

# Contents

<i>Preface</i>	<i>page ix</i>
<b>1 Introduction: basics of QCD perturbation theory</b>	<b>1</b>
1.1 The QCD Lagrangian	1
1.2 A review of Feynman rules for QCD	3
1.2.1 QCD Feynman rules	6
1.3 Rules of light cone perturbation theory	7
1.3.1 QCD LCPT rules	10
1.3.2 Light cone wave function	12
1.4 Sample LCPT calculations	14
1.4.1 LCPT “cross-check”	14
1.4.2 A sample light cone wave function	17
1.5 Asymptotic freedom	19
<b>2 Deep inelastic scattering</b>	<b>22</b>
2.1 Kinematics, cross section, and structure functions	22
2.2 Parton model and Bjorken scaling	27
2.2.1 Warm-up: DIS on a single free quark	27
2.2.2 Full calculation: DIS on a proton	29
2.3 Space–time structure of DIS processes	38
2.4 Violation of Bjorken scaling; the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution equation	43
2.4.1 Parton distributions	43
2.4.2 Evolution for quark distribution	45
2.4.3 The DGLAP evolution equations	53
2.4.4 Gluon–gluon splitting function*	56
2.4.5 General solution of the DGLAP equations	60
2.4.6 Double logarithmic approximation	63
Further reading	72
Exercises	72

<b>3</b>	<b>Energy evolution and leading logarithm-<math>1/x</math> approximation in QCD</b>	<b>74</b>
3.1	Paradigm shift	74
3.2	Two-gluon exchange: the Low–Nussinov pomeron	76
3.3	The Balitsky–Fadin–Kuraev–Lipatov evolution equation	82
3.3.1	Effective emission vertex	83
3.3.2	Virtual corrections and reggeized gluons	88
3.3.3	The BFKL equation	92
3.3.4	Solution of the BFKL equation	95
3.3.5	Bootstrap property of the BFKL equation*	103
3.3.6	Problems of BFKL evolution: unitarity and diffusion	107
3.4	The nonlinear Gribov–Levin–Ryskin and Mueller–Qiu evolution equation	112
3.4.1	The physical picture of parton saturation	112
3.4.2	The GLR–MQ equation	115
	Further reading	121
	Exercises	121
<b>4</b>	<b>Dipole approach to high parton density QCD</b>	<b>123</b>
4.1	Dipole picture of DIS	123
4.2	Glauber–Gribov–Mueller multiple-rescatterings formula	129
4.2.1	Scattering on one nucleon	130
4.2.2	Scattering on many nucleons	133
4.2.3	Saturation picture from the GGM formula	139
4.3	Mueller’s dipole model	141
4.3.1	Dipole wave function and generating functional	141
4.3.2	The BFKL equation in transverse coordinate space	153
4.3.3	The general solution of the coordinate-space BFKL equation*	159
4.4	The Balitsky–Kovchegov equation	163
4.5	Solution of the Balitsky–Kovchegov equation	172
4.5.1	Solution outside the saturation region; extended geometric scaling	172
4.5.2	Solution inside the saturation region; geometric scaling	176
4.5.3	Semiclassical solution	178
4.5.4	Traveling wave solution	181
4.5.5	Numerical solutions	184
4.5.6	Map of high energy QCD	188
4.6	The Bartels–Kwiecinski–Praszalowicz equation*	189
4.7	The odderon*	192
	Further reading	195
	Exercises	196
<b>5</b>	<b>Classical gluon fields and the color glass condensate</b>	<b>198</b>
5.1	Strong classical gluon fields: the McLerran–Venugopalan model	198
5.1.1	The key idea of the approach	198
5.1.2	Classical gluon field of a single nucleus	200

5.1.3	Classical gluon distribution	205
5.2	The Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner evolution equation	215
5.2.1	The color glass condensate (CGC)	215
5.2.2	Derivation of JIMWLK evolution	216
5.2.3	Obtaining BK from JIMWLK and the Balitsky hierarchy	224
	Further reading	226
	Exercises	226
<b>6</b>	<b>Corrections to nonlinear evolution equations</b>	<b>228</b>
6.1	Why we need higher-order corrections	228
6.2	Running-coupling corrections to the BFKL, BK, and JIMWLK evolutions	229
6.2.1	An outline of the running-coupling calculation	230
6.2.2	Impact of running coupling on small- $x$ evolution	235
6.2.3	Nonperturbative effects and renormalons*	240
6.3	The next-to-leading order BFKL and BK equations	242
6.3.1	Short summary of NLO calculations	243
6.3.2	Renormalization-group-improved NLO approach*	245
	Further reading	248
	Exercises	249
<b>7</b>	<b>Diffraction at high energy</b>	<b>250</b>
7.1	General concepts	250
7.1.1	Diffraction in optics	250
7.1.2	Elastic scattering and inelastic diffraction	253
7.2	Diffraction dissociation in DIS	255
7.2.1	Low-mass diffraction	256
7.2.2	Nonlinear evolution equation for high-mass diffraction	262
	Further reading	270
	Exercises	271
<b>8</b>	<b>Particle production in high energy QCD</b>	<b>272</b>
8.1	Gluon production at the lowest order	272
8.2	Gluon production in DIS and $pA$ collisions	274
8.2.1	Quasi-classical gluon production	274
8.2.2	Including nonlinear evolution	284
8.3	Gluon production in nucleus–nucleus collisions	290
	Further reading	291
	Exercises	292
<b>9</b>	<b>Instead of conclusions</b>	<b>293</b>
9.1	Comparison with experimental data	293
9.1.1	Deep inelastic scattering	294

9.1.2	Proton(deuteron)–nucleus collisions	295
9.1.3	Proton–proton and heavy ion collisions	297
9.2	Unsolved theoretical problems	303
	Further reading	306
<b>Appendix A: Reference formulas</b>		<b>307</b>
A.1	Dirac matrix element tables	307
A.2	Some useful integrals	307
A.3	Another useful integral	310
<b>Appendix B: Dispersion relations, analyticity, and unitarity of the scattering amplitude</b>		<b>312</b>
B.1	Crossing symmetry and dispersion relations	312
B.2	Unitarity and the Froissart–Martin bound	316
	<i>References</i>	319
	<i>Index</i>	336