by k'', statistically averaged particle extinction matrix by $\mathbf{K}(\vartheta, \varphi)$, statistically averaged particle phase matrix by $\mathbf{Z}(\vartheta, \varphi; \vartheta', \varphi')$, and the primary source function by $\mathbf{I}_0(\mathbf{r}, \vartheta, \varphi)$, polarized RTE is as follows (see Mishchenko *et al.* 2006; Mishchenko 2008b; Freimanis 2011a):

$$\begin{aligned}\frac{d\mathbf{I}(\mathbf{r},\vartheta,\varphi)}{ds} &- \mathbf{U}_1\mathbf{I}(\mathbf{r},\vartheta,\varphi)\frac{d\psi}{ds} \\ &= -k''\mathbf{I}(\mathbf{r},\vartheta,\varphi) - n_0\mathbf{K}(\vartheta,\varphi)\mathbf{I}(\mathbf{r},\vartheta,\varphi) \\ &+ n_0\int_{4\pi}\mathbf{Z}(\vartheta,\varphi;\vartheta',\varphi')\mathbf{I}(\mathbf{r},\vartheta',\varphi')\sin\vartheta' d\vartheta' d\varphi' + \mathbf{I}_0(\mathbf{r},\vartheta,\varphi), \qquad (1.1)\end{aligned}$$

where $d\mathbf{I}/ds$ is the derivative of Stokes vector along the path of propagation, $d\psi/ds$ is the speed of rotation of polarization reference basis vectors around the direction of propagation, and $\mathbf{U}_1$ is the transformation matrix of Stokes vector upon infinitesimal rotation of the reference system. It is assumed that the polarization reference plane is that going through the spatially variable polar axis $\vartheta = 0$ and the direction of propagation of radiation; as a result, in curvilinear coordinate system this plane generally rotates.

The left-hand side of Eq. (1.1) is a differential operator with special expression in each particular coordinate system; the right-hand side is one and the same in all coordinate systems. The aim of this study is to find explicit expressions for the differential operator of RTE in some coordinate systems which can be potentially used to study planetary nebulae, protoplanetary nebulae and the related objects.

2. Summary of the results

Standard procedures described in Freimanis (2011a) were applied to the following orthogonal coordinate systems:

(a) Circular cylindrical system (Korn & Korn (1968)) with the polar axis $\vartheta = 0$ directed in either z or radial direction;

(b) Prolate spheroidal system (Korn & Korn (1968));

(c) Elliptic conical system, with alternative parameterization as defined in Freimanis (2011b);

(d) Classic toroidal system (Korn & Korn (1968));

(e) Simple toroidal system (tokamak-shaped, see Gray (1999)).

Clear expressions for the differential operator of polarized RTE in all these cases were found.

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