$\alpha\Lambda$-dynamos

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Abstract: In contrast to $\alpha\omega$-dynamos, where the angular velocity is arbitrarily prescribed, we consider here $\alpha\Lambda$-dynamos, for which the differential rotation and meridional circulation are solutions of the momentum equation. The non-diffusive parts of the Reynolds stress tensor are parameterized by the Λ-effect. In earlier investigations we have shown that the turbulent magnetic diffusivity has to be much smaller than the eddy viscosity, otherwise the dynamo is not oscillatory or else the contours of constant angular velocity are cylindrical, contrary to observations. In the present paper we investigate the effects of compressibility.

1. The basic equations

We consider here the initial value problem of a conducting fluid in a rotating sphere with radius R. We solve the hydromagnetic equations, starting with a rigid rotation with angular velocity, Ω. A weak magnetic seed-field is present initially. We are interested in the evolution of the velocity and magnetic fields on time scales that are much longer than the sound travel time and can therefore adopt the anelastic approximation. The equations governing the generation of magnetic field and differential rotation are

$$\frac{\partial \boldsymbol{B}}{\partial t} = \operatorname{curl}(\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B} - \eta_t \mu_0 \boldsymbol{J}), \tag{1}$$

$$\rho \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\nabla p + \boldsymbol{J} \times \boldsymbol{B} - \rho \boldsymbol{g} - \operatorname{Div}(\rho \mathcal{Q} - \mathcal{B}), \tag{2}$$

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$$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = -\mathrm{div}\, \boldsymbol{F}, \tag{3}$$

with $\mathrm{div}\, \rho \boldsymbol{u} = \mathrm{div}\, \boldsymbol{B} = 0$. $\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{u} \cdot \nabla$ denotes the advective derivative, $\boldsymbol{J} = \mathrm{curl}\, \boldsymbol{B}/\mu_0$ is the electric current, where μ_0 is the induction constant, η_t is the turbulent magnetic diffusivity, and $\mathcal{Q}$ and $\mathcal{B}$ are respectively the Reynolds and Maxwell stress tensors. Axisymmetry is assumed and, as in Paper I (Brandenburg *et al.*, 1990), we neglect $\mathcal{B}$.

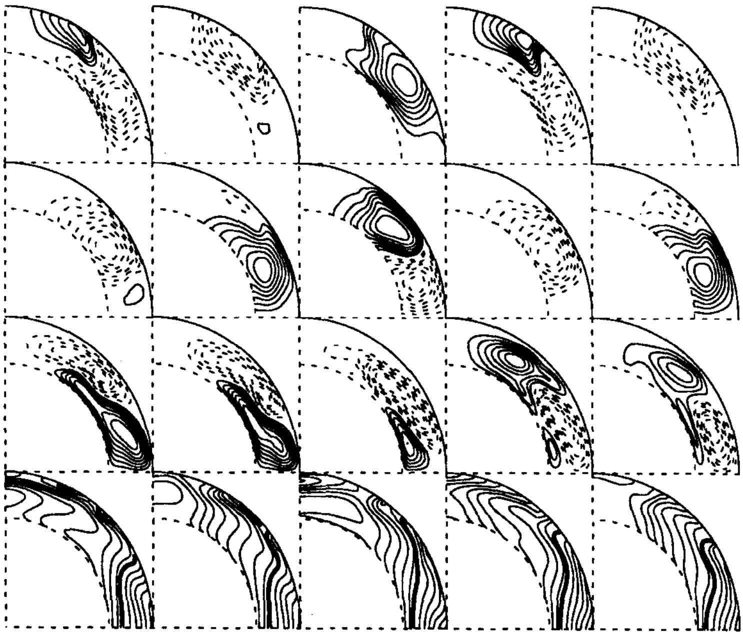

Fig. 1. The effect of compressibility on the magnetic cycle: Snapshots showing the variation of magnetic and velocity field through approximately one magnetic cycle period. In the upper row are the field lines of the poloidal magnetic field, in the second row contours of constant toroidal field strength, in the third row stream lines of the meridional motion, and in the last row contours of constant angular velocity.

2. Results

The Taylor-Proudman theorem shows that for idealized rotating flows the surfaces of constant angular velocity are cylinders parallel to the rotation axis. In many of our preliminary models we found this property to hold approximately at Taylor numbers $\mathrm{Ta} \gtrsim 10^5$, even though the strict conditions of the theorem did not apply. It is possible that stratification may give rise to deviations from cylindrical Ω-contours even for high Taylor numbers.

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We are mainly interested in oscillatory dynamo solutions of $\alpha\omega$-type, because such solutions are of interest for understanding the solar cycle. Previous investigations have shown that for such solutions $|C_\Omega|$ must be sufficiently large ($|C_\Omega| \gtrsim 10^3$). This can be achieved only when $\mathrm{Pr}_M^2\mathrm{Ta}$ is larger than $10^{7\cdots8}$ ($\mathrm{Pr}_M = \nu_t/\eta_t$). We assume here the simplest form for the Λ-effect, taking $V^{(0)} = 1$ and $V^{(1)} = H^{(1)} = 0$ (see Paper I for the notation). The compressible case is characterized by two more dimensionless numbers: ξ, and $\tilde{\xi}$, where ξ is the surface pressure scale height relative to the stellar radius, and $\tilde{\xi}$ is the superadiabatic gradient at the surface. We take $\xi = 0.01$, and $\tilde{\xi} = 2 \times 10^{-6}$. Approximately twelve pressure scale heights are covered in the model. The field geometry for $C_\alpha = 10$ and $\mathrm{Ta} = 10^8$ is displayed in Fig. 1. When the magnetic field is included, the Ω contours differ substantially from the non-magnetic case, in particular at the surface where the density becomes small and $\boldsymbol{J} \times \boldsymbol{B}/\rho$ is large. The meridional circulation is also strongly affected by the presence of the field. We note that our Ω-contours are still basically parallel to the rotation axis through much of the shell. The influence of the magnetic field on the mean surface temperature is shown in Fig. 2.

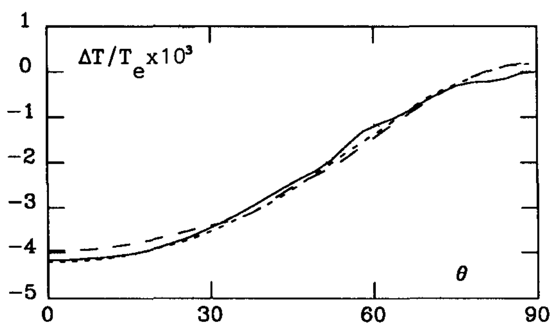

Fig. 2. Temperature difference at the surface relative to the equator temperature for the same model as in Fig. 1 at different instants during the magnetic cycle. The cyclic modulation of the temperature is about 0.03%, whilst the pole-equator difference is about 0.4%.

We have compared this model with an equivalent one where stratification is absent. The corresponding field geometry for such a model is displayed in Fig. 3. Clearly, cyclic variations of the differential rotation and the meridional circulation are much weaker than in the compressible, stratified case. It is surprising that the dynamo period is longer when stratification is absent. This is in contrast to results of Glatzmaier (1985), who found longer cycle periods for his models with stratification than for the incompressible models of Gilman and Miller (1981). Usually, periods become shorter as nonlinear feedbacks grow. In fact, in our compressible case the feedback is very strong, in particular in the upper layers, where the density is low. The disagreement with Glatzmaier's result may be due to the fact that the model discussed here has a highly supercritical dynamo number.

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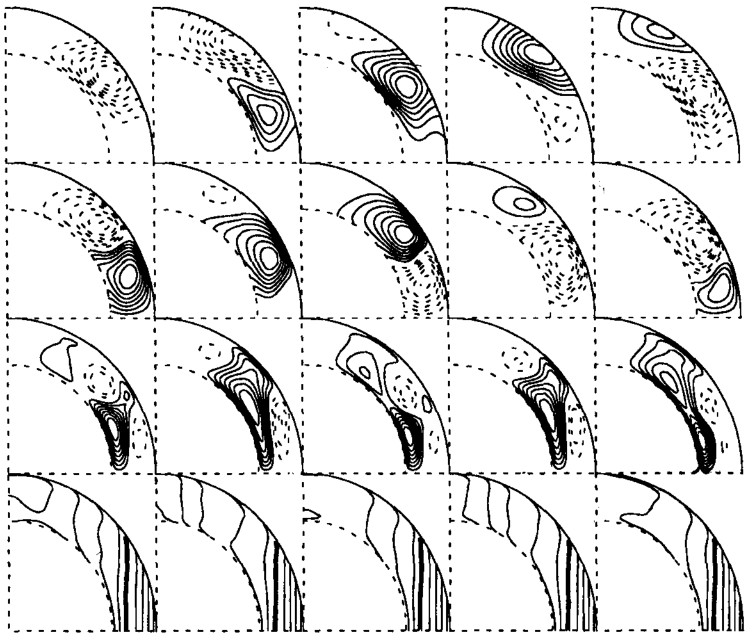

Fig. 3. Same as Fig. 1, but without thermodynamics and without stratification. Note that the modulation of meridional flow and angular velocity is much less than in the compressible case. (The heavy Ω-contour denotes the basic rotation velocity.) Also the cycle period is longer than in the compressible case. For the present choice of parameters ($V^{(0)} > 0$ and $C_\alpha > 0$), dynamo waves migrate polewards.

3. Conclusion

The present investigation demonstrates the problems of dynamo models where the differential rotation is a selfconsistent solution of the dynamo. Other effects might be included to make the model more realistic, e.g. a decrease in eddy viscosity at the base of the convective layer, or including an anisotropy in the eddy viscosity of α-effect.

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