ON A METHOD OF OBTAINING HALF-YEARLY AND QUARTERLY PREMIUMS FROM THE ANNUAL PREMIUM.

To the Editor of the Assurance Magazine.

SIR,—Allow me to draw the attention of your readers to a method of obtaining half-yearly and quarterly premiums from the annual premium. As it is one of much utility to the practical computer, and I do not remember to have met with it in any of the works in use among actuaries, it may not be altogether unworthy of a place in your columns.

The subject has been treated by the late Mr. Orchard in his exceedingly valuable work, *Single and Annual Assurance Premiums*, where at p. vii. of the Introduction he gives the expression for finding c_x, the quantity to be added to half the yearly premium to form the half-yearly premium. That expression is as follows:—

$$\frac{1-d(1+x)}{2}(r_x+\cdot 75-r_{x+1})=\frac{\mathrm{A}_x(r_x+\cdot 75-r_{x+1})}{2}=c_x;$$

which, translated into symbols more generally adopted, is as follows:—

$$\frac{1-d(1+a_x)}{2}\left(\frac{1}{\cdot 75+a_x}-\frac{1}{1+a_x}\right)=\frac{\mathrm{A}_x}{2}\left(\frac{1}{\cdot 75+a_x}-\frac{1}{1+a_x}\right)=c_x.$$

The arithmetical operation is a very simple one, and the value of the increment c_x is easily obtained with the assistance of a table of reciprocals; but when these results are collected into a tabular form, there is an inconvenience in the use of them which is not altogether unimportant—viz., that before entering the table it is necessary to find from the annual premium the corresponding annuity, which annuity is made the argument of the table of c_x.

The method now submitted enables us, by a very obvious substitution in one of the terms of the above quoted expressions, to form the values of c_x in terms of the annual premium, instead of in terms of the single premium, and to use the annual premium directly as the argument to the tabular results. It is as follows:—

$$\text{Since } \frac{1-d(1+a_x)}{1+a_x}=\pi_x=\text{annual premium,}$$

$$\pi_x(1+a_x)=1-d(1+a_x);$$

and substituting the former of these equivalents for the latter, we obtain a more elegant and convenient expression for the required value of c_x.

The annual premium then being $\frac{\pi_x(1+a_x)}{1+a_x}=\pi_x$, and the annual amount of the half-yearly premium by the usually adopted approximation being $\frac{\pi_x(1+a_x)}{\cdot 75+a_x}=\pi_x+c_x$, it is required to find the value of their difference, viz.,

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$$\frac{\pi_x(1+a_x)}{\cdot75+a_x}-\frac{\pi_x(1+a_x)}{1+a_x}=c_x.$$

Dividing by $\pi_x(1+a_x)$, and again multiplying by that quantity, we obtain

$$\pi_x\cdot\left(\frac{1+a_x}{\cdot75+a_x}-1\right)=\pi_x\left(\frac{(1+a_x)-(\cdot75+a_x)}{\cdot75+a_x}\right)=\pi_x\frac{\cdot25}{\cdot75+a_x}=c_x.$$

Then the half-yearly premium $=\frac{\pi_x+c_x}{2}$ will be

$$\frac{\pi_x+\pi_x\left(\frac{\cdot25}{\cdot75+a_x}\right)}{2}=\frac{\pi_x}{2}\left(1+\frac{\cdot25}{\cdot75+a_x}\right).$$

And generally, when the premium is payable m times a year, the addition to the annuity will be $\left(1-\frac{m-1}{2m}\right)=\frac{m+1}{2m}$; and substituting this value in the above equation, it becomes

$$\pi_x\left\{\frac{(1+a_x)-\left(\frac{m+1}{2m}+a_x\right)}{\frac{m+1}{2m}+a_x}\right\}=\pi_x\frac{\frac{m-1}{2m}}{\frac{m+1}{2m}+a_x}=c_x,$$

and the premium for the mth portion of a year will be

$$\frac{\pi_x}{m}\left(1+\frac{\frac{m-1}{2m}}{\frac{m+1}{2m}+a_x}\right)$$

Reference has been made above to the facility with which the half-yearly values (to which Mr. Orchard confined his table) may be calculated by his method. The one here given may, however, lay claim to attention upon the ground that the arithmetical operation is thereby still further simplified. An example worked by both methods will enable the reader to form his own estimate of the comparative merits of each of them.

Example.—To find the half-yearly premium when the annual premium is £4·100.

By Mr. Orchard's method.

To $\pi_x=4{\cdot}100$ the corresponding annuity* is 13·260.

Then 13·260+·75=14·010;	its reciprocal	·071377
13·260+1 =14·260	,, ,,	·070126
		·001251

* *i.e.*, The argument of Mr. Orchard's tables.

$$\begin{array}{rrl} & & \cdot001251 \\ A_x \text{ corresponding to annuity* } 13\cdot260 = 58\cdot466 & & \\ \text{and } \dfrac{58\cdot466}{2} = & \overline{33292} & \text{inverted} \\ & 2502 & \\ & 1126 & \\ & 25 & \\ & 4 & \\ \hline & \cdot03657 & \end{array}$$

$$\frac{\pi_x}{2} = \frac{4\cdot100}{2} = \begin{array}{r} 2\,050 \\ \hline 2\cdot087 \end{array}$$

By the method above adduced.

To $\pi_x = 4\cdot100$, the corresponding annuity* is $13\cdot260$,

$$\text{and } \frac{1}{13\cdot260 + \cdot75} = \cdot071377 \text{ and } \frac{\cdot071377}{4} + 1$$
$$= 1\cdot01784$$

$$\pi_x = \frac{4\cdot100}{2} = \begin{array}{r} 502 \text{ inverted} \\ \hline 2036 \\ 51 \\ \hline 2\cdot087 \end{array}$$

I have calculated a table of half-yearly and quarterly premiums, at 3 per cent., by this method; which I think will be found generally useful, being true for all cases given, irrespective of the rate of mortality, and not first requiring the finding of the corresponding annuity, as is the case with Mr. Orchard's table as already referred to. A copy of the table so calculated is subjoined.

I am, Sir,

Your obedient servant,

SAMUEL L. LAUNDY.

Eagle Life Office,
Dec. 1, 1863.

See Note, *ante*, p. 233.

Table of Half-yearly and Quarterly Premiums, at 3 per Cent., for any Rate of Mortality, derived from the Annual Premiums by means of the expression

$$\frac{\pi_x}{m}\cdot\left(1+\frac{\frac{m-1}{2m}}{\frac{m+1}{2m}+a_x}\right).$$

Yearly.	Half-yearly	Quarterly.	Yearly.	Half-yearly	Quarterly.	Yearly.	Half-yearly.	Quarterly.	Yearly.	Half-yearly.	Quarterly.
1·00	0·505	0·254	3·30	1·676	0·845	5·60	2 861	1·446	7·90	4·060	2·058
·05	·530	·266	·35	·702	·858	·65	·887	·459	·95	·086	·072
·10	·556	·279	·40	·727	·871	·70	·913	·473	8·00	·112	085
·15	·581	·292	·45	·753	·884	·75	·939	·486	·05	·138	·099
·20	·606	·305	·50	·779	·897	·80	·965	·499	·10	·165	·112
·25	.632	·317	·55	·804	·910	·85	·991	·512	·15	·191	·126
·30	·657	·330	·60	·830	·923	·90	3·016	·525	·20	217	·139
·35	·682	·343	·65	·855	·936	·95	·042	·539	·25	·243	·153
·40	·708	·356	·70	·881	·949	6·00	·068	·552	·30	·270	·166
·45	·733	·369	·75	·907	·962	·05	·094	·565	·35	·296	·180
·50	·758	·381	·80	·932	·975	·10	·120	·578	·40	·322	·193
·55	·784	·394	·85	·958	·988	·15	·146	·592	·45	·349	·207
·60	·809	·407	·90	·984	1·001	·20	·172	·605	·50	·375	·220
·65	·834	·420	·95	2·009	·014	·25	·198	·618	·55	·401	·234
·70	·860	·433	4·00	·035	·027	·30	·224	·631	·60	·427	·247
·75	·885	·445	·05	·061	·040	·35	·250	·645	·65	·454	·261
·80	·911	·458	·10	·087	·053	·40	·276	·658	·70	·480	·274
·85	·936	·471	·15	·112	·066	·45	·302	·671	·75	·506	·288
·90	·962	·484	·20	·138	·079	·50	·328	·684	·80	·533	·301
·95	·987	·497	·25	·164	·092	·55	·354	·698	·85	·559	·315
2·00	1·012	·509	·30	·189	·105	·60	·380	·711	·90	·585	·328
·05	·038	·522	·35	·215	·118	·65	·406	·724	·95	·612	·342
·10	·063	·535	·40	·241	·131	·70	·432	·738	9·00	·638	·355
·15	·089	·548	.45	·267	·144	·75	·459	·751	·05	·664	·369
·20	·114	·561	·50	·292	·157	·80	·485	·764	·10	·691	·382
·25	·140	·574	·55	·318	·170	·85	·511	·778	·15	·717	·396
·30	·165	·586	·60	·344	·183	·90	·537	·791	·20	·744	·409
·35	·191	·599	·65	·370	·196	·95	·563	·804	·25	·770	·423
·40	·216	·612	·70	·396	·210	7·00	·589	·818	·30	·796	·437
·45	·242	·625	·75	·421	·223	·05	·615	·831	·35	·823	·450
·50	·267	·638	·80	·447	·236	·10	·641	·844	·40	·849	·464
·55	293	·651	·85	·473	·249	·15	·667	·858	·45	·876	·477
·60	·318	·664	·90	·499	·262	·20	·693	·871	·50	·902	·491
·65	·344	·677	·95	525	·275	·25	·720	·884	·55	·929	·505
·70	·369	·690	5·00	·550	·288	·30	·746	·898	·60	·955	·518
·75	·395	·702	·05	·576	·301	·35	·772	·911	·65	·981	·532
·80	·420	·715	·10	·602	·314	·40	·798	·924	·70	5·008	·545
·85	·446	·728	·15	·628	·328	·45	·824	·938	·75	·034	·559
·90	·471	·741	·20	·654	·341	·50	·850	·951	·80	·061	·573
·95	·497	·744	·25	·680	·354	·55	·876	·965	·85	·087	·586
3·00	·522	·767	·30	·706	·367	·60	·903	·978	·90	·114	·600
·05	·548	·780	·35	·731	·380	·65	·929	·991	·95	·140	·614
·10	·574	·793	·40	·757	·393	·70	·955	2·005	10·00	5·167	·627
·15	·599	·806	·45	·783	·407	·75	·981	·018			
·20	·625	819	·50	809	·420	·80	4·007	·032			
·25	·650	·832	·55	·835	·433	85	·034	·045			

Proportional Parts.

Δ	13	25
·001	·000,3	·000,5
2	,5	1,0
3	,8	1,5
4	1,0	2,0
5	1,3	2,5
6	1,6	3,0
7	1,8	3,5
8	2,1	4,0
9	2,3	4,5

Δ	14	26
·001	·000,3	·000,5
2	,6	1,0
3	,8	1,6
4	1,1	2,1
5	1,4	2,6
6	1,7	3,1
7	2,0	3,6
8	2,2	4,2
9	2,5	4,7

Δ	15	27
·001	000,3	·000,5
2	,6	1,1
3	,9	1,6
4	1,2	2,2
5	1,5	2,7
6	1,8	3,3
7	2,1	3,8
8	2,4	4,3
9	2,7	4,9